THE ASYMPTOTIC BEHAVIOR AND THE CONVERGENCE OF THE SOLUTION OF A REACTION-DIFFUSION DIFFERENCE SCHEME IN A CIRCULAR REGION*

CHEN SUI-YANG (陈绥阳)
(Lanzhou University Lanzhou, China)

GUO BEN-YU (郭本瑜)
(Shanghai University of Science and Technology Shanghai, China)

§ 1. Introduction

Ludwig, Jones and Holling[1] proposed an ordinary differential equation to describe the budworm density in the forest. Ludwig, Aronson and Weinberger[2] considered the spatial effect of the budworm, by adding a diffusion term in the original model. They also studied this problem for a region of infinite strip in detail. Recently, Guo Ben-yu. Mitchell and Sleeman[3] and Guo Ben-yu, Sleeman, Mitchell[4] considered this problem for circular and rectangular regions respectively and very precise results were obtained. Guo Ben-yu and Mitchell[5] also studied the asymptotic behavior and the convergence of a reaction-diffusion difference scheme in an infinite strip.

In this paper we consider the linear and nonlinear reaction-diffusion difference equations, the existence of the positive solution of the steady problem and the asymptotic behavior of the solution of the unsteady problem. Finally, we prove the convergence of the approximate solution.

§ 2. The Difference Scheme for the Linear Problem

In this section we consider a linear model whose boundary condition means that the exterior is a lethal environment for the budworm. Assume that Ω is a bounded open domain in R^2 and $U(x, t)$ is the scaled density of the budworm population. Then $U(x, t)$ satisfies the equation

$$\begin{cases} \dfrac{\partial U}{\partial t} - \Delta U = U, & x \in \Omega,\ 0 < t < \infty, \\ U(x, t) = 0, & x \in \partial\Omega,\ 0 \leqslant t < \infty, \\ U(x, 0) = U_0(x), & x \in \Omega, \end{cases} \tag{2.1}$$

where $U_0(x)$ is a given function and $U_0(x) = 0$ on $\partial\Omega$.

If $U_0(x) = U_0(\rho)$ where $\rho = |x|$ and if Ω is a circular domain with the radius l, then

* Received June 20, 1986.

$$\begin{cases} \frac{\partial U}{\partial t}+\varepsilon PU=U, & 0<\rho<1,\ 0<t<\infty \\ \frac{\partial U}{\partial \rho}(0,t)=0, & U(1,t)=0,\ 0\leqslant t<\infty, \\ U(\rho,0)=U_0(\rho), & 0<\rho<1 \end{cases} \tag{2.2}$$

with $\varepsilon=\frac{1}{l^2}$ and $P=-\frac{\partial^2}{\partial\rho^2}-\frac{1}{\rho}\frac{\partial}{\partial\rho}$.

Let h and τ be the mesh sizes of the space and time respectively, where $Nh=1$, N being a positive integer. We define

$$\Omega_h=\{\rho/\rho=h,\ 2h,\ \cdots,\ (N-1)h\}$$

and $\overline{\Omega}=\Omega_h+\partial\Omega_h$ where $\partial\Omega_h$ is the boundary of Ω_h.

Let $\eta^k(\rho)$ be the value of the mesh function η at the point $\rho=jh$ and $t=k\tau$, and $\eta^k_\rho(\rho)$, $\eta^k_{\bar\rho}(\rho)$ and $\eta^k_{\hat\rho}(\rho)$ denote respectively the forword, the backward and the central difference quotients of $\eta^k(\rho)$ with respect to ρ. Similarly, $\eta^k_t(\rho)$ denotes the forward difference quotient of $\eta^k(\rho)$ with respect to t. We define

$$P_h\eta^k(\rho)=-\eta^k_{\rho\bar\rho}(\rho)-\frac{1}{\rho}\eta^k_{\hat\rho}(\rho).$$

Let $u^k(\rho)$ be the approximation to $U(\rho,k\tau)$. The Crank-Nicolson scheme for solving (2.2) is

$$\begin{cases} u^k_t(\rho)+\frac{\varepsilon}{2}P_hu^k(\rho)+\frac{\varepsilon}{2}P_hu^{k+1}(\rho)=\frac{1}{2}u^k(\rho)+\frac{1}{2}u^{k+1}(\rho). & \rho\in\Omega_h,\ k\geqslant 0, \\ u^k_{\hat\rho}(0)=0,\ u^k(1)=0, & k\geqslant 0, \\ u^0(\rho)=U_0(\rho), & \rho\in\Omega_h. \end{cases} \tag{2.3}$$

The corresponding steady equation is

$$\begin{cases} \varepsilon P_hv(\rho)=v(\rho),\ \rho\in\Omega_h, \\ v_\rho(0)=0 \quad v(1)=0. \end{cases} \tag{2.4}$$

§ 3. The Discrete Green Function

To study the behavior of the solution of (2.4), we define the discrete Green function as

$$\begin{cases} P_hG_h(\rho,\rho')=\frac{1}{h^2}\delta(\rho,\rho'), & \rho\in\Omega_h, \\ G_{h,\rho}(0,\rho')=0, & G_h(1,\rho')=0, \end{cases} \tag{3.1}$$

where $\rho'\in\overline{\Omega}_h$ and $\delta(\rho,\rho')$ is a Kronecker function.

Let

$$G_h(\rho')=(G_h(h,\rho'),\ \cdots,\ G_h((N-1)h,\rho'))^*,$$
$$\delta(\rho')=(\underbrace{0,\ \cdots,\ 0}_{(j'-1)},\ 1,\ \underbrace{0,\ \cdots,\ 0}_{(N-1-j')})^*,\ \rho'=j'h.$$

Then from (3.1) we obtain

$$BG_h(\rho')=\frac{1}{h^2}\delta(\rho'),$$