

# 3-D Total Generalized Variation Method for Dynamic Cardiac MR Image Denoising<sup>\*</sup>

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## Abstract

Total Generalized Variation (TGV) regularization model is one of the most effective methods for MR image denoising. However, for 3D dynamic MR image, the TGV regularization model cannot use the correlated information of each slice. Therefore, in order to effectively denoising the dynamic MR image, 3D Total Generalized total Variation (3D-TGV) is proposed to denoise different kinds noise in the dynamic MR image. Experimental results show that, compared with the Total Variation (TV) and Total Generalized Variation (TGV), the proposed 3D TGV method has a better performance, and can significantly improve the denoising effect, with higher Signal-to-noise Ratio (SNR) and fewer artifacts.

*Keywords:* 3D Total Generalized Variation (3D-TGV); Dynamic MR Imaging; Denoising; Total Variation (TV)

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## 1 Introduction

With the increasing demands of the magnetic resonance imaging in clinical applications, how to minimize the effect of the noise in magnetic resonance imaging plays an important role. As we know, many denoising procedures employ some sort of regularization method to improve the quality of magnetic resonance imaging. For example, Rudin et al. [1] applied the total variation regularization model (Total Variation, TV) to solve the denoising problem occurred in the procedure of magnetic resonance imaging, which is based on the partial differential equations (PDF) variation denoising model. The main benefit of TV model is that it can remove random

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noise very well, while preserving the edges in the image. However, the TV method needs meet the requirement that the MR images are piecewise constant. Due to inhomogeneities of the exciting B1 field of high field systems with 3T, this requirement cannot be met in practical MRI examinations [2]. Additionally, due to the assumption of piecewise constancy, the TV-based denoising method often leads to staircasing artifacts and results in patchy and cartoon-like images [3], which appears unnatural. To overcome the shortcoming, Higher Degree Total Variation (HDTV) [4], Group-sparsity Total Variation (GSTV) method [5], and Total Generalize Variation (TGV) [6] are used to replace TV as the penalty [7, 8] for denoising MR image. In most of the existing dynamic MR image denoising methods, 2D/1D transforms were applied to solve the 3D dynamic problem, which treats the 3D data as a series of 2D images and then unfold the 3D dataset into a 2D matrix to explore the spatiotemporal redundancy [9–12], the quality of the denoising image will be significantly affected. In order to improve the denoising performance of dynamic MR image, an extend TGV method, i.e., 3D-TGV, is proposed to directly denoise the 3D MR image data, which can significantly improve edge-preserving and noise-removing not only 2D-TGV but also 3D-TV. In this paper, some experiments were designed to prove the superiority of the proposed method in denoising dynamic MR image.

## 2 Method and Theory

### 2.1 TV Denoising Method

#### (1) 2D-TV method.

As we know, denoising the contaminated MR image can be acted as an inverse problem, and the noisy image which is defined as follows:

$$f(x) = u(x) + v(x) \quad (1)$$

where  $u(x)$  is the image to be restored,  $v(x)$  is the noise and  $f(x)$  is the noisy image. The famous total variation filtering, known as the ROF model [1], minimizes the cost functional as follows

$$u = \arg \min_{u \in BV(\Omega)} \{F(u) + \lambda R(u)\} \quad (2)$$

where  $F(u)$  is the data fidelity term,  $R(u)$  is the regularization term, and  $\lambda > 0$  is a regularization parameter to balance  $F(u)$  and  $R(u)$ .  $\lambda$  is determined by both the nature of image and the intensity of noise. The 2D total variation is used as regularization term, which is defined as

$$TV(u) = \int_{\Omega} |\nabla u| dx = \sup_v \left( \int_{\Omega} u \operatorname{div} v \mid v \in C_c^1(\Omega, R^2), |v| \leq 1 \right) \quad (3)$$

The total variation of  $u$  is also known as the Bounded Variation (BV) of  $u$ , and BV denotes the space of functions  $u$  for which the  $TV(u)$  is finite.

#### (2) 3D-TV method.