Numerical Stability of Integration Methods Used in Cloth Simulation

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Abstract: For better performance, efficient numerical methods describing physical behaviour of clothes are needed for any cloth simulation system. In this paper we report a quantitative research performed on the stability of the most widely used integration techniques in cloth simulation. Advantages and weaknesses of various integration techniques are listed for trade-off among stability, accuracy and speed. In this light, we offer a solution for choosing numerical methods when dealing with simulation problems.

Keywords: Cloth simulation, numerical integration, euler, midpoint, runge-kutta, verlet, stability.

1. Introduction

The physical model employed is of essential importance in the design of a stable cloth simulation system. Among the available models, there are continuous models [1] and discrete models [2]. Among which, the mass-spring system is widely-used [3] since it is the most intuitive and simple model for efficient animation of clothes. The mass-spring model assumes that a cloth is composed of mass-points and springs. Each spring connects two mass points, and the structure of the spring connection is based on the geometry topology of the cloth mesh. The motion of the cloth is determined by interactive forces between the masses. The modeling requires numerically solving an ordinary differential equation (ODE) system as illustrated in Eq. 1. In order to obtain the evolution of cloth animation along time, the numerical system has to be integrated numerically. Various numerical integration methods are available for this task.

$$\begin{cases} x'(t) = f\left(t, x(t)\right) \\ x(t_0) = x_0 \end{cases} \tag{1}$$

The main concern of our work is to briefly review the integration techniques. The algorithms are analyzed in Section 3. Section 4 shows experimental results of the integration methods. Finally, conclusions are drawn from the experimental results.

2. Related work

According to Volino *et al.* [4], integration techniques are classified into four major categories: explicit methods, implicit methods, low-order methods and high-order methods. Provot [3] used explicit Euler method to integrate the Newton equation and obtain the positions and velocities at all instances of time. As stated by Baraff and Witkin [5] explicit methods are ill-suited to solve stiff equations because they require many small steps to stably advance the simulation forward in time. Therefore the iteration times are increased dramatically slowing down the simulation procedure. The 4th order Runge-Kutta method is often a better compromise between accuracy and speed. For higher order methods, the stability is poor for non-linear condition. They cannot efficiently handle situation with collision detection and response. In order to increase the stability, the stiffness of the system has to be lowered down which causes artifacts namely the so-called "Super Elasticity".

Pioneered by Baraff and Witkin [5], implicit integrations have been widely used for cloth simulation because the stability is better than the explicit methods. In their approach, modified conjugate gradient (CG) method was used to alleviate the computation. In order to increase the speed, Desbrun *et al*. [6] make some approximations in implicit technique alleviating the burden to solve the linear system by precomputed filter. It did speed up the simulation however it still suffers from heavy computation because the inverse matrix may not be sparse. Based on this work Kang *et al*. [7] did some further simplification to eliminate solving the linear system. Nevertheless the physical correctness was not considered. Specific materials may not be modeled correctly in this way. Eberhardt *et al*. [8] proposed the use of IMEX (implicit-explicit) methods to solve the arising problems. IMEX methods split ODEs (ordinary differential equations) into stiff parts

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and non-stiff parts solved by implicit and explicit integration respectively. The criteria for evaluating integration techniques are convergence, accuracy, stability and efficiency [9]. Various integration methods have been compared with regard to their speed, accuracy and efficiency [4,10,11].

A post correction method was used to restrict the spring elongations to a maximum value [3,6,12]. Unfortunately physical soundness is not guaranteed by this modification. Baraff and Witkin [5] proposed a very efficient method to enforce constraints in a CG method. The acceleration is filtered at the constrained direction under each iteration by mass modification. However these methods did not present how to determine the order in which the springs are adjusted. The result of the inverse dynamics process is highly dependent on this order. Tsiknis *et al*. [13] proposed an order-independent method for strain limiting using element-by-element creating a disjoint set of particles then stitch them together. In order to speed up massive simulation, the author adopted physics-aware subdivision scheme.

3. Algorithms

3.1 Physical model of garment

The present work employs a mass-spring system [3] for garments based on triangular mesh. Tension-shear springs and bend springs are constructed according to the topology of the mesh. Each edge of the triangular mesh that connected two vertices is treated as springs for tension and shearing, and the line that connected two vertices across each edge is considered as bending spring as illustrated in Figure 1.

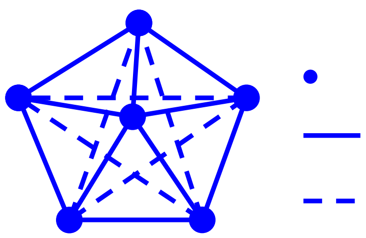

Figure 1 Mass-spring system with Tension-Shear. springs (solid lines), and bending springs (dash lines).

The forces exerted on the masses are classified into external forces and internal forces. External force $\boldsymbol{F}_{ex}$ of each particle including omnipresent gravity and air resistance is expressed as

$$\boldsymbol{F}_{ex} = m\boldsymbol{g} + \left(a + b\|\boldsymbol{v}_i\|\right)\boldsymbol{v}_i \quad .$$

where m denotes mass of each particle, $\boldsymbol{g}$ is acceleration of gravity, a represents linear air resistance coefficient and b is quadric air resistance coefficient, $\boldsymbol{v}_i$ is velocity of ith mass.

Internal force $\boldsymbol{F}^{inter}_{ij}$ are caused by non-linear springs between two masses i and j at position $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ in the form of:

$$\boldsymbol{F}^{inter}_{ij} = k_{ij}\left(\|\boldsymbol{x}_i - \boldsymbol{x}_j\| - l_{ij}\right)\frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|} + d_{ij}\frac{<\boldsymbol{v}_i - \boldsymbol{v}_j, \boldsymbol{x}_i - \boldsymbol{x}_j>}{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2}\left(\boldsymbol{x}_i - \boldsymbol{x}_j\right)$$

k_{ij} denotes elastic modulus of this spring and l_{ij} is its rest length. d_{ij} is damping coefficient. The type of spring determines its spring constant. Structural springs usually possess higher spring constant values than shear and bending ones. Damping forces are added to account for energy dissipation due to internal friction. These forces damp out the garment's kinetic energy according to its relative velocity. The most popular model used as damping force $\boldsymbol{F}^{d}_{ij}$ of each particle is

$$\boldsymbol{F}^{d}_{ij} = -d_{ij}\left(\boldsymbol{v}_i - \boldsymbol{v}_j\right).$$

The linear terms are particularly suited for numerical integration. In order to overcome the artifacts caused by high damping forces, we rectify the damping model according to the method reported by Hauth *et al* [9]. The linear damping force was projected onto the direction of the spring to alleviate this phenomenon. Random sequential order has been taken which mitigates the order-dependent problems from this point of view.

3.2 Numerical methods

To animate the mass-spring model mentioned above the numerical integration methods are needed for computing the new velocity and position. The numerical integrators considered in this study are explicit Euler, Symplectic, Midpoint, RK4, Verlet, implicit method [5] and Semi-implicit method [6]. The algorithms are as follows: Explicit Euler methods are described in Eq.2 [3]. The accuracy of this method is $O\left(h^2\right)$.

$$\begin{pmatrix}\Delta \boldsymbol{x} \\ \Delta \boldsymbol{v}\end{pmatrix} = h\begin{pmatrix}\boldsymbol{v}_0 \\ \boldsymbol{M}^{-1}\boldsymbol{f}_0\end{pmatrix} \tag{2}$$