AN EXTRAPOLATION METHOD FOR BEM

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Abstract

This paper gives an asymptotic expansion of the error on the mesh point for Galerkin approximation of integral equations of the first kind. The extrapolation formula and some numerical results are given.

1. Introduction

It has been shown in [1]-[3] that the Richardson extrapolation can be applied to the elliptic Ritz projection with linear finite elements and increase the accuracy on mesh points z from

$$u_h(z) = u(z) + O(h^2|\ln h|)$$

to

$$\tilde{u}_h(z) = \frac{1}{3}\left[4u_{\frac{h}{2}}(z) - u_h(z)\right] = u(z) + O(h^3|\ln h|)$$

or

$$(O(h^4|\ln h|)),$$

where T_h is uniform triangulation and $T_{\frac{h}{2}}$ is generated from T_h by dividing each triangle into four congruent subtriangles.

In this paper, the above basic results are extended to boundary finite elements for integral equations of the first kind.

2. The Extrapolation for BEM

Let us consider the following boundary value problem

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = u_0 & \text{on } \Gamma, \end{cases} \tag{2.1}$$

where Ω is a bounded domain with smooth boundary Γ. The equivalent integral equation is

$$\begin{aligned} \int_\Gamma q(x)E(x;y)ds_x = & \int_\Gamma u_0(x)\frac{\partial}{\partial n_x}E(x;y)ds_x + \frac{1}{2}u_0(y) \\ & -\int_\Omega f(x)E(x;y)dx \quad \forall\, y \in \Gamma, \end{aligned} \tag{2.2}$$

$$u(y) = \int_\Gamma q(x)E(x;y)ds_x - \int_\Gamma u_0(x)\frac{\partial}{\partial n_x}E(x;y)ds_x + \int_\Omega f(x)E(x;y)dx \quad \forall\, y\in\Omega, \tag{2.3}$$

where $E(x;y)$ is a fundamental solution of Laplace equation, i.e.

$$\triangle_x E(x:y) + \delta(x-y) = 0$$

and $q = \left.\frac{\partial u}{\partial n}\right|_\Gamma$. For two–dimensional problems, it is known that

$$E(x;y) = -\frac{1}{2\pi}\ln|x-y|. \tag{2.4}$$

The integral equation (2.2) can be expressed by

$$(Aq)(y) = \int_\Gamma q(x)E(x;y)ds_x = F(y) \quad \forall y\in\Gamma \tag{2.5}$$

where

$$F(y) = \int_\Gamma u_0(x)\frac{\partial}{\partial n_x}E(x;y)ds_x + \frac{1}{2}u_0(y) - \int_\Omega f(x)E(x;y)dx.$$

The corresponding variational problem is

$$\begin{cases} \text{find } q\in H^{-\frac{1}{2}}(\Gamma)=V \quad \text{such that} \\ (Aq,q') = (F,q') \quad \forall q'\in V. \end{cases} \tag{2.6}$$

Let us consider the equation

$$\begin{cases} -\int_\Gamma \ln|t-s|e_\Gamma(s)dl_s = u_\Gamma, \\ \int_\Gamma e_\Gamma(s)dl_s = 1, \end{cases} \tag{2.7}$$

where $e_\Gamma(s)$ and $c_\Gamma = \exp(-u_\Gamma)$ are defined as the equilibrium distribution and the transfinite diameter respectively [7]. It has been proved in [7] that when $c_\Gamma \neq 1$(i. e. $u_\Gamma \neq 0$), the solution of (2.6) exists and is unique.

Let L denote the arc length of Γ and let us identify functions on Γ with L–periodic functions on the real axis [9] :

$$x(s+L) = x(s), \quad q(s+L) = q(s) \quad \text{etc. for all } s\in R.$$

We consider the grid points x_j on Γ defined by

$$x_j = x(jh), \quad j = 0,\pm1,\pm2,\dots, \quad h := \frac{L}{N+1}$$

and the periodic grid function q with values $q_h^j = q_h(x_j)$ such that

$$q_h^{j+(N+1)} = q_h^j.$$

Suppose $V_h(\Gamma_h)$ is a piecewise linear function space on $[0,L]$, where $\Gamma_h = \bigcup_i \Gamma_i$, $\Gamma_i = x_i\widehat{x_{i+1}}$ and c can represent different constants which are independent of h.