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AN ADAPTIVE FINITE ELEMENT METHOD FOR THE WAVE SCATTERING BY A PERIODIC CHIRAL STRUCTURE*

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Abstract

The electromagnetic wave propagation in the chiral medium is governed by Maxwell's equations together with the Drude-Born-Fedorov (constitutive) equations. The problem is simplified to a two-dimensional scattering problem, and is formulated in a bounded domain by introducing two pairs of transparent boundary conditions. An a posteriori error estimate associated with the truncation of the nonlocal boundary operators is established. Based on the a posteriori error control, a finite element adaptive strategy is presented for computing the diffraction problem. The truncation parameter is determined through sharp a posteriori error estimate. Numerical experiments are included to illustrate the robustness and effectiveness of our error estimate and the proposed adaptive algorithm.

Mathematics subject classification: 35Q61, 65N15, 65N30, 78A45.
Key words: Maxwell's equations, A posteriori error analysis, Adaptive algorithm, Scattering.

1. Introduction

Consider a time-harmonic electromagnetic plane wave incident on a periodic chiral structure in $\mathbb{R}^3$. The medium inside the structure is chiral and nonhomogeneous. In particular, two homogeneous regions are separated by the periodic structure. In this paper, we restrict ourselves to the special case, i.e., by assuming that the chiral structure is periodic in x_1 direction and invariant in x_2 direction, the scattering problem may be simplified to a two-dimensional one. The more general diffraction problem by chiral gratings in $\mathbb{R}^3$ will be discussed in a separate work.

Recently, chiral materials have been studied intensively in the electromagnetic theory literature. In general, the electromagnetic fields inside the chiral medium are governed by Maxwell equations and a set of constitutive equations known as the Drude-Born-Fedorov constitutive equations, in which the electric and magnetic fields are coupled. The property of the chiral media is completely characterized by the electric permittivity ε, the magnetic permittivity μ and the chirality admittance β. On the other hand, periodic structures(grating) have received considerable attention in the past several years because of important applications in integrated optics, optical lenses, antireflective structures, lasers, etc. For the model equations, the

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physical background and computational aspects, there exist lots of results in the literature for the electromagnetic scattering problem in periodic and non-periodic chiral structures. we refer to [3, 4, 15, 27, 30, 41] and the references therein. It should be noted that a good introduction and review on the electromagnetic diffractive in chiral medias can be found in Lakhtakia [30] and Lakhtakia, Varadan and Varadan [31]. Recently, thin chiral coatings and the low frequency behavior of the scatting problems in chiral media are studied by Ammari and Nédélec [2] and Ammari et al [5]. The existence and uniqueness of solutions to the scattering problem are established for biperiodic chiral media in Ammari and Bao [1] and for nonperiodic chiral media in Ammari and Nédélec [3]. The related work on the variational formulations and the numerical analysis for the scattering problems in chiral environment can found in Ammari and Bao [1], Zhang and Ma [43], and Zhang et al [42, 44]. Also see [6, 9, 10, 12, 13, 20, 22, 23, 25, 26] for other related mathematical results and practical applications of Maxwell's equation in general media.

A posteriori error estimates, which measure the actual discrete errors without knowledge of the limit solutions, is computable quantities in terms of discrete solution. Ever since the pioneering work of Babuška and Rheinboldt [8], the adaptive finite element methods based on the a posteriori error estimates have become a class of important numerical tools for solving many model equations, especially for those which have physical features of multiscale phenomenon. We refer to [11, 17, 24, 33–35, 40] for numerical analysis and scientific computations. For the convergence and the quasi-optimality of adaptive finite element methods, some typical works can be found in Dörfler [24], Verfürth [40], Monk, Nochetto, and Siebert [35, 36], Binev, Dahmen and DeVore [16], Mekchay and Nochetto [32], Stevenson [39], Cascon, Kreuzer, Nochetto, and Siebert [17] and Stevenson [39]. In particular, Chen and Wu [20] proposed a new numerical approach with combinations of adaptive finite element method and perfectly matched layer(PML) technique for a 1D grating problem. Based on this numerical tool, great progress has been made in convergence analysis as well as algorithm design for a large class of the scattering problems. We can refer to [11, 14, 18, 19, 21, 28, 29, 45] and the references therein. This approach is very attractive in the scattering problems, mainly because PML can be used to deal with the difficulty in truncating the unbounded domain and the adaptive finite element method can very efficiently capture the local singularities.

The purpose of this paper is to extend our previous work on 1D linear grating problem(cf., [46]) to 1D chiral grating problem. In our approach, the first step is to reduce the problem from an infinite domain into a bounded domain by introducing nonlocal boundary operators, the so-called DtN operators. Then the nonlocal boundary operators are approximately truncated by taking sufficiently many terms of the corresponding infinite series expansions. A finite element formulation with the truncation operators is established for solving the diffractive problem. Finally, we obtain an a posteriori error estimate between the exact solution and finite element solution. The a posteriori error estimate consists of two parts, finite element discretization error and the truncation error of boundary operators. It is easy to see that the truncation error is exponentially decaying when the parameter N with being dependent on the truncation is increased. The adaptive finite element algorithm is also designed to determine the parameter N and choose elements for refinement. The numerical examples are included to show the feasibility and effectiveness of our adaptive algorithm. In the future, we hope that the algorithm can be applied to solve other scientific problems defined on unbounded domain, even those problems that could not be solved with the PML techniques can be solved with our algorithm.

The organization of this paper is as follows. In Section 2, we introduce some notation used in this paper and give the variational formulation for the model problem with the transparent