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AN IMPROVED VARIATIONAL MODEL AND ITS NUMERICAL SOLUTIONS FOR SPECKLE NOISE REMOVAL FROM REAL ULTRASOUND IMAGES*

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Abstract

Ultrasound imaging technique is one of the most non-invasive, practically harmless to the human body, accurate, cost effective and real-time techniques in medical diagnosis. However, ultrasound images suffer from the so-called speckle noise because of the imaging principle. The speckle noise reduces the quality and visibility of ultrasound images, thereby decreasing overall reliability of the images and interfering with the clinical diagnosis. In this paper, we propose a novel variational model under a combination of total variation regularization and Weberized total variation regularization and prove the existence and uniqueness of the minimizer for the variational problem. In order to efficiently solve the associated Euler-Lagrange equation consisting of nonlinear partial differential equation, we apply a finite difference method and develop several numerical techniques for solving the resulting discrete system. Numerical experiments on various synthetic and real ultrasound images not only confirm that our improved model is effective, but also it can provide significant improvement over evaluated models. Moreover, they also show that our proposed multigrid method has great potential applications to medical ultrasound imaging technique in delivering fast, accurate, and visually pleasing restoration results.

Mathematics subject classification: 68U10, 65F10, 65K10.
Key words: Image restoration, Multigrid method, Speckle noise, Ultrasound image, Weber's law.

1. Introduction

Research in the field of medical imaging has produced several different techniques for clinical diagnosis such as ultrasound (US), computed tomography (CT), magnetic resonance imaging (MRI), and so on. Each technique has its own advantages and disadvantages. One may be more appropriate than other depending on syndrome and/or disease severity. In particular, US imaging has been considered as one of the most non-invasive, practically harmless to the human body, portable, accurate, cost effective and real-time techniques for visualizing the human body's internal structures (e.g. soft organs such as liver, kidney, spleen, uterus, and heart) and movements (e.g., blood flowing through vessels and fetal development in pregnant women). These features not only help the clinicians in almost all stages of patient care: from disease detection to treatment guidance and monitoring, but also make the US imaging the most prevalent diagnostic tool in nearly all hospitals around the world.

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Basically, the US imaging is a coherent imaging system. All US images are obtained by using high-frequency sound waves, inaudible to the human ear. As the sound waves are transmitted through body tissues, they are partially reflected by the boundary between two tissue structures back to the US machine in different ways, depending on the difference in acoustic impedance of the two tissues at the interface. The time-of-flight (TOF) and the energy of the reflected echo are recorded and transformed into video or photographic images. Note that the measurement of TOF determines the gray level of each image pixel, whereas the measurement of the echo energy provides coordinates identification of the analyzed tissues. We also note that US imaging cannot be used to create the images of bones because they are too dense to penetrate. In addition, the intestinal tract and normal lung tissue are not easily identified with this medical imaging technique because air or gas can interfere with the production of the US images.

Due to the coherent nature of the US imaging systems, the quality and visibility of the US images are limited by the noise, which originates both from physical phenomena underlying the image acquisition process and imperfections of the US imaging system. The so-called *speckle noise* (SN) is found commonly in the US images of the soft organs whose underlying structures are too small to be resolved by large wavelengths. As pointed out in [42], SN is a type of the multiplicative noise (MN), which is a random granular appearance that marks small differences in gray levels and obscures small structures. The SN occurs when there is large number of scatterers with random phase within the resolution cell of an US beam. This interfering phenomenon arises when two or more waves traveling to the probe from the scatterers interfere with each other, constructively or destructively, producing bright and dark spots. The pattern of the SN depends on the probe characteristics, such as the transducer (the US device making the sound waves and receiving the echoes) frequency and the distance from the maximum-pressure point to the transducer. In particular, the SN at high (acoustic) frequencies is less granular than at low frequencies while, its size rises as the distance of the probe source increases.

In the literature, various methods have been proposed and studied for SN reduction from US images. These methods include adaptive filtering [30, 37, 41, 42], wavelet techniques [1, 29, 38], anisotropic diffusion methods [17, 62], and variational methods [2, 4, 32, 34–36, 45, 50]. Among these SN removal techniques, the variational methods are well-established mathematical theory to offer superior image restoration quality. However, much improvement on computational complexity is a major challenge to develop fast, accurate, and stable numerical algorithms for solving associated variational problems.

In the next section, we present a mathematical framework and briefly introduce a variational formulation of US image denoising problems.

1.1. Mathematical framework

Let $z : \Omega \subset \mathbb{R}^2 \to V \subset \mathbb{R}$ be an observed noisy image, where Ω is a rectangle of $\mathbb{R}^2$. The goal of image denoising problems is to restore (or recover) the original image $u : \Omega \subset \mathbb{R}^2 \to V \subset \mathbb{R}$ from the noisy image z. According to the maximum likelihood principle [25], most image denoising problems involve solving the following variational problem:

$$\min_{u \in \mathcal{U}} \left\{ \mathcal{J}_\alpha(u) = \mathcal{D}(u, z) + \alpha \mathcal{R}(u) \right\}, \tag{1.1}$$

where $\mathcal{D}(u, z)$ is the fidelity or data fitting term deriving from the assumption on the distribution of the noise in the observed noisy image, which is used to penalize the inconsistency between