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Analytical Determination of Electric Voltage for Pressure-driven Flow Through Complex Microchannels

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Abstract

An analytical determination of electric voltage generated by the pressure-driven flow through complex microchannels was analyzed based on the fractal theory. The pressure-driven flow through a complex microchannel with consideration of electrokinetic phenomena is described by the momentum and Poisson-Boltzmann (P-B) equations. The solution of induced electric field strength and electric voltage across complex microchannels are obtained using the fractal theory and technique, which are the function of dimensionless electroosmotic radius and the porosity. The results obtained show that the analytical results are agreed well with the experimental data.

Keywords: Fractal Theory; Electrical Double Layer; Electric Voltage; Porosity

1 Introduction

Complex microchannels are widely used in several engineering areas including thermal insulation, heat exchangers, filtration and separation of particles, composite fabrication, and medical science. Most solid surfaces carry electrostatic charges which produces an electrical surface potential. If the liquid contains a very small number of ions in the liquid. The rearrangement of the charges on the solid surface and the balancing charges in the liquid is called the Electrical Double Layer (EDL). It has been proved that the flows in micro-scale are quite different from those in macro-scale, suggesting that the electrokinetic phenomena caused by EDL effects must be taken into account in micro-channel flows [1-5].

The disordered nature of pore structure in complex microchannels shows the use of a fractal structure formed by both micro-pores inside the porous fibrous materials [6]. Applications of the fractal theory to analyze transport properties of complex microchannels in science and engineering have received increasing attention in the past two decades [7, 8]. A fractal geometry model for evaluating permeability of porous textile is studied, which is used in liquid composite molding [9]. A mathematical model for the coupled heat and mass transfer in complex microchannels is established based on the fractal characters of the pore size distribution [10].

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The notion of generating electrical power in this way is an old topic, yet has received renewed attention in the context of the voltage generation by pressure-driven through complex microchannels. Pressure-driven flow through a microchannel induces a flow of charges in the double layer at the channel walls, generating an electrical current. The efficiency of electrical power generation is discussed in individual rectangular nano-channels by means of streaming currents and the pressure-driven transport of counter ions in the EDL [11]. A mathematical model is presented to describe resistance effects of the EDL in porous textiles on the coupled heat and liquid moisture transfer [12]. The fractal dimensions and the micro-structural parameters of fractal complex microchannels are analyzed to study the mass transfer for fluid flow [13].

There were rarely works analyzing the electric voltage generated by pressure-driven flow through complex microchannels on the basis of the fractal analysis of pore microstructures. Taking into account the electrokinetic effect, the momentum equation describing the pressure-driven flow as well as the P-B description of induced electric potential distributions is presented. The fractal model of the electric voltage across complex microchannels generated by pressure-driven flow is derived based on the fractal characters of complex microchannels. It is found that the electric voltage is a function of porosity, the dimensionless local averaging net charge density, the dimensionless electroosmotic radius, length of the complex microchannel, the pressure across the complex microchannel, the solid surface zeta potential and so on. Furthermore, the predicted model for voltage shows good agreements with experimental data, illustrating that the analytical determination for the electric voltage across the complex microchannels is satisfactory, which can be applied to measure the actual voltage across complex microchannels and adjust the magnitude through changing the concerning parameters in microfluidic system.

2 Analysis of Mathematical Model

Firstly we considered the steady flow of an electrolyte solution through a single microchannel. Based on the P-B equation, the electric potential ψ can be expressed as [6]:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr}\right) = -\frac{\rho_e}{\varepsilon} \tag{1}$$

The net charge density is given as [14]:

$$\rho_e = -2\chi \mathrm{en}_0 \sinh\left(\frac{\chi e\psi}{k_b T}\right) \tag{2}$$

where n_0 and χ are the bulk ionic concentration and the valence of ions, respectively, ε is the permittivity or the dielectric constant, e is the charge of an electron, k_b is the Boltzmann constant and T is the absolute temperature. By using following non-dimensional quantities $\bar{\psi} = \frac{\psi}{\zeta}$, $\bar{r} = \frac{r}{r_c}$, $\bar{\rho}_e = \frac{\rho_e}{-\varepsilon\zeta/r_c^2}$, where ζ denotes the Zeta potential on the surfaces of channelwall and r_c is the pore radius of single microchannel, $k = \kappa r_c$ and k is the dimensionless electroosmotic radius, where $\kappa = \sqrt{2n_0 e^2\chi^2/\varepsilon k_b T}$.

Eq. (1) can be expressed as:

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d\bar{\psi}}{d\bar{r}}\right) = k^2 \sinh\left(\bar{\psi}\right) \tag{3}$$