Journal of Fiber Bioengineering & Informatics 4:1 (2011) 59–69
http://www.jfbi.org | doi:10.3993/jfbi04201107

Investigation on Effective Heat Conductivity of Fibrous Assemblies by Fractal Method

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Abstract

A fractal model for predicting the effective heat conductivity of fibrous assemblies was established. In the model, fiber-to-fiber contact influence on solid heat conductivity was taken into consideration. Radiative heat conductivity was also considered to get the effective heat conductivity. The effective heat conductivity was proved to be related to the following parameters, including the pore area fractal dimension, tortuosity fractal dimension, maximum and minimum pore diameters, solid conductivity, air conductivity, porosity and the ratio of the number of perpendicular channels to the total number of channels. Experiment was conducted to verify the model, and good agreement was found between the experimental and theoretical results.

Keywords: Effective Heat Conductivity; Fibrous Assembly; Fractal; Fiber-to-fiber Contact; Radiative Heat Conductivity

1 Introduction

The study of the effective heat conductivity of fibrous assemblies has received continuous attention [1-3] due to their various applications in clothing and engineering. Numerous researchers have worked on the effective heat conductivity calculation of fibrous assemblies [4-6]. These studies are all based on the assumption that the fibrous assemblies are a continuous medium, which makes it difficult to consider the influence of microstructure of pores, and thus the application of these existing theories has some fundamental limitations.

Fractal theory has been applied to study the thermal conductivity of porous media [7-9]. Chen et al [8] developed a fractal model to study effective heat conductivity of soil. Later, Yu et al [9] proposed fractal models to calculate the effective thermal conductivity of mono- and bi-dispersed porous media, such as sandstone and particles etc. The effective heat conductivity of wood [10], foam [11] and other objects were also discussed by some researchers [12, 13]. The above models

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April 2011

established were based on the assumption that the objects investigated are exactly self-similar, which is not in accordance with real porous media. For real porous media, the microstructure are statistically self-similar. Kou et al [14] investigated the effective heat conductivity of fibrous materials under this condition. The model assumed that the air and fibers are in parallel arrangement in the fibrous materials, which neglects the fiber-to-fiber contact effect on the effective heat conductivity. The previous models also did not take radiative heat conductivity into consideration.

In this paper, we will use the fractal theory to calculate the effective heat conductivity of fibrous assemblies. Fiber-to-fiber contact effect on the property was taken into consideration. What is more, radiative heat conductivity was also considered. Experiment is conducted to verify the model.

2 The Effective Heat Conductivity Calculation by Fractal Method

2.1 Microstructure and Fractal Description of Porous Media

As is known, an object measurement is related to its dimension and is invariant with the unit of measurement used. In general, ordered objects such as points, lines, surfaces and cubes can be described by Euclidean geometry using integer dimension 0, 1, 2 and 3, respectively. However, it is found that numerous objects in nature, such as rough surfaces, coastlines, mountains, rivers, lakes and islands are disordered and irregular, and they cannot be described by the Euclidean geometry because of the scale-dependent measures of length, area and volume. These objects are called fractals, and the dimensions of such objects are non-integral and defined as fractal dimensions. A fractal object measurement $M(L)$ is related to the length scale$L$ by the following power form [15].

$$M(L) \sim L^{D_f} \tag{1}$$

Where the "$\sim$" should be read as "scale as". M can be the length of a line, the area of a surface, the volume of a cube, or the mass of an object. D_f is the fractal dimension of the object, $0 < D_f < 2$ in two dimensions. For real porous media, the size distribution of pores satisfies the fractal power law [8, 9],

$$N(L \geq p_{\min}) = \left(\frac{p_{\max}}{p}\right)^{D_f} \tag{2}$$

Where p, $p_{\min}$ and $p_{\max}$ is the pore size, minimum pore size and maximum pore size respectively.

The number of pores within the infinitesimal range p to $p+dp$ can be deduced by a differentiating equation (2) with respect to p.

$$-dN = D_f p_{\max}^{D_f} p^{-(1+D_f)} dp \tag{3}$$

Dividing Eq. 3 by 2, Eq. 4 is obtained:

$$\frac{-dN}{N} = D_f p_{\min}^{D_f} p^{-(1+D_f)} dp = f(p)dp \tag{4}$$