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A GENERAL CLASS OF ONE-STEP APPROXIMATION FOR INDEX-1 STOCHASTIC DELAY-DIFFERENTIAL-ALGEBRAIC EQUATIONS*

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Abstract

This paper develops a class of general one-step discretization methods for solving the index-1 stochastic delay differential-algebraic equations. The existence and uniqueness theorem of strong solutions of index-1 equations is given. A strong convergence criterion of the methods is derived, which is applicable to a series of one-step stochastic numerical methods. Some specific numerical methods, such as the Euler-Maruyama method, stochastic θ-methods, split-step θ-methods are proposed, and their strong convergence results are given. Numerical experiments further illustrate the theoretical results.

Mathematics subject classification: 34K50, 60H35, 65L80, 65L20.
Key words: Stochastic delay differential-algebraic equations, One-step discretization schemes, Strong convergence.

1. Introduction

Differential algebraic equations (DAEs) are often used to model some actual problems in science and technology, such as automatic control, electric circuits, multibody dynamics, computer aid design and so on. The numerical algorithms and their analysis play the key roles in the research of this kind of equations, and the related work refers to the monographs of Ascher & Petzold and Hairer & Wanner (cf. [3, 6]). Sometimes, however, the actual problems could be influenced on delay factor or stochastic perturbation (cf. [11, 25]). Hence, it is necessary to consider these impacts when one establishes a real model of DAEs. In this way, when a delay argument is presented, DAEs can be classed into the deterministic delay differential algebraic equations (DDDAEs) and stochastic delay differential-algebraic equations (SDDAEs).

A few methods have been proposed to solve DDDAEs numerically. For instance, Ascher and Petzold [2] studied retarded and neutral equations and presented a series of convergence results for linear multistep and Runge-Kutta methods. Hauber [7] analyzed convergence of the collocation methods for DDDAEs with state-dependent delay. Luzyanina and Roose [10] investigated the periodic solutions of semi-explicit DDDAEs and their collocation methods. Zhu and Petzold [26] presented some analytical and numerical stability criteria of Hessenberg-type DDDAEs, where multistep methods and Runge-Kutta methods were concerned. Moreover, the stability properties of numerical methods applied to delay differential equations without the algebraic restriction also refer to the paper [19, 23, 24] and the references therein.

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For stochastic differential algebraic equations without delay (SDAEs), some research results have been presented. For example, Schein and Denk applied a two-step scheme to solve linear implicit SDAEs with additive noise in [16]. Penski [15] developed a numerical method with strong order one to compute a circuit simulation model of SDAEs and analyzed the method's mean-square stability. In [8,9], the authors proposed a class of stiffly accurate stochastic Runge-Kutta methods for nonlinear index-1 SDAEs with scalar noise and investigated their mean-square stability. Furthermore, some closely related works can be seen in the papers [1, 17, 20].

Compared with the studies for DDDAEs and SDAEs, the research on numerical methods for SDDAEs is still in their infancy. To our knowledge, for index-1 SDDAEs, Xiao and Zhang [21,22] derived some existence and uniqueness results and the Euler-Maruyama methods. Whereas, most of the existing stochastic numerical methods are only for stochastic delay differential equations (SDDEs) or stochastic differential equations (SDEs) without algebraic constrain, see, e.g., [4, 5, 12–14, 18] and their references.

In this paper, we will deal with the index-1 SDDAEs with delay $\tau > 0$:

$$\begin{aligned} dx(t) &= f(t, x(t), x(t-\tau), y(t), y(t-\tau))dt \\ &\qquad + g(t, x(t), x(t-\tau), y(t), y(t-\tau))dW(t), \qquad t \in [t_0, T], \qquad (1.1\text{a}) \\ 0 &= u(t, x(t), x(t-\tau), y(t)), \qquad t \in [t_0, T], \qquad (1.1\text{b}) \end{aligned}$$

whose initial values $x(t) = a(t)$ and $y(t) = b(t)$ for $t \le t_0$. The paper is organized as follows. In section 2, we make some basic definitions, and investigate existence and uniqueness of the strong solutions of SDDAEs (1.1). In section 3, we develop a class of general one-step discretization methods for solving SDDAEs (1.1) and derive the methods' strong convergence criteria. In section 4, some specific numerical methods are proposed for SDDAEs and SDAEs, such as the Euler-Maruyama method, stochastic θ-methods and split-step θ-methods. We apply the obtained results to specific numerical methods and hence some new convergence results of the methods are given. Connection and comparison between the obtained results and the existed ones are given. Finally, with several numerical experiments, we further illustrate the theoretical results.

2. Existence and Uniqueness of Strong Solutions of Index-1 SDDAEs

To give a clear statement to the index-1 SDDAEs, we first introduce some notations. Let $(\Omega, \mathscr{A}, P)$ denote a complete probability space with a right-continuous filtration $\{\mathscr{A}_t\}_{t\ge 0}$, in which each $\mathscr{A}_t$ $(t \ge 0)$ contains all P-null sets in $\mathscr{A}$, and $W(t) = (W_1(t), \ldots, W_d(t))^T$ be the d-dimensional standard Wiener process defined on space $(\Omega, \mathscr{A}, P)$. Throughout the paper, $|\cdot|$ denotes the Euclid norm for a vector and the trace norm for a matrix. For an integrable random variable ξ, we define

$$E(\xi) := \int_\Omega \xi dP, \qquad E_t(\xi) := E(\xi|\mathscr{A}_t), \qquad \|\xi\|_{L^p} := (E|\xi|^p)^{\frac{1}{p}}.$$

Moreover, $C(\mathcal{J}; R^d)$ is the Banach space consisting of all continuous R^d-valued functions φ defined on $\mathcal{J}$ with the norm $\|\varphi\| = \sup_{t\le t_0} |\varphi(t)|$, $\mathcal{L}^p(\mathcal{J}; R^d)$ the family of R^d-valued $\mathcal{F}_t$-adapted processes $\{f(t)\}_{t\in\mathcal{J}}$ such that $\int_a^b |f(t)|^p dt < \infty$ holds almost surely (a.s. for short), and $\mathcal{M}^p(\mathcal{J}; R^d)$ the family of processes $\{f(t)\}_{t\in\mathcal{J}}$ in $\mathcal{L}^p(\mathcal{J}; R^d)$ such that $E \int_a^b |f(t)|^p dt < \infty$.