CONVERGENCE OF DIFFERENCE METHODS FOR INVERSE PROBLEMS OF A ONE-DIMENSIONAL HYPERBOLIC SYSTEM OF FIRST ORDER*[1)]

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Abstract

In this paper, the difference methods for solving the inverse problem of a one-dimensional hyperbolic system of first order are discussed. Some difference schemes are constructed and the convergence of these schemes is proved.

§ 1. Introduction and Summary

In [2], the inverse problem of a one-dimensional linear hyperbolic system of first order is discussed. This problem can be transformed into a semilinear initial-value problem by using a relation obtained from the propagation of singularity. The theorems of existence and stability are proved there. In this paper, we discuss the difference methods for solving this inverse problem as a semilinear initial-value problem.

Consider the following system

$$\begin{cases} \dfrac{\partial W}{\partial t}+c^{-1}(x)\dfrac{\partial P}{\partial x}=0, \\ \dfrac{\partial P}{\partial t}+c(x)\dfrac{\partial W}{\partial x}=0, \end{cases} \quad x>0,\ t>0 \tag{1.1}$$

with the initial conditions

$$W(x,0)=P(x,0)=0 \tag{1.2}$$

and the boundary conditions

$$\begin{cases} W(0,t)=\delta(t)+W_0(t), \\ P(0,t)=\delta(t)+P_0(t). \end{cases} \tag{1.3}$$

The inverse problem is to determine W, P and c satisfying (1.1) and (1.2) from the given data (1.3) and a given constant $c(0)$, here we assume $c(0)=1$.

Set $D=P+cW$ and $U=P-cW$. Then (1.1) becomes

$$\begin{cases} \dfrac{\partial D}{\partial t}+\dfrac{\partial D}{\partial x}=\beta(x)\cdot(D-U), \\ \dfrac{\partial U}{\partial t}-\dfrac{\partial U}{\partial x}=\beta(x)\cdot(D-U), \end{cases} \quad x>0,\ t>0, \tag{1.4}$$

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where

$$\beta(x) = \frac{c'(x)}{2c(x)} \tag{1.5}$$

and the corresponding initial and boundary conditions become

$$D(x, 0) = U(x, 0) = 0 \tag{1.6}$$

and

$$\begin{cases} D(0, t) = 2\delta(t) + D_0(t), \\ U(0, t) = U_0(t), \end{cases} \tag{1.7}$$

where $D_0(t) = P_0(t) + W_0(t)$, and $U_0(t) = P_0(t) - W_0(t)$. So we need only to solve (1.4) under the conditions (1.6) and (1.7). Obviously the solution of (1.4) with (1.6) satisfies

$$D(x, t) = U(x, t) = 0, \quad \text{for } x > t > 0. \tag{1.8}$$

By the theory of propagation of singularity (see [6], Ch. 6), we can get the important relation

$$U(x, x) = \beta(x) \exp \int_0^x \beta(s)\, ds, \quad x \geq 0 \tag{1.9}$$

and D can be decomposed as

$$D(x, t) = 2\delta(t-x) \exp \int_0^x \beta(s)\, ds + \tilde{D}(x, t), \tag{1.10}$$

where $\tilde{D}(x, t)$ has a discontinuity of the second kind on $x = t$ (see Appendix).

Now we consider our problem only in the domain $S_{(0,T)} = \{(x, t) \mid t > x, 0 < x < T\}$. Then the original inverse problem is transformed to the following initial value problem:

$$\begin{cases} \dfrac{\partial D}{\partial t} + \dfrac{\partial D}{\partial x} = \beta(x) \cdot (D - U), \\ \dfrac{\partial U}{\partial t} - \dfrac{\partial U}{\partial x} = \beta(x) \cdot (D - U) \end{cases} \tag{1.11}$$

with the initial conditions (in the x-direction)

$$\begin{cases} D(0, t) = D_0(t), \\ U(0, t) = U_0(t), \end{cases} \tag{1.12}$$

where $\beta(x)$ is determined by $U(x, x) = \beta(x) \exp \int_0^x \beta(s)\, ds$.

Set

$$d(x) = \exp \int_0^x \beta(s)\, ds. \tag{1.13}$$

Then by (1.5), we have

$$d(x) = \exp \int_0^x \frac{c'}{2c}\, ds = \sqrt{c(x)},$$

i.e.

$$d^2(x) = c(x).$$

On the other hand,