THE DEFERRED CORRECTION PROCEDURE FOR LINEAR MULTISTEP FORMULAS*

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Abstract

A general approach of deferred correction procedure based on linear multistep formulas is proposed. Several deferred correction procedures based on backward differentiation formulas (Gear's method), which allow us to develop L-stable algorithms of order up to 4 and $L(\alpha)$-stable algorithms of order up to 7, are derived. Preliminary numerical results indicate that this approach is indeed efficient.

1. Introduction

The original idea of deferred correction was first proposed by Fox[4] to improve the accuracy of the basic solution. Significant improvements and extensions have been made since then. The concept of deferred correction now has a much wider meaning than the original idea of Fox. The various techniques of deferred correction have been widely used to obtain solutions of ordinary differential equations with improved orders of accuracy (see [5]—[8], [1]). The main reason why the technique of deferred correction arouses so much interest is that it often has more computational advantages than that of Richardson extrapolation. Particularly, in dealing with stiff systems, a deferred correction procedure or a local extrapolation one based on an underlying method that is usually of a low order and a high stability should preserve the good stability properties of this method. In this connection it is well known that local extrapolation is not praiseworthy.

In this paper we consider the problem of deriving an efficient deferred correction procedure based on linear multistep formulas. The key to the settlement of the question lies in choosing an appropriate correction term such that when the procedure is applied to the usual scalar test equation $y'=\lambda y$, with a constant step size h, the correction term is a rational function of λh, not a polynomial in λh as is usually the case. The procedures proposed in this paper can not only raise the order of accuracy of the basic solution but also improve the stability properties of the underlying formulas.

In Section 2 we explain our general approach and give some examples for simple deferred correction. In Section 3 we derive several deferred correction procedures based on BDF, which allow us to develop L-stable algorithms of order up to 4 and $L(\alpha)$-stable algorithms of order up to 7. Finally in Section 4 we present preliminary numerical results which will indicate that these algorithms

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are indeed efficient.

2. The General Approach

In this section we shall be concerned with the numerical solution of the initial value problem

$$y'=f(t,y),\qquad y(t_0)=y_0, \tag{2.1}$$

obtained by using linear multistep formula

$$LM(h,p,y_{n+k})=\sum_{j=0}^{k}\alpha_j y_{n+j}-h\sum_{j=0}^{k}\beta_j f_{n+j}=0 \tag{2.2}$$

with $\alpha_k=1$ and the local truncation error (LTE)

$$T_k=C_{p+1}h^{p+1}y^{(p+1)}(t_n)+O(h^{p+2}). \tag{2.3}$$

The basic idea of deferred correction is to compute another improved numerical solution y_{n+k} by adding in a correction term computed from the numerical solution $\bar{y}_{n+k}$ of

$$LM(h,p,\bar{y}_{n+k})=0.$$

For this purpose we choose a formula

$$LO(h,q,y_{n+k})=\sum_{j=0}^{k}\bar{\alpha}_j y_{n+j}-h\sum_{j=0}^{k}\bar{\beta}_j f_{n+j}=0,\quad q>p,\ \bar{\alpha}_k=1, \tag{2.4}$$

with an order higher than that of $LM(h,p,\bar{y}_{n+k})=0$ such that LO can be split into two parts, namely

$$LO(h,q,y_{n+k})=LM(h,p,y_{n+k})+L\varepsilon(h,p,y_{n+k}).$$

Obviously the LTE associated with

$$L\varepsilon(h,p,y_{n+k})=0 \tag{2.5}$$

is $(-T_k)$. Therefore $L\varepsilon$ may serve as the deferred correction term which is to be found and the solution $\bar{y}_{n+k}$ of $LM(h,p,\bar{y}_{n+k})=0$ can be improved to y_{n+k} by means of the correction formula

$$LM(h,p,y_{n+k})=-L\varepsilon(h,p,\bar{y}_{n+k}). \tag{2.6}$$

As we shall see in the following such a choice of correction term (2.5) is often unsatisfactory for solving stiff systems since this form of deferred correction procedure usually destroys the good stability properties of the underlying formula.

However, this choice of (2.5) (called simple correction term) unifies the deferred correction procedure and the popular predictor corrector into one procedure and makes it easy to derive some implicit multistep formulas.

As an alternative, in dealing with stiff systems we consider a rational deferred correction procedure (see [1])

$$LM(h,p,\bar{y}_{n+k})=0, \tag{2.7a}$$

$$LM(h,p,y_{n+k})=-P_l(hJ)(I-h\beta_k J)^{-m}L\varepsilon(h,p,\bar{y}_{n+k}), \tag{2.7b}$$

where J is an approximation to the Jacobian matrix $\frac{\partial f}{\partial y}$, P_l is a polynomial of degree l and l, m are integers.

As we shall see in Section 3, if the following conditions