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Dynamical Behaviors of a new Chaotic System and its Adaptive Control

Guoliang Cai [+], Zhenmei Tan

Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, P.R. China

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Abstract. In this letter, a new chaotic system is discussed. Some basic dynamical properties, such as Lyapunov exponents, Poincaré mapping are studied. Based on Lyapunov stability theory, the new chaotic system is controlled by the method of adaptive backstepping. Numerical simulations show effectiveness and feasibility of this approach.

Keywords: new chaotic system, dynamical properties, chaotic control

1. Introduction

Since in 1963, Lorenz found the first chaotic attractor [1] in a three-dimensioned (3D) autonomous system when he studied atmospheric convection. In the past years, chaos has been found in many engineering systems, and more and more chaotic systems have been found [2-8].

Sensitivity to initial conditions is a fundamental characteristic of a chaotic system, so chaos control is crucial in application of chaos. It attracted a great deal of attention from various fields since Huber published the first paper on chaos control in 1989. Until now, many different methods have been proposed to achieve chaos control, such as OGY method [9], adaptive method [10], nonlinear feedback approach [11] and backstepping design [12]. Chaos has been found to be useful or has a great potential to be useful, in many disciplines such as information processing, collapse prevention of power systems, high-performance circuits and devices, thorough liquid mixing with low power consumption, and biomedical engineering applications in the research of human brain and heart. Recently, it has been noticed that purposefully creating chaos can be a key issue in many technological applications such as communication, encryption, etc.

In this paper, we propose a new nonlinear chaotic system, in which have four parameters, and two nonlinear terms. Its nonlinear term in the third equation is different from that of Lorenz system, Chen system, and Lü system. Some basic properties of the new system are studied. Based on Lyapunov stability theory, with the adaptive backstepping techniques, parameter identification and control of the uncertain new chaotic system can be achieved simultaneously with only one controller. This method overcomes the singularity problem caused by nonlinear terms in new system, and it has a great potential in application.

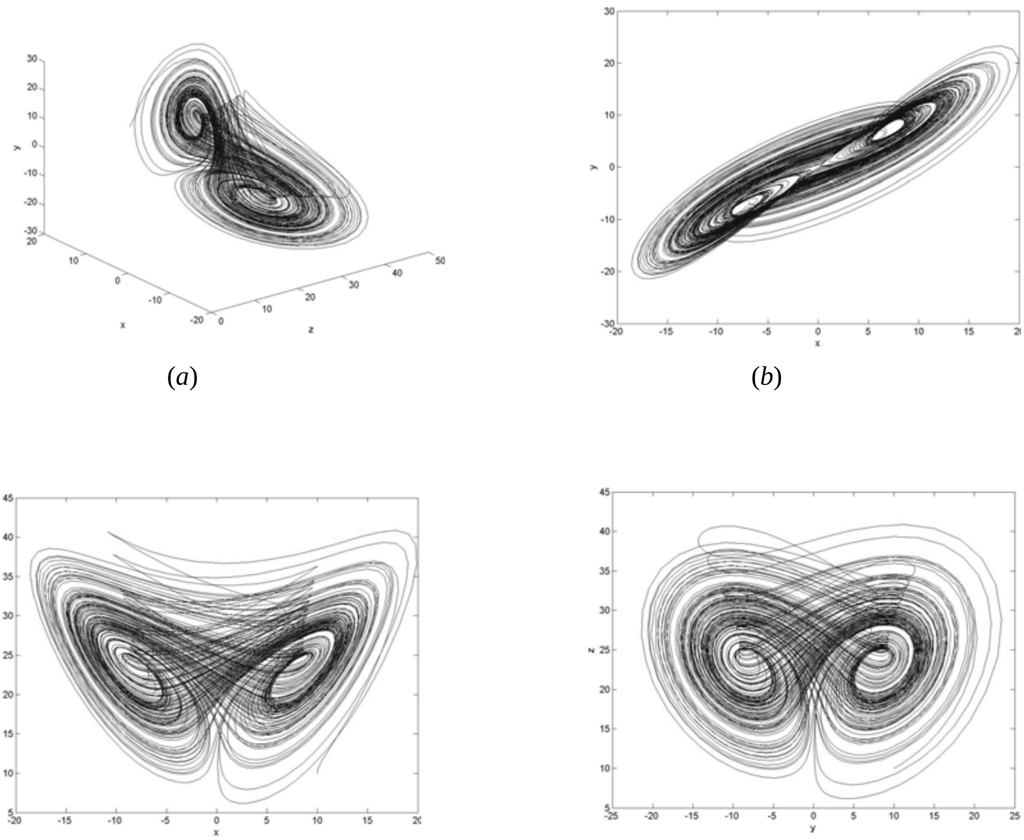
2. A new chaotic system

A new nonlinear chaotic system is proposed in this letter, the autonomy differential equations that describe the system are

$$\dot{x} = a(y-x), \dot{y} = bx + cy - xz, \dot{z} = y^2 - hz \tag{1}$$

System (1) has a chaotic attractor shown in Fig. 1, when a = 27.5, b = 3.5, c = 19.3, h = 2.9.

+ Corresponding author. Tel.: +86-511-8879 1998; fax: +86-511-8879 1467.
E-mail address: glcai@ujs.edu.cn..



(*a*) (*b*)

(*c*) (*d*)

Fig. 1: The new attractor. (*a*) Three-dimensional view; (*b*) *x-y* phase plane strange attractor; (*c*) *x-z* phase plane strange attractor; (*d*) *y-z* phase plane strange attractor.

2.1. Some basic properties

2.1.1. Symmetry and invariance

Note that the invariance of the system (1) under the transformation $(x, y, z)\rightarrow(-x, -y, z)$,i.e. under reflection in the *z*-axis. The symmetry persists for all values of the system parameters.

2.1.2. Dissipativity and the existence of attractor

For system (1), one has

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a + c - h = -11.1$$

So system (1) is dissipative, with an exponential contraction rate: $dv/dt=e^{-11.1.}$. That is a volume element V_0 is contracted by the flow into a volume element V0e-11.1t in time t. This means that each volume containing the system trajectory shrinks to zero as $t\rightarrow\infty$ at an exponential rate -11.1. In fact, numerical simulations have shown that system orbits are ultimately confined into a specific limit set of zero volume, and the system asymptotic motion settles onto an attractor.

2.2. Equilibria and stability Equilibria and stability

The equilibria of system (1) can be easily found by solving the three equations $\dot{x} = \dot{y} = \dot{z} = 0$, which lead to $a(y-x)=0, bx+cy-xz=0$, and $y^2 - hz = 0$. It can be easily verified that there are three equilibria: S_0 (0, 0, 0), S_1(8.1314,8.1314,22.8), S_2(-8.1314,-8.1314,22.8), in which two equilibria S_1 and S_2 are symmetrically placed with respect to the *z*-axis.

For equilibrium S_0 (0, 0, 0), system (1) is linearized, the Jacobian matrix is defined as

$$J_0=\begin{pmatrix} -a & a & 0 \\ b-z & c & -x \\ 0 & 2y & -h \end{pmatrix}=\begin{pmatrix} -27.5 & 27.5 & 0 \\ 3.5 & 19.3 & 0 \\ 0 & 0 & -2.9 \end{pmatrix}$$

To gain its eigenvalues, we let $|\lambda I - J_0| = 0$

These eigenvalues that corresponding to equilibrium S_0 (0, 0, 0) are respectively obtained as follows:

λ_1= -29.4734, λ_2= 21.2734, λ_3= -2.9.

Here λ_2 is a positive real number, λ_1 and λ_3 are two negative real number. Therefore, the equilibrium S_0 (0, 0, 0) is a saddle point. So this equilibrium point S_0 (0, 0, 0) is unstable.

For equilibrium point S_1, has a Jacobian matrix equal to

$$J_1=\begin{pmatrix} -a & a & 0 \\ b-z & c & -x \\ 0 & 2y & -h \end{pmatrix}=\begin{pmatrix} -27.5 & 27.5 & 0 \\ -19.3 & 19.3 & -8.1314 \\ 0 & 16.2628 & -2.9 \end{pmatrix}$$

To gain its eigenvalues, we let $|\lambda I - J_1| = 0$

These corresponding eigenvalues of S_1 are

λ_1 = -15.7967, λ_2= 2.3483+14.9899*i*, λ_3= 2.3483-14.9899*i*.

Results show that λ_1 is a negative real number, λ_2 and λ_3 form a complex conjugate pair and their real parts are positive, so equilibrium point S_1 is a saddle-focus point, this equilibrium point is unstable.

For equilibrium point S_2 we think over corresponding linearization of state Eqs. (1), it has a Jacobian matrix equal to

$$J_2=\begin{pmatrix} -a & a & 0 \\ b-z & c & -x \\ 0 & 2y & -h \end{pmatrix}=\begin{pmatrix} -27.5 & 27.5 & 0 \\ -19.3 & 19.3 & 8.1314 \\ 0 & -16.2628 & -2.9 \end{pmatrix}$$

We let $|\lambda I - J_2| = 0$

These corresponding eigenvalues of S_2 are

λ_1= -15.7967, λ_2= 2.3483+14.9899*i*, λ_3= 2.3483-14.9899*i*.

Here λ_1 is a negative real number, λ_2 and λ_3 become a pair of complex conjugate eigenvalues with positive real parts. The equilibrium point S_2 is a saddle-focus point, this equilibrium point S_2 is also unstable.

2.3. Lyapunov exponents and Lyapunov dimension

As is well known, the Lyapunov exponents measure the exponential rates of divergence or convergence of nearby trajectories in phase space, according to the detailed numerical as well as theoretical analysis, the largest value of positive Lyapunov exponents of this chaotic system is obtained as λ_{L1}=2.0713. It is related to the expanding nature of different direction in phase space. Another one Lyapunov exponent λ_{L2}=0. It is related to the critical nature between the expanding and the contracting nature of different direction in phase space.

While negative Lyapunov exponent λ_{L3}= -13.1671. It is related to the contracting nature of different direction in phase space.

So, we can obtain the Lyapunov dimension of chaos attractors of system (1), it is described as

$$D_L=j+\frac{1}{|\lambda_{Lj+1}|}\sum_{i=1}^{j}\lambda_{Li}=2+\frac{\lambda_{L1}+\lambda_{L2}}{|\lambda_{L3}|}=2+\frac{2.0713+0}{|-13.1671|}=2.1573$$

The largest value of Lyapunov exponents of system (1) is large than zero, and the Lyapunov dimension of chaos attractors of system (1) is fractal dimension, therefore, there is really chaos in this system.

2.4. Waveform, Spectrum and Poincaré mapping

The waveforms of *x*(*t*) in time domain are shown in Fig. 2. The waveform of *x*(*t*) is nonperiodic. Its spectrum is continuous as shown in Fig. 3.

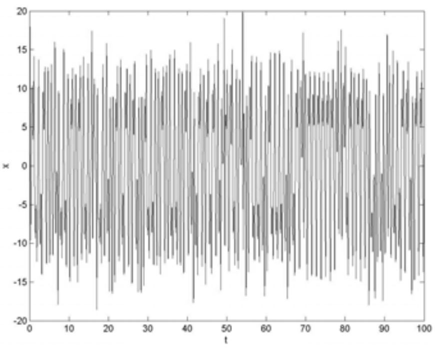


Fig. 2: $x(t)$ waveform

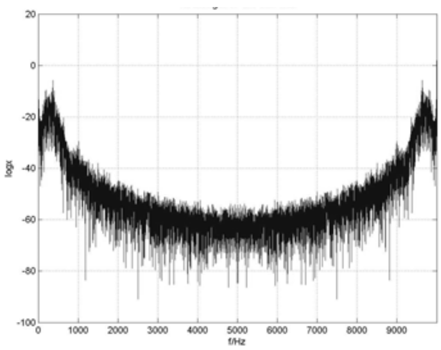


Fig. 3: Spectrum of $|x|$

Fig. 4 shows the Poincaré mapping. It is clear that some sheets are folded.

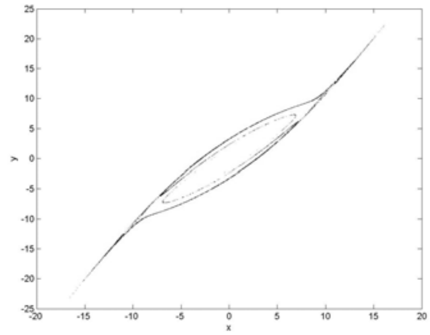


Fig. 4: The Poincaré map of x-y plane.

3. Adaptive backstepping control

The controlled system of system (1) is described as follows:

$$\dot{x} = a(y-x), \dot{y} = bx + cy - xz, \dot{z} = y^2 - hz + u \tag{2}$$

Where a, b, c, and h are unknown parameters to be identified, here we suppose $a > c > 0$, u is a controller to be designed.

There are three steps in the backstepping design procedure. At step i, an intermediate control function α_i should be developed using an appropriate Lyapunov function $V_i(t)$.

Step 1. Now we consider the first equation in system (2). We have

$$\dot{x} = ay - ax = a\overline{y} + a\alpha_1 - ax \tag{3}$$

Where $\overline{y} = y - \alpha_1(x)$, with α_1 being an artificial control to be defined later. Using α_1 as a control to stabilize the x_1-subsystem defined by Eq. (3), we can choose the following Lyapunov function:

$$V_1(t) = 2^{-1}x^2 \tag{4}$$

Calculating the derivative of $V_1(t)$ along system (2), we have

$$\dot{V}_1(t) = x\dot{x} = ax\overline{y} - a\left(1-\alpha_1\right)x^2 \tag{5}$$

Since $a > 0$, we can choose $\alpha_1 = px$, where $p < 1$. We have

$$\dot{x} = a\overline{y} - a\left(1-p\right)x \tag{6}$$

$$\dot{V}_1(t) = ax\overline{y} - a\left(1-p\right)x^2 \tag{7}$$

Step 2. In this step, we deal with the singularity problem caused by xz in the second equation of system (2).

Defining $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{h}$ as the estimates of a, b, c and h, and introducing the parameters errors $\overline{a} = \hat{a} - a$, $\overline{b} = \hat{b} - b$, $\overline{c} = \hat{c} - c$, $\overline{h} = \hat{h} - h$.

Then the derivative of $\bar{y}$ is expressed as:

$$\begin{aligned}\dot{\bar{y}} &= \dot{y} - \dot{\alpha}_1 = \dot{y} - p\dot{x} \\ &= -x(\bar{z} + \alpha_2) + \bar{b}x + p\bar{c}x + \bar{a}p(1-p)x - (ap-c)\bar{y} - (\bar{b} - b)x \\ &\quad -(p\bar{c} - pc)x - (\bar{a} - a)p(1-p)x \end{aligned} \tag{8}$$

Where $\bar{z} = z - \alpha_2$, with α_2 also being an artificial control to be defined later.

According to Eqs. (6) and (8), we obtain the

$$\begin{cases}\dot{x} = a\bar{y} - a(1-p)x \\ \dot{\bar{y}} = -x(\bar{z} + \alpha_2) + \bar{b}x + p\bar{c}x + \bar{a}p(1-p)x - (ap-c)\bar{y} - (\bar{b} - b)x \\ \quad -(p\bar{c} - pc)x - (\bar{a} - a)p(1-p)x \end{cases} \tag{9}$$

Using α_2 as a control to stabilize the $(x, \bar{y})$-subsystem (9). In the following we can consider a Lyapunov function $V_2(t)$ defined by

$$V_2(t) = V_1(t) + 2^{-1}\bar{y}^2 + 2^{-1}\bar{a}^2 + 2^{-1}\bar{b}^2 + 2^{-1}\bar{c}^2 \tag{10}$$

Its derivative is given by

$$\begin{aligned}\dot{V}_2(t) &= -a(1-p)x^2 - (ap-c)\bar{y}^2 - x\bar{y}\bar{z} + \bar{a}(\dot{\bar{a}} - (1+p-p^2)x\bar{y}) \\ &+\bar{b}(\dot{\bar{b}} - x\bar{y}) + \bar{c}(\dot{\bar{c}} - px\bar{y}) + x\bar{y}(-\alpha_2 + \hat{a}(1+p-p^2) + \hat{b} + p\hat{c})\end{aligned} \tag{11}$$

Choosing

$$\alpha_2 = \hat{a}(1+p-p^2) + \hat{b} + p\hat{c} \tag{12}$$

$$\begin{cases}\dot{\bar{a}} = (1+p-p^2)x\bar{y} - m\bar{a} \\ \dot{\bar{b}} = x\bar{y} - n\bar{b} \\ \dot{\bar{c}} = px\bar{y} - r\bar{c}\end{cases} \tag{13}$$

Where controlling parameters $m > 0$, $n > 0$ and $r > 0$. Then Eq. (11) can be rewritten as

$$\dot{V}_2(t) = -a(1-p)x^2 - (ap-c)\bar{y}^2 - x\bar{y}\bar{z} - m\bar{a}^2 - n\bar{b}^2 - r\bar{c}^2 \tag{14}$$

We can choose an appropriate p such that $ap-c > 0$. Since $a > c > 0$, it is sufficient to choose $\frac{c}{a} < p < 1$.

We will cancel the third term in the next step. By using Eq. (11), the Eq. (8) can be rewritten in the form

$$\dot{\bar{y}} = -x\bar{z} - \bar{a}(1+p-p^2)x - \bar{b}x - p\bar{c}x - ax - (ap-c)\bar{y} \tag{15}$$

Step 3. The derivative of $\bar{z}$ can be expressed as

$$\dot{\bar{z}} = \dot{z} - \dot{\alpha}_2 = y^2 - hz - (\partial\alpha_2 / \partial a)\dot{\bar{a}} - \left(\partial\alpha_2 / \partial\bar{b}\right)\dot{\bar{b}} - (\partial\alpha_2 / \partial\bar{c})\dot{\bar{c}} + u \tag{16}$$

Then we can get the following system by Eqs. (6), (15) and (16)

$$\begin{cases}\dot{x} = a\bar{y} - a(1-p)x \\ \dot{\bar{y}} = -x\bar{z} - \bar{a}(1+p-p^2)x - \bar{b}x - p\bar{c}x - ax - (ap-c)\bar{y} \\ \dot{\bar{z}} = y^2 - hz - (\partial\alpha_2 / \partial\bar{a})\dot{\bar{a}} - (\partial\alpha_2 / \partial\bar{b})\dot{\bar{b}} - (\partial\alpha_2 / \partial\bar{c})\dot{\bar{c}} + u\end{cases} \tag{17}$$

In the following we will choose an appropriate input u to stabilize the system (17). Considering the following Lyapunov function

$$V_3(t) = V_2(t) + 2^{-1}\bar{z}^2 + 2^{-1}\bar{h}^2 \tag{18}$$

Then we can obtain the derivative of $V_3(t)$

$$\begin{aligned}\dot{V}_3(t) &= \dot{V}_2(t) + \bar{z}\dot{\bar{z}} + \bar{h}\dot{\bar{h}} \\ &= -a(1-p)x^2 - (ap-c)\bar{y}^2 + \bar{h}(\dot{\bar{h}} + z\bar{z}) - m\bar{a}^2 - m\bar{b}^2 - r\bar{c}^2 + \bar{z}(u + y^2 \\ &\quad -\hat{h}z - x\bar{y} - (\partial\alpha_2 / \partial\bar{a})\dot{\bar{a}} - (\partial\alpha_2 / \partial\bar{b})\dot{\bar{b}} - (\partial\alpha_2 / \partial\bar{c})\dot{\bar{c}})\end{aligned} \tag{19}$$

We choose the following updated law

$$\dot{\bar{h}} = -z\bar{z} - k\bar{h} \tag{20}$$

Where $k > 0$.

Letting

$$u = -q\bar{z} + x\bar{y} - y^2 + \hat{h}z + (\partial\alpha_2/\partial\bar{a})\dot{\bar{a}} + (\partial\alpha_2/\partial\bar{b})\dot{\bar{b}} + (\partial\alpha_2/\partial\bar{c})\dot{\bar{c}} \tag{21}$$

Where $q > 0$.

According to Eqs. (12) and (13), the Eq(21) can be rewritten as:

$$\begin{aligned} u = &-y^2 - q\bar{z} + (h+\bar{h})(\bar{z}+\alpha_2) + x\bar{y}(p^2 + 2 + (1+p-p^2)^2) \\ &- m\bar{a}(1+p-p^2) - n\bar{b} - pr\bar{c} \end{aligned} \tag{22}$$

So we have

$$\dot{V}_3(t) = -a(1-p)x^2 - (ap-c)\bar{y}^2 - q\bar{z}^2 - m\bar{a}^2 - n\bar{b}^2 - r\bar{c}^2 - k\bar{h}^2 \tag{23}$$

i.e. $\dot{V}_3(t)$ is negative definite.

According to Eqs. (12),(13) and (21), Eq. (17) can be rewritten as

$$\dot{\bar{z}} = x\bar{y} + (\bar{h}-q)\bar{z} + (1+p-p^2)(\bar{a}+a)\bar{h} + (\bar{b}+b)\bar{h} + p(\bar{c}+c)\bar{h} \tag{24}$$

We can get the following ($x, \bar{y}, \bar{z}, \bar{a}, \bar{b}, \bar{c}, \bar{h}$)-system defined by Eqs. (6), (13), (15), (20) and (24):

$$\begin{cases} \dot{x} = a\bar{y} - a(1-p)x \\ \dot{\bar{y}} = -x\bar{z} - \bar{a}(1+p-p^2)x - \bar{b}x - p\bar{c}x - ax - (ap-c)\bar{y} \\ \dot{\bar{z}} = x\bar{y} + (\bar{h}-q)\bar{z} + (1+p-p^2)(\bar{a}+a)\bar{h} + (\bar{b}+b)\bar{h} + p(\bar{c}+c)\bar{h} \\ \dot{\bar{a}} = (1+p-p^2)x\bar{y} - m\bar{a} \\ \dot{\bar{b}} = x\bar{y} - n\bar{b} \\ \dot{\bar{c}} = px\bar{y} - r\bar{c} \\ \dot{\bar{h}} = -\bar{z}^2 - \bar{z}((\bar{a}+a)(1+p-p^2) + \bar{b} + b + p(\bar{c}+c)) - k\bar{h} \end{cases} \tag{25}$$

For system (25), according to Eqs. (4), (9) and (18), the Lyapunov function

$$V = V_3 = 2^{-1}x^2 + 2^{-1}\bar{y}^2 + 2^{-1}\bar{z}^2 + 2^{-1}\bar{a}^2 + 2^{-1}\bar{b}^2 + 2^{-1}\bar{c}^2 + 2^{-1}\bar{h}^2$$

Since V_3 is a positive function and $\dot{V}_3$ is negative definite, it follows that the system (25) is globally asymptotically stabilized at the equilibrium (0, 0, 0, 0, 0, 0, 0).

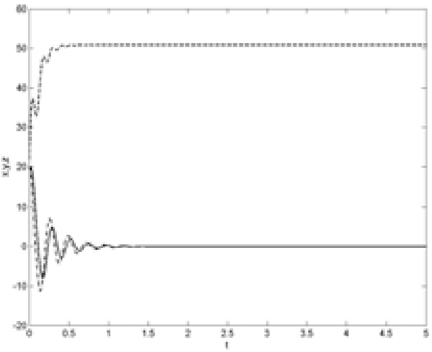
In view of x, $\bar{y} = y - \alpha_1(x)$ and $\alpha_1 = px$, we know that the states x and y go to zero asymptotically. From $\bar{z} = z - \alpha_2$ and Eq. (12), we have $z \to a(1+p-p^2) + b + pc$, as $t \to \infty$, i.e. z is bounded. At the same time, from Eq. (21), we can conclude that the control u is also bounded.

4. Numerical simulation

To verify the effectiveness of the above methods, computer simulations were carried out for the controlled new system. We select the initial conditions (x_0, y_0, z_0)=(20,20,20),($\hat{a}_0, \hat{b}_0, \hat{c}_0, \hat{h}_0$)=(10,10,10,10), ($m$,n ,r ,k)=(10,10,10,100), p=0.8, and q=10.

Figure 5 shows that states $x(t)$ and $y(t)$ of the controlled system (2) are asymptotically regulated to x= 0 and y = 0, and the state z remain bounded.

Figure 6 and Figure 7 show that the parameter estimates $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{h}$ and the control u are all bounded.



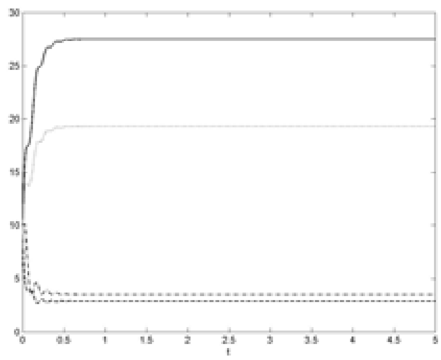


Fig. 5: *x*(-),*y*(-.) and *z*(--) of the controlled system (1) Fig. 6: estimated parameters $\overline{a}$ (-), $\overline{b}$ (-.), $\overline{c}$ (···), $\overline{h}$ (--)

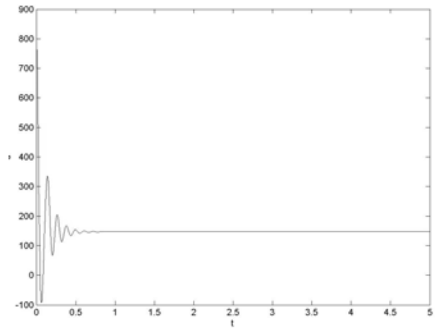


Fig. 7: control action *u*

5. Conclusion and discussion

A new nonlinear chaotic system is introduced and discussed in this paper. Some basic dynamical properties, such as Lyapunov exponents, Poincaré mapping, fractal dimension, continuous spectrum and chaotic behaviors of this new attractor are studied. The found of the new nonlinear chaotic system is an innovation in theory. With the adaptive backstepping techniques, control of the uncertain new chaotic system and parameter identification can be achieved simultaneously with only one controller. The numerical experiments show the feasibility and effectiveness of the method.

The new attractor and its forming mechanism need further study and exploration. A great deal of achievements will be obtained in the near further.

6. Acknowledgements

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7. References

[1] Lorenz E N, Deterministic nonperiodic flow. *J Atmos Sci*, 1963, **20**: 130-141.

[2] G. R.Chen and Ueta T, Yet another chaotic attractor. *Int J Bifure Chaos*, 1999, **9**: 1465-1466.

[3] J. H. Lü and G. R.Chen, A new chaotic attractor coined. *Int J Bifure Chaos*, 2002, **12**: 659-661.

[4] J. H. Lü, G. R.Chen and D. Z.Cheng , Bridge the gap between the Lorenz system and the Chen system. *Int J Bifure Chaos*, 2002, **12**: 2971-2926.

[5] C. X. Liu, T. Liu, L. Liu and K. Liu, A new chaotic attractor, *Chaos. Solitons & Fractals*, 2004, **22**: 1031-1038.

[6] M. Sun, L. X.Tian and Y. Fu, An energy resources demand-supply system and its dynamical analysis. *Chaos, Solitons & Fractals*, 2007, **32**: 168-180.

[7] G. Y. Qi, S. Z.Du, G. R. Chen, Z. Q. Chen and Z. Z.Yuan, On a four-dimensional chaotic system, *Chaos, Solitons & Fractals*. 2005,**23**:. 1671-1682.

[8] L. Liu, Y. C. Su and C. X.Liu , A new chaotic system and its circuit emulation, *Acta Phys Sin*. 2006,**55**: 3933-3937.

[9] Ott E, Grebogi C and Yorke J A ,Controlling chaos, *Phys Rev Lett*. 1990,**64**: 1196-2204.

[10] G. L.Cai , J. J.Huang and L. X. Tian and Q. C. Wang, Adaptive control and slow manifold analysis of a new chaotic system, *Int J Nonlinear Sci*. 2006,**2**: 50-60.

[11] Alasty A and Salarieh H, Nonlinear feedback control of chaotic pendulum in presence of saturation effect, *Chaos, Solitons & Fractals*. 2007,**31**: 292-304.

[12] Y. Y.Yu and S. C.Zhang Adaptive backstepping control of the uncertain Lü system, *Chin Phys Soc*.2002,**11**:1249-1253.