ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 1, No. 1, 2006, *pp*. 53-60

Image Inpainting Based on Smooth Level Lines

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(*Received April 01, 2005, accepted Oct. 23, 2005*)

Abstract. This paper gives a new inpainting model based on curve and horizontal line evolution. By connecting the level lines that reach the edges and let the connected lines have minimum curvature, the model can finish the task successfully. What's more, the algorithm has no request on the topology of the inpainting area and can deal with all areas simultaneously.

Keywords: image inpainting, level line, curve evolution, total variation

1. Introduction

Image inpainting is to repair the damaged area in an image and make it intact. It has many important applications in the area of digital image processing, visual analysis and film industry, including repairing the old photos, removing the scratches in the old films, removing the redundant texts and objects in some scenes, image magnifying, image encoding, disocclusion and so on. Image inpainting has many affiliations with other subjects in the field of image processing, Such as image interpolation, image replacement, texture composition and error concealing. There exist many technologies to inpaint image. The frequent used methods include nonlinear filter, Bayesian methods, wavelets and spectrum analyzing methods and study and growth method mainly used in texture image.

Image inpainting is first proposed as a formal research subject by Bertalmio，Sapiro，Caselles and Ballester [4]. They put forward an effective algorithm and afterwards, they gives an improved model [5], extending the grey and grads information into the inpainting area. Tony Chan, Shen and Esedoglu give a systematic research and analysis and theorize the subject. They use the total variation and Mumford-Shah model to impaint image. As these methods violate the connection principle and producing corner, they advanced a CDD model by analyzing the curvatures on some special points, and put forward a Mumford–Shah–Euler model combining Euler elastic model [11][7]. The disocclusion method proposed by Masou and Morel can also be used to image inpainting.

The methods mentioned above are mainly applied to non-texture images. As to the processing methods appropriate for texture image, you can refer to [14][15]. But these methods require the users to give the texture to be filled with. When the structure is complex, it needs lots of work and time-consuming. Though the search work can sometime be finished automatically, it is still time-consuming and need offering a lot of parameters.

Recently, Bertalmio and Vese [10] proposed a method to discompose image to texture and structure part, and to repair the two parts using corresponding theories. Sung Ha Kang，Tony Chan and Stefano Soatto[3] put forward a method repairing big regions in image. This method needs apparent structure characteristics and referenced images.

When referring to image inpainting, it's inevitably to discuss the inherent statistical model. The common partial differential equations can only be applied to non-texture image. The algorithm this paper proposed based on partial differential equation, so the model present here can be used only to repair non-texture image.

2. Image processing and curve evolution based on level

2.1. curve evolution

In [16], snake model is introduced, and use it as an active contour model to segment image. The basic idea of

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active contour model or Snake model is: to evolve a curve at the limit of a given image; at first we put the initial line in the vicinity of the object to be detected, and then the curve will shrink to the object, and stop at the edge of the object.

Osher–Sethian proposed a level set method [17] which involves curves in an inherent form. Motion curves C denoted inherently by a Lipschitz function ϕ: $C = \{(x, y) \mid \phi(x, y) = 0\}$, we can define ϕ as a function of the distance of every point in its domain to the curve C, inside the curve we use negative value, and, outside, positive.

Let curve be defined as $C(t) = \{x(t), y(t)\}, t \in [0,1]$. Thus, the general form of the curve involve can be denoted as: $\frac{dC}{dt} = v\vec{N}$, $\vec{N}$ is the normal line of curve C, this is, the curve move at the speed of v along the direction of the normal.

Suppose evolve ϕ, and its zero set indicates the evolving curve, then the evolving curve can be denoted as $\frac{\partial \phi}{\partial t} = v|\nabla \phi|$. It's easy to get their relation [9]:

$$\frac{d\phi}{dt} = < \nabla \phi, C_t > = < \nabla \phi, v\vec{N} > = v < \nabla \phi, \frac{\nabla \phi}{|\nabla \phi|} > = v|\nabla \phi| 。$$

When the level set function evolves, its inherent curve automatically changes its topological structure. There exists a special form, this is, mean-curvature evolving model, where $v = div(\nabla \phi(x, y) / |\nabla \phi(x, y)|)$, is curvature of the level line crossing the point (x, y). The equation becomes:

$$\begin{cases} \frac{\partial \phi}{\partial t} = div\left(\nabla \phi(x, y) / |\nabla \phi(x, y)|\right)|\nabla \phi| & t \in (0, \infty), (x, y) \in R^2 \\ \phi(0, x, y) = \phi_0(x, y) & (x, y) \in R^2 \end{cases}$$

The active contour curve evolving model based on mean-curvature motion model, which is used to image segmentation [18], can be denoted as:

$$\begin{cases} \frac{\partial \phi}{\partial t} = g\left(|\nabla u_0|\right)|\nabla \phi|\left(div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) + v\right) & t \in (0, \infty) \\ \phi(0, x, y) = \phi_0(x, y) & (x, y) \in R^2 \end{cases}$$

Where $v \geq 0$ is constant; ϕ_0 is the initial level set function; u_0 is the image to be segmented; $g\left(|\nabla u_0|\right)$ is the edge detecting function, $g(z) > 0$ and is decreasing function, depending on the grade of the image, and $\lim_{z \to \infty} g(z) = 0$. The ideal condition is that $g\left(|\nabla u_0|\right) = 0$ at the edges, thus the curves stop evolving at the edges.

2.2. Image processing based on level line

Level line plays an important role in image understanding and representation. Let $u(x)$ be the grey value of image u at point x. For every grey value λ, the level set $x_\lambda u$ can be defined as: $x_\lambda u = \{x, u(x) \geq \lambda\}$, and level line can be defined as the boundary of $x_\lambda u$. Then, an image can be constructed by all its level lines. And this representation cannot be influenced by illumination intensity. As man is sensitive only to grey levels in the image, this representation is important. Grey value is not a reliable characteristic, for the difference in grey values of two pixels varies along with illumination conditions. So we should use these special relations among level lines when we process natural images. This level line representation allows image contrast varying. Based on this analysis, Masnou and Morel propose a disocclusion method [1], they connect corresponding level lines which cross the "T" points that covering objects intercourse. They realize that by minimizing the following function:

$$\begin{cases}\int_\Omega | \, |Dv|(1+|curv|)dx \\ v = u \qquad\qquad x \in D\end{cases}, \quad \begin{cases}\int\int |Dv|() \\ v = u\end{cases}$$

Ω is the domain of image， v is the set of all possible image functions. u is the given image, D is the covering area, $curv$ the curvature of level line.

2.3. Image inpainting based on level lines

In the following, we use the idea of curve evolution to smooth the level lines, and then we can repair images. Sethian and Malladi [12][13] give some methods to smooth all the level lines in smooth images. They make the corresponding level lines evolve in some ways that are influenced by the intensity variation of grey values. These methods can be used to image smoothing, denoising, enhancement, shape-evolving and representation, and at the same time, remain important edges in images.

Let the inpainting domain of the image is D, E is a close domain in D^c [6], E contains the image information in the vicinity of the inpainting area. Just like other inpainting methods, we extend the image information in E to the inpainting area. Like curve evolution and at the restriction of the image information in E, we make all the level lines evolve in the inpainting domain, and stop in some conditions. That is, we use a restricted level line evolution to repair image, the equation is:

$$\begin{cases} u_t = K|\nabla u| & x \in D \\ u = u_0 & x \in E \end{cases} \tag{3.1}$$

where k is the curvature of the level line at point (x,y), and

$$k = div\left(\nabla u / |\nabla u|\right) = \frac{u_x{}^2 u_{xx} + 2u_x u_y u_{xy} + u_y{}^2 u_{yy}}{u_{xx}{}^2 + u_{yy}{}^2}.$$

Level lines evolution of this form let the level lines to evolve at the speed proportional to the curvature and stop when the curvature of level lines is zero. Thus, this method can connect the level lines that reach the inpainting domain, and finish the task of repairing images.

The method proposed here repairs images by calculating and minimizing the curvature of every point in the inpainting domain directly. This inpainting method based on level lines can allow the repaired image have sharp edges. Just like the optimization questions of some other grade decent methods, the choice of initial values is very important. In practical use, we first initialize the inpainting domain by smooth inpainting method. The equation is:

$$\begin{cases} u_{i,j}^{k+1} = 1/4\,(u_{i-1,j}^k + u_{i+1,j}^k + u_{i,j-1}^k + u_{i,j+1}^k) & (i,j) \in D \\ u_{i,j}^{k+1} = u_{i,j}^0 & (i,j) \in E \end{cases}。$$

On the one hand, smooth inpainting method cannot be influenced by the geometric information, and it can extend grey values around inpainting domain into its inside smoothly, therefore, it fully uses the grey information. On the other hand, this algorithm is very simple and converges fast. It gives an analogy of the repaired image and uses it as the input of next processing. We then use the level line evolution equation (3.1) to iterate the image, so that connected level lines have minimum curvature.

Tony Chen and Shen in [6] proposed three important principles of low-level image inpainting algorithms:

(1) An inpainted image is total determined by the information E in the vicinity of inpainting domains.

(2) They can connect broken edges when they are comparably narrow.

(3) Robust to noise, that is, they can extend image information to inpainting domains in despite of noises.

The algorithm proposed here naturally satisfies the first one. It also satisfies the second one, and it let the inpainted image have sharp edges(from fig.2).

In most circumstances, original image contains noise, and an appropriate process is to embed denoising in the inpainting process. This process is natural when man's eye obtain information from image with noises, and at the same time it can extend them to inpainting areas.

To overcome the influence of noise for image inpainting and considering the above discussion, we establish a inpainting model. Inside inpainting domains, it can inpaint image using level-line smoothing inpainting, and outside, it can evoke Rudin-Osher-Fatemi TV denoising model[2]:

$$\frac{\partial u}{\partial t}(or0) = \nabla\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda(u - u_0) \qquad x \in D^c 。$$

The two processes can be united into one equation:

$$\frac{\partial u}{\partial t}(or0) = \nabla\left(\frac{\nabla u}{|\nabla u|}\right)G(k,x) + \lambda_e(x)(u - u_0) \qquad x \in \Omega, \quad \Omega \text{ is the domain of image.}$$

Where $G(k,x) = \begin{cases} 1 & x \in D^c \\ |\nabla u| & x \in D \end{cases}$, $\lambda_e = \begin{cases} \lambda & x \in D^c \\ 0 & x \in D \end{cases}$,

Just like the denoising model discussed in [2], λ can be estimated according to the noise in the image. At $\partial\Omega$, the boundary condition is determined by TV denoising model. Thus, a natural choise is Neumann boundary condition[2].

2.4. Experiment and Result

This paper proposes an algorithm of iterating a partial differential equation to mend images. This algorithm is simple, and time complexity is low, far less than above presented ones. But its repair results is comparable to some known algorithms, and looks better when repairing some images.

Figure 1, figure 2 and figure3 show the inpainting effect of our algorithm. From figure 1, we can see that our algorithm nicely repairs the texts and scratches in the image. Figure 2 is the enlargement of inpainting area of old image, and it compares the inpainting results of some inpainting algorithms with the result of our algorithm. We can see that the result of our algorithm makes the inpainting image has clearer edges. When the damaged edge is a bit long, our inpainting result has sharper edge than the one of TV model. Figure 3 comes from the paper[4], and gives our inpainting result.

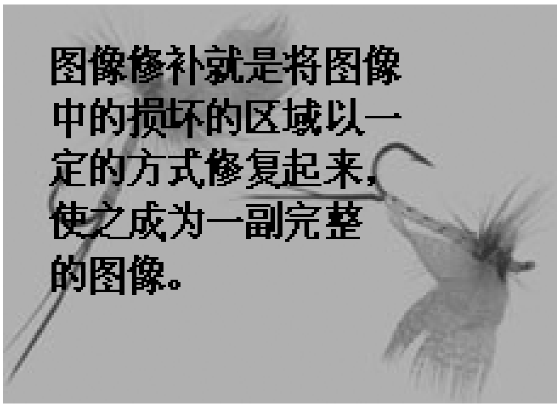
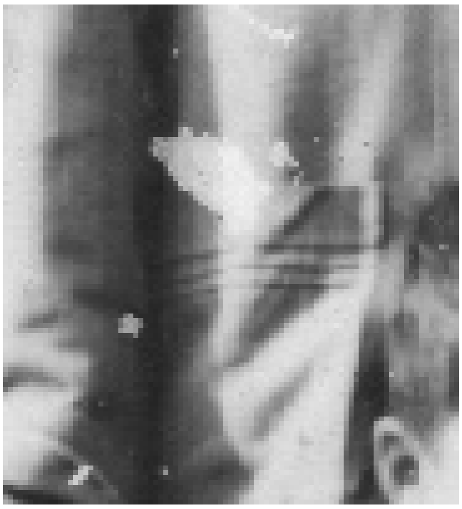


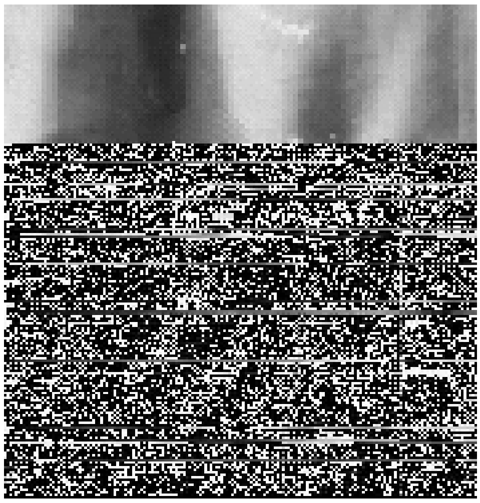
Image with texts

Inpainted image

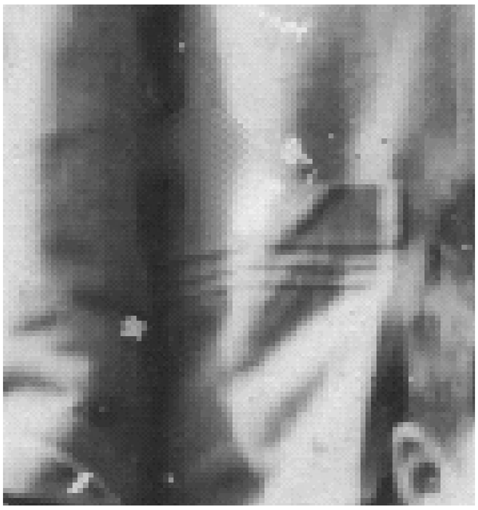
Fig. 1. The effect of removing texts in image



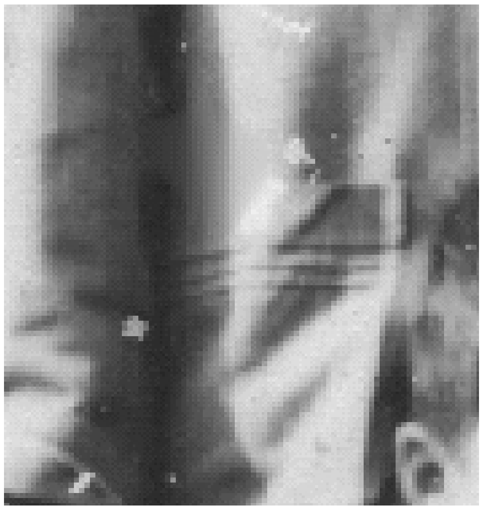
（1）original image



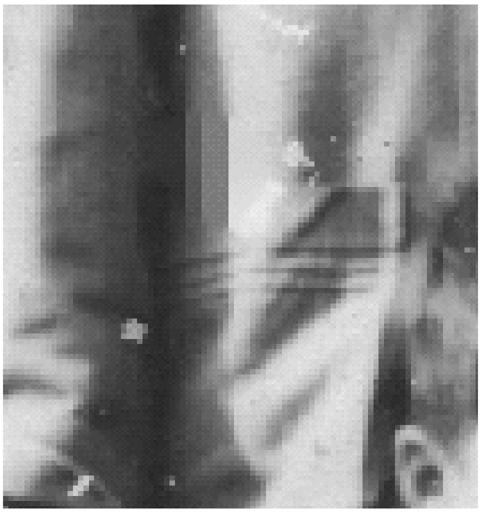
（2）smooth inpainting



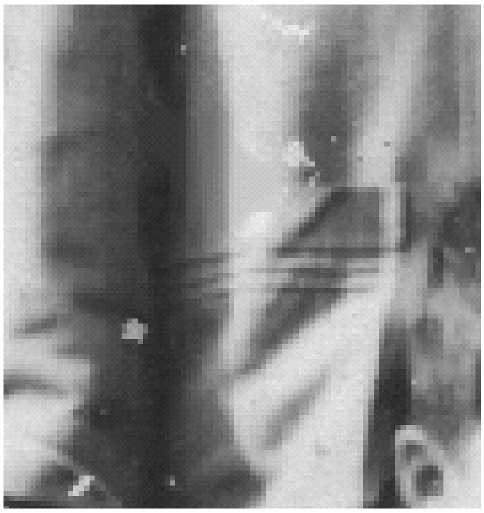
(3) Bertalmio algorithm



(4) TV inpainting model



(5) CDD algorithm



(6) Our algorithm

Fig.2. Contrast of image inpainting algorithms

Original image: damaged old photo

the result using Bertalmio algorithm

Result of our algoritm

Fig. 3 the comparison between Bertalmio's algorithm and ours

3. Acknowledgements

The support from Gao ShiSong during programming are deeply appreciated. We also thank Doctor Jiang to review the paper and give several comments and suggestions to improve the quality of the paper.

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