Journal of Computational Mathematics, Vol.8, No.3, 1990, 289–297.

NONLINEAR INTEGRAL EQUATION OF INVERSE SCATTERING PROBLEMS OF WAVE EQUATION AND ITERATION*

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Abstract

In this paper, we reduce the 1–D, 2–D and 3–D inverse scattering problems of the wave equation into the nonlinear integral equation. The iteration for solving the above integral equations has been considered.

§1. Introduction

There are several works about the inverse scattering problem of the wave equation [1–5]. In [4] and [5], the characteristic iteration for solving 1–D, 2–D and 3–D inverse scattering problems to determine potential have been given. In this paper, by the method of [4] and [5], we reduce the above inverse problem to the nonlinear integral equation. In the 1–D case, the integral equation is of second kind. In 2–D and 3–D cases, the integral equations are Radon's interesting integral geometry problem.

The integral equation in this paper will be useful for the theoretical and numerical analysis and application of the above inversion.

The iteration for solving the above integral equation is considered. Moreover, we perform several simulative numerical calculations in the 1–D and 2–D scattering inversions and get excellent numerical results.

We shall first deal with the nonlinear integral equation in 3–D. Then we will describe the nonlinear integral equation in 1–D and 2–D. A description of the parallel iteration for solving the above nonlinear integral equations is given in Section 4. Our numerical results are presented and discussed in Section 5.

§2. 3–D Inverse Scattering Potential Problem and Its Nonlinear Integral Equation

2.1 Basic equation and its scattering inversion

$$\frac{\partial^2 u}{\partial t^2} - \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + q(x,y,z)u = 0,$$
$$(x,y,z) \in R^2 \times R^+, \quad t > 0, \tag{2.1}$$
$$u(x,y,z,t) = 0, \qquad (x,y,z) \in R^2 \times R^+, \quad t \le 0, \tag{2.2}$$

* Received November 14, 1987.

$$\frac{\partial u}{\partial z}(x,y,0,t)=\delta(x,y,t),\quad (x,y)\in R^2,\quad t\geq 0. \tag{2.3}$$

To recover $q(x,y,z)$ from measured data on the surface boundary

$$u(x,y,0,t)=f(x,y,t)=-\frac{1}{2\pi}\frac{\delta(t-\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}+f_S(x,y,t) \tag{2.4}$$

will be called the inverse scattering potential problem of 3–D wave equation, where $\delta(\cdot)$ is the generalized delta function, and $q(x,y,z)\geq 0$ is a continuous function.

2.2 The properties of the solution

Lemma 2.1. *For $q(x,y,z)\in C(R^2\times R^+)$, the solution of (2.1)–(2.3) can be decomposed to*

$$u(x,y,z,t)=-\frac{1}{2\pi}\frac{\delta(t-\sqrt{x^2+y^2+z^2})}{\sqrt{x^2+y^2+z^2}}+v(x,y,z,t), \tag{2.5}$$

where $v(x,y,z,t)$ satisfies

$$\frac{\partial^2 v}{\partial t^2}-\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+q(x,y,z)v=\frac{1}{2\pi}q(x,y,z)\frac{\delta(t-\sqrt{x^2+y^2+z^2})}{\sqrt{x^2+y^2+z^2}},$$

$$(x,y,z)\in R^2\times R^+,\quad t>0, \tag{2.6}$$

$$v(x,y,z,t)=0,\quad (x,y,z)\in R^2\times R^+,\quad t\leq 0, \tag{2.7}$$

$$\frac{\partial v}{\partial z}(x,y,0,t)=0,\quad (x,y)\in R^2,\quad t\geq 0. \tag{2.8}$$

Proof. Let

$$u_1=-\frac{1}{2\pi}\frac{\delta(t-\sqrt{x^2+y^2+z^2})}{\sqrt{x^2+y^2+z^2}}.$$

Since u_1 is a solution of (2.1)–(2.3) when $q(x,y,z)=0$, by substituting (2.5) into (2.1)–(2.3), (2.6)–(2.8) can be obtained immediately.

2.3 Nonlinear Integral Equation of 3–D

Theorem 2. *Suppose that $q(x,y,z)\in C(R^2\times R^+)$. Then the 3–D inverse scattering problem for determining $q(x,y,z)$ from (2.1)–(2.4) will be reduced to the following nonlinear integral equation:*

$$\frac{1}{2\pi^2[t^2-(x^2+y^2)]}\iint_{S[(x,y);t]}[q(\xi,\eta,\varsigma)(\xi^2+\eta^2+\varsigma^2)\sin\theta]d\theta d\phi$$
$$=\frac{1}{\pi}\iint_{D[(x,y);t]}q(\xi,\eta,\varsigma)\frac{v(\xi,\eta,\varsigma;t-\sqrt{(\xi-x)^2+(\eta-y)^2+\varsigma^2})}{\sqrt{(\xi-x)^2+(\eta-y)^2+\varsigma^2}}d\xi d\eta d\varsigma+f_S(x,y,t), \tag{2.9}$$

where $S[(x,y);t]$ denotes the half ellipsoid and $D[(x,y);t]$ is its body.