ON NUMERICAL METHODS FOR ROBUST POLE ASSIGNMENT IN CONTROL SYSTEM DESIGN (II)*1)

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Abstract

L. R. Fletcher[1] has pointed out that how to best exploit any freedom of choice in regard to the eigenvalues of a closed-loop system is an unsolved problem for robust pole assignment in control system design. This paper suggests a numerical method to solve this problem. Numerical results show that the method is feasible.

§ 1. Introduction

Throughout this paper we shall use the same notational convention as in [7].

The following robust assignment problem has been investigated (Ref. [1], [2], [3], [7]):

Problem RPA. Given a real $n\times n$ matrix A, a real full rank $n\times m$ matrix $B(m<n)$ and a set $\mathscr{L}$ of n complex numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$, closed under complex conjugation, find a real $m\times n$ matrix F and a non-singular $n\times n$ matrix X satisfying

$$(A+BF)X=X\Lambda, \tag{1.1}$$

where $\Lambda=\operatorname{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$, such that the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $A+BF$ are as insensitive to perturbations in the matrix $A+BF$ as possible.

This paper investigates an unsolved problem proposed by L. R. Fletcher[1].

L. R. Fletcher has pointed out that in practice the eigenvalue spectrum $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ "is usually required to be contained in some region of the complex plane rather than to be precisely some given complex numbers. Numerical experimentation indicates that some eigenvalue spectra are much more sensitive than others to perturbations in A, B and F so that choosing Λ to minimize this sensitivity is an important practical issue about which virtually nothing is known." ([1, p. 169]) Hence, the problem of how to best exploit any freedom of choice in regard to the eigenvalue spectrum is well worth investigating.

The unsolved problem may be formulated as follows:

Problem RPA 1. Given a real $n\times n$ matrix A, a real full rank $n\times m$ matrix $B(m<n)$, p segments $\mathscr{L}_1, \mathscr{L}_2, \cdots, \mathscr{L}_p$ lying in the real axis $\mathbb{R}$ and regions $\mathscr{D}_1, \mathscr{D}_2, \cdots, \mathscr{D}_q$ of the complex plane $\mathbb{C}$, where

$$\mathscr{L}_j=\{\xi\in\mathbb{R}: \lambda_{j,1}\leqslant\xi\leqslant\lambda_{j,2}\}, \quad j=1, 2, \cdots, p, \tag{1.2}$$

$$\mathscr{D}_j=\{z=\xi+i\eta\in\mathbb{C}: \mu_{j,1}\leqslant\xi\leqslant\mu_{j,2}, \nu_{j,1}\leqslant\eta\leqslant\nu_{j,2}\}, \quad j=1, 2, \cdots, q \tag{1.3}$$

* Received March 18, 1986.
1) The Projects Supported by National Natural Science Foundation of China.

and $p+2q=n$, find n numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$, a nonsingular $n\times n$ matrix X and a real $m\times n$ matrix F satisfying the equation (1.1) and

$$\lambda_j \in \mathscr{L}_j, \quad j=1, 2, \cdots, p, \tag{1.4}$$

$$\lambda_{p+j} \in \mathscr{D}_j, \ \lambda_{p+q+j}=\bar{\lambda}_{p+j}, \quad j=1, 2, \cdots, q, \tag{1.5}$$

such that the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $A+BF$ are as insensitive to perturbations in the matrix $A+BF$ as possible.

Clearly, Problem RPA and Problem RPA1 are both inverse algebraic eigenvalue problems.

The aim of this paper is to suggest a numerical method for solving Problem RPA1. For simplicity we consider only the case where the pair (A, B) is controllable ([1]) and $p=n$. Moreover, for convenience we write

$$\lambda_j=\lambda_j(t_j)=\lambda_{j,1}+(\lambda_{j,2}-\lambda_{j,1})\sin^2 t_j, \quad t_j\in \mathbb{R} \ \forall j. \tag{1.6}$$

The idea and technique described in this paper may be used to solve Problem RPA1 in the case of $p<n$.

The procedure of the numerical method for solving Problem RPA1 consist of two basic steps ([2], [7]):

Step A—X. Compute the decomposition

$$B=(U_0^{(B)}, U_1^{(B)})\begin{pmatrix} Z \\ 0 \end{pmatrix}, \tag{1.7}$$

where $(U_0^{(B)}, U_1^{(B)})$ is a real orthogonal matrix and Z nonsingular;

Construct orthogonal bases comprised by the columns of matrices $S_j(t_j)$ and $\hat{S}_j(t_j)$ for the space $\mathscr{S}_j(t_j)\equiv\mathscr{N}(U_1^{(B)T}(A-\lambda_j(t_j)I))$ and its complement, $\hat{\mathscr{S}}_j(t_j)$ for $\lambda_j(t_j)\in\mathscr{L}_j$, $j=1, 2, \cdots, n$;

Select vectors $x_j=S_j(t_j)w_j\in\mathscr{S}_j(t_j)$, $j=1, 2, \cdots, n$ such that $X=(x_1, x_2, \cdots, x_n)$ is well-conditioned.

Step F. Find the matrix $M=A+BF$ by solving $MX=X\Lambda$ and compute F explicitly from

$$F=Z^{-1}U_0^{(B)T}(M-A). \tag{1.8}$$

Obviously, to find a well-conditioned matrix X is the key of the above mentioned procedure. In the next section we reduce the problem for finding a well-conditioned matrix X to an unconstrained optimization problem. In Section 3 we deduce a formula of the gradient vector for the objective function described in Section 2, and in Section 4 we use the DFP algorithm to solve the unconstrained optimization problem. Numerical results are given in Section 5.

§ 2. An Optimization Problem

Let

$$X=(x_1, x_2, \cdots, x_n), \quad Y=X^{-T}=(y_1, y_2, \cdots, y_n), \tag{2.1}$$

$$c_j=\|x_j\|_2\|y_j\|_2\geqslant 1, \quad j=1, 2, \cdots, n \tag{2.2}$$

and

$$c=(c_1, c_2, \cdots, c_n)^T, \tag{2.3}$$

where