ON THE STABILITY OF FINITE-DIFFERENCE SCHEMES OF HIGHER-ORDER APPROXIMATE ONE-WAY WAVE EQUATIONS*

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Abstract

The finite difference migration, proposed and developed by J. F. Claerbout[1], is now widely used in seismic data processing. The method has a limitation that the events are not dipping too much. Guanquan ZHANG derived a new version of higher-order approximation of one-way wave equation in the form of systems of lower-order equations[2]. For these systems he constructed some suitable difference schemes and developed a new algorithm of finite-difference migration for steep dips[3]. In this paper, we discuss the stability of these difference schemes by the method of energy estimation.

§1. Equations and Difference Schemes

For steep dip migration the following system of lower-order equations can be used[2]

$$\begin{cases} \dfrac{\partial p}{\partial z}=\sum\limits_{l=1}^{m/2}\dfrac{\partial q_l}{\partial t}, & (1.1a)\\ \dfrac{1}{c^2}\dfrac{\partial^2 q_l}{\partial t^2}=\alpha_{m,l}^2\dfrac{\partial^2 q_l}{\partial x^2}+\beta_{m,l}\dfrac{\partial^2 p}{\partial x^2}, \quad l=1,2,\cdots,m/2. & (1.1b)\end{cases}$$

The initial and boundary conditions are

$$\begin{cases} p|_{z=0}=\varphi(x,t), & |x|<X,\ 0<t\leqslant T,\\ p=q_l=\dfrac{\partial q_l}{\partial t}=0, & |x|=X \text{ or } t=0,\end{cases}$$

where m is an even integer, $l=1,2,\cdots,\dfrac{m}{2}$,

$$\alpha_{m,l}=\cos(l\pi/(m+1)),$$

$$\beta_{m,l}=\prod_{j=1}^{m-1}(\alpha_{m,l}-\alpha_{m-1,j})\Big/\prod_{j\neq l}^{m}(\alpha_{m,l}-\alpha_{m,j}).$$

It can be easily verified[2] that

$$\beta_{m,l}>0,\quad \sum_{l=1}^{m/2}\beta_{m,l}=1/2. \tag{1.1c}$$

From (1.1a), (1.1b) and (1.1c) one obtains

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t\,\partial z}-\frac{1}{2}\frac{\partial^2 p}{\partial x^2}-\sum_{l=1}^{m/2}\alpha_{m,l}^2\frac{\partial^2 q_l}{\partial x^2}=0. \tag{1.1d}$$

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Obviously, problems (1.1d), (1.1b) and (1.1a), (1.1b) are equivalent. For $m=2$, (1.1a), (1.1b) can be simplified to

$$\begin{cases} \dfrac{\partial^2 p}{\partial z\,\partial t}=\dfrac{c^2}{2}\dfrac{\partial^2 p}{\partial x^2}+\dfrac{c^2}{4}\dfrac{\partial^2 q}{\partial x^2}, & (1.2a)\\ \dfrac{\partial p}{\partial z}=\dfrac{\partial q}{\partial t}. & (1.2b)\end{cases}$$

The approximations of $p(k\Delta x, j\Delta t, n\Delta z)$, $q(k\Delta x, (j-1/2)\Delta t, (n+1/2)\Delta z)$, $q(k\Delta x, j\Delta t, (n+1/2)\Delta z)$, $q_l(k\Delta x, (j-1/2)\Delta t, (n+1/2)\Delta z)$ are denoted respectively by

$$p^n_{k,j},\ q^{n+1/2}_{k,j-1/2},\ q^{n+1/2}_{k,j},\ q^{n+1/2}_{lk,j-1/2}.$$

Δ^+_x, Δ^-_x, and δ^2 are defined by

$$\Delta^+_x p^n_{k,j}=p^n_{k+1,j}-p^n_{k,j},\quad \Delta^-_x p^n_{k,j}=p^n_{k,j}-p^n_{k-1,j},\quad \delta^2 p^n_{k,j}=\Delta^+_x\Delta^-_x p^n_{k,j}.$$

Δ^+_t, Δ^-_t, Δ^+_z, Δ^-_z are similarly defined. If we let Δ denote Δ^+, we have the identities

$$\Delta(u_jv_j)=u_{j+1}\Delta v_j+(\Delta u_j)v_j=u_j\Delta v_j+(\Delta u_j)v_{j+1}$$
$$=\frac{1}{2}[(u_j+u_{j+1})\Delta v_j+(v_j+v_{j+1})\Delta u_j], \tag{1.3}$$
$$(u_j+u_{j+1})\Delta u_j=\Delta(u_j^2). \tag{1.4}$$

We can approximate (1.2) by the difference scheme

$$\text{I:}\begin{cases} \Delta^+_t\Delta^+_z(1+\alpha\delta^2)p^n_{k,j}/\Delta t\Delta z=r^n_k\Delta^+_x\Delta^-_x(p^n_{k,j}+p^{n+1}_{k,j}+p^n_{k,j+1}+p^{n+1}_{k,j+1})/\Delta x^2\\ \quad +r^n_k\Delta^+_x\Delta^-_x(q^{n+1/2}_{k,j}+q^{n+1/2}_{k,j+1})/\Delta x^2, & (1.5a)\\ \Delta^+_t q^{n+1/2}_{k,j}/\Delta t=\Delta^+_z(p^n_{k,j}+p^n_{k,j+1})/2\Delta z, & (1.5b)\end{cases}$$

with the initial and boundary conditions

$$\begin{cases} p^0_{k,j}=\varphi(k\Delta x, j\Delta t),\\ p^n_{k,j}=q^{n+1/2}_{k,j}=0, \quad |k|=K,\\ p^n_{k,j}=q^{n+1/2}_{k,j}=0, \quad j=0.\end{cases} \tag{1.5c}$$

The interval $[-X, X]$ is divided into $2K$ equal parts, $\Delta x=X/K$.

We can also use the following scheme

$$\text{II:}\begin{cases} \Delta^+_t\Delta^+_z(1+\alpha\delta^2)p^n_{k,j}/\Delta t\Delta z-r^n_k\Delta^+_x\Delta^-_x(p^n_{k,j}+p^{n+1}_{k,j}+p^n_{k,j+1}\\ \quad +p^{n+1}_{k,j+1})/\Delta x^2-2r^n_k\Delta^+_x\Delta^-_x q^{n+1/2}_{k,j+1/2}/\Delta x^2=0, & (1.6a)\\ \Delta^+_t q^{n+1/2}_{k,j-1/2}/\Delta t=\Delta^+_z p^n_{k,j}/\Delta z, & (1.6b)\end{cases}$$

with the initial and boundary conditions

$$\begin{cases} p^0_{k,j}=\varphi(k\Delta x, j\Delta t),\\ p^n_{k,j}=q^{n+1/2}_{k,j-1/2}=0, \quad |k|=K,\\ p^n_{k,j}=q^{n+1/2}_{k,j+1/2}=0, \quad j=0.\end{cases} \tag{1.6c}$$

(1.1) can also be approximated by the difference scheme

$$\text{III:}\begin{cases} \Delta^+_t\Delta^+_z(1+\alpha\delta^2)p^n_{k,j}/\Delta t\Delta z=r^n_k\Delta^+_x\Delta^-_x(p^n_{k,j}+p^{n+1}_{k,j}+p^n_{k,j+1}+p^{n+1}_{k,j+1})/\Delta x^2\\ \quad +\sum\limits_{l=1}^{m/2}\alpha^n_{lk}\Delta^+_x\Delta^-_x q^{n+1/2}_{lk,j+1/2}/\Delta x^2, & (1.7a)\\ \Delta^+_t\Delta^-_t q^{n+1/2}_{lk,j+1/2}/\Delta t^2=\alpha^n_{lk}\Delta^+_x\Delta^-_x q^{n+1/2}_{lk,j+1/2}/\Delta x^2\\ \quad +\beta^n_{lk}\Delta^+_x\Delta^-_x(p^n_{k,j}+p^{n+1}_{k,j}+p^n_{k,j+1}+p^{n+1}_{k,j+1})/\Delta x^2, & (1.7b)\end{cases}$$

with the initial and boundary conditions