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Modified simple equation method to the nonlinear Hirota Satsuma KdV system

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Abstract. In this article, we use the modified simple equation method to construct the exact traveling wave solutions for some nonlinear PDE's in mathematical physics namely the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations. Based on this formulation, solitary solutions can be easily obtained by using the proposed method. Some known solutions obtained by the tanh-coth method are recovered as special cases. The proposed method is direct and more powerful than the other methods.

Keywords: The modified simple equation method , Exact solutions, Nonlinear evaluation equations, The nonlinear Hirota Satsuma equations.

1. Introduction

The nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to engineering, biology, chemistry, mechanics, etc. The exact solutions to nonlinear partial differential equations (NLPDEs) play an essential role in the nonlinear science, in that they may provide much physical information and help one to understand the mechanism that governs these physical models. Many powerful methods have been presented to construct the each solutions such as the Backlund transform [1], the homogeneous balance method [2], the extended tanh-function method [3,4], the F-expansion method [5], the exp–function expansion method [6], the generalized Riccati equation [7,8], the sub-ODE method [9,10], the extended sine-cosine method[11], the complex hyperbolic function method [12,13], the (G'/G)-expansion method [14,15,16], the modified rational Jacobi elliptic functions method [17], and so on.

In this article, we use the modified simple equation method [18,19,20,21,22] to calculate the exact solutions of some NLPDEs in mathematical physics, namely the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations.

2. Summary of the modified simple equation method

In this section, we would like to outline the main steps of this method as follows:
We consider the following nonlinear partial differential equation

$$U(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, ...) = 0. \tag{2.1}$$

Step 1. We use the following travelling wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x + ct, \tag{2.2}$$

where c is an arbitrary constant to be determined latter. The transformation (2.2) permits us to convert the PDE (2.1) to the ODE in the following form

$$P(u, u', u'', ...) = 0, \tag{2.3}$$

where P is a polynomial in u and its total derivatives with respect ξ.

Step 2. We suppose that equation (2.3) has the following solution form

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^k, \tag{2.4}$$

where A_k are arbitrary constants to be determined latter, such that $A_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined latter.
Step 3. We determine the positive integer *N* of the formal polynomial solution equation (2.4) by balancing the highest nonlinear terms and the highest-order derivatives in equation (2.3).

Step 4. We substitute (2.4) into (2.3); we calculate all the necessary derivatives $u', u'', \ldots$, and then we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\frac{\psi'(\xi)}{\psi(\xi)}$ and its derivatives. In this polynomial, we equate all the coefficients of it with zero. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Eq. (2.1).

3. Applications

The AODV is a reactive routing protocol that combines the advantages of both protocols, Dynamic Source Routing (DSR) and DSDV [22]. If a node using AODV protocol for communication, send a message to a destination node for which it does not have a valid route to, it initiates a route discovery process to locate the destination node [23].
In this section, we use the modified simple equation method to construct the exact solutions for the the coupled Hirota – Satsuma KdV equations and the generalized coupled Hirota – Satsuma KdV equations which are very important in the mathematical physics and have been bailed attention by many researcher in physics and engineering.

3.1. Example 1. The coupled Hirota – Satsuma KdV equations
In this section, we study the coupled Hirota – Satsuma KdV equations**[23,24]**

$$\begin{aligned} u_t &= \frac{1}{4}u_{xxx} + 3uu_x - 6vv_x, \\ v_t &= -\frac{1}{2}v_{xxx} - 3uv_x. \end{aligned} \tag{3.1}$$

We use the travelling wave transformation,

$$u = U(\xi), \quad v = V(\xi), \tag{3.2}$$

where $\xi = x - \lambda t$. The transformation (3.2) permit us to convert Eqs.(3.1) into the following ordinary differential equations

$$\begin{aligned} &\lambda U + \frac{1}{4}U'' + \frac{3}{2}U^2 - 3V^2 + C_1 = 0, \\ &-\lambda V' + \frac{1}{2}V'' + 3UV' = 0, \end{aligned} \tag{3.3}$$

where C_1 is the integration constant. Balancing the highest order derivative U'' and the nonlinear term U^2 and V^2 in Eq.(3.3), we get $N = M = 2$. Consequently, the solutions of Eq. (3.3) takes the following form

$$\begin{aligned} U &= a_0 + a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + a_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2, \\ V &= b_0 + b_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + b_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2, \end{aligned} \tag{3.4}$$

where $a_0, b_0, a_1, b_1, b_2,$ are constants to be determined later, and $\psi(\xi)$ is an arbitrary function to be determined later. Substituting (3.4) into (3.3) and equating all the coefficients of $\psi(\xi)$ to be zero, we obtain

$$\lambda a_0 - 3b_0^2 + \frac{3}{2}a_0^2 + C_1 = 0,$$
$$-3b_2^2 + \frac{3}{2}a_2 + \frac{3}{2}a_2^2 = 0,$$
$$(3a_1a_0 + \lambda a_1 - 6b_1b_0)\psi' + \frac{1}{4}a_1\psi''' = 0, \tag{3.5}$$
$$-3b_1^2\psi'^2 - \frac{3}{4}a_1\psi'\psi'' + 3a_2a_0\phi'^2 + \frac{1}{2}a_2\psi'^2 + \lambda a_2\psi'^2 - 6b_2b_0\psi'^2 + \frac{1}{2}a_2\psi'\psi''' + \frac{3}{2}a_1^2\psi'^2 = 0,$$
$$-\frac{5}{2}a_2\psi'^2\psi'' + 3a_1a_2\psi'^3 + \frac{1}{2}a_1\psi'^3 - 6b_2b_1\psi'^3 = 0$$

and

$$6a_2b_2 + 12b_2 = 0,$$
$$(3b_1a_0 - \lambda b_1)\psi'' + \frac{1}{2}b_1\psi^{(4)} = 0,$$
$$-2b_1\psi'''\psi' - \frac{3}{2}b_1\psi''^2 + 3b_2\psi''\psi''' + b_2\psi'\psi^{(4)} + (-3a_0b_1 + \lambda b_1)\psi'^2 - (2\lambda b_2 - 6b_2a_0 - 3a_1b_1)\psi'\psi'' = 0,$$
$$(6b_1 + 6a_1b_2 + 3a_2b_1)\psi'^2\psi'' - (3b_1a_1 + 6b_2a_0 - 2\lambda b_2)\psi'^3 - 12b_2\psi'\psi''^2 - 7b_2\psi'^2\psi''' = 0,$$
$$-(3b_1 + 3a_2b_1 + 6a_1b_2)\psi^{(4)} + (27b_2 + 6a_2b_2)\psi'^3\psi'' = 0, \tag{3.6}$$

With the help of Maple software package, we solve Eq. (3.5) and (3.6) to yield the following families:
Family 1.

$$a_2 = -1, \qquad a_1 = A_1, \qquad a_0 = -\frac{1}{6}A_1^2 + \frac{\lambda}{3},$$
$$b_2 = 0, \qquad b_1 = \pm\frac{1}{6}\sqrt{3(A_1^2 - 8\lambda)}, \qquad b_0 = \pm\frac{A_1}{12}\sqrt{3(A_1^2 - 8\lambda)},$$
$$c_1 = \frac{-\lambda^2}{2} + \frac{1}{48}A_1^4 - \frac{A_1^2\lambda}{6}, \qquad \psi(\xi) = \frac{1}{A_1}e^{A_1(\xi + A_2)} + A_3 \tag{3.7}$$

where A_1, A_2, A_3 and λ are arbitrary constants. The solution of Eq. (3.5) takes the following form

$$U = -\frac{1}{6}A_1^2 + \frac{\lambda}{3} + A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right) - A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right)^2,$$
$$V = \pm\frac{A_1}{12}\sqrt{3(A_1^2 - 8\lambda)} \mp \frac{A_1}{6}\sqrt{3(A_1^2 - 8\lambda)}\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right), \tag{3.8}$$

In the special case when $A_3 = 1/A_1$ the solitary wave solutions take the form:

$$U = \frac{\lambda}{3} - \frac{1}{6}A_1^2 + \frac{1}{4}A_1^2\,sech^2\left[\frac{1}{2}A_1(\xi + A_2)\right],$$
$$V = \mp\frac{A_1}{12}\sqrt{3(A_1^2 - 8\lambda)}\,tanh\left[\frac{1}{2}A_1(\xi + A_2)\right], \tag{3.9}$$

Family 2.

$$a_2 = -2, \qquad a_1 = 2A_1, \qquad a_0 = -\frac{1}{6}A_1^2 + \frac{\lambda}{3},$$

$$b_2 = 1, \qquad b_1 = -A_1, \qquad b_0 = \frac{1}{12}A_1^2 - \frac{2\lambda}{3}, \qquad (3.10)$$

$$c_1 = \frac{5\lambda^2}{6} - \frac{1}{48}A_1^4, \qquad \psi(\xi) = \frac{1}{A_1}e^{A_1(\xi+A_2)} + A_3$$

where A_1, A_2, A_3 and λ are arbitrary constants. The solution of Eq. (3.5) takes the following form

$$U = -\frac{1}{6}A_1^2 + \frac{\lambda}{3} + 2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right) - 2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right)^2,$$

$$V = \frac{1}{12}A_1^2 - \frac{2\lambda}{3} - A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right) + A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right)^2, \qquad (3.11)$$

In the special case when $A_3 = 1/A_1$ the solitary wave solutions take the form:

$$U = \frac{\lambda}{3} - \frac{1}{6}A_1^2 + \frac{1}{2}A_1^2 \sec h^2\left[\frac{1}{2}A_1(\xi + A_2)\right],$$

$$V = \frac{1}{12}A_1^2 - \frac{2\lambda}{3} - \frac{A_1^2}{4}\sec h^2\left[\frac{1}{2}A_1(\xi + A_2)\right], \qquad \textbf{(3.12)}$$

Family 3.

$$a_2 = -2, \qquad a_1 = 2A_1, \qquad a_0 = -\frac{1}{6}A_1^2 + \frac{\lambda}{3},$$

$$b_2 = -1, \qquad b_1 = A_1, \qquad b_0 = -\frac{1}{12}A_1^2 + \frac{2\lambda}{3}, \qquad (3.13)$$

$$c_1 = \frac{5\lambda^2}{6} - \frac{1}{48}A_1^4, \qquad \psi(\xi) = \frac{1}{A_1}e^{A_1(\xi+A_2)} + A_3$$

The solution of Eq. (3.5) takes the following form

$$U = -\frac{1}{6}A_1^2 + \frac{\lambda}{3} + 2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right) - 2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right)^2,$$

$$V = -\frac{1}{12}A_1^2 + \frac{2\lambda}{3} + A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right) - A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1 + e^{A_1(\xi+A_2)}}\right)^2, \qquad (3.14)$$

In the special case when $A_3 = 1/A_1$ the solitary wave solutions take the form:

$$U = \frac{\lambda}{3} - \frac{1}{6}A_1^2 + \frac{1}{2}A_1^2 \sec h^2\left[\frac{1}{2}A_1(\xi + A_2)\right],$$

$$V = -\frac{1}{12}A_1^2 + \frac{2\lambda}{3} + \frac{A_1^2}{4}\sec h^2\left[\frac{1}{2}A_1(\xi + A_2)\right], \qquad \textbf{(3.15)}$$

Family 4.

$$a_2 = -1, \quad a_0 = \frac{\lambda}{3}, \quad b_1 = \pm\frac{1}{3}\sqrt{-6\lambda}, \quad c_1 = \frac{-\lambda^2}{2}, \quad \psi(\xi) = A_1\xi + A_2$$

$$a_1 = b_2 = b_0 = 0, \qquad (3.16)$$

The solution of Eq. (3.5) takes the following form

$$U=\frac{\lambda}{3}-\left(\frac{A_1}{A_1\xi+A_2}\right)^2,$$
$$V=\pm\frac{1}{3}\frac{\sqrt{-6\lambda}A_1}{A_1\xi+A_2}, \tag{3.17}$$

3.2 Example 2. The generalized Hirota – Satsuma KdV equations

In this section, we study the generalized Hirota – Satsuma KdV equations **[23,24]**

$$u_t=\frac{1}{4}u_{xxx}+3uu_x+3(-v^2+w)_x,$$
$$v_t=-\frac{1}{2}v_{xxx}-3uv_x, \tag{3.18}$$
$$w_t=-\frac{1}{2}w_{xxx}-3uw_x,$$

We use the travelling wave transformation,

$$u=U(\xi),\qquad v=V(\xi),\qquad w=W(\xi) \tag{3.19}$$

where $\xi=x-\lambda t$. The transformation (3.19) permit us to convert Eqs.(3.18) into the following ordinary differential equations

$$\lambda U+\frac{1}{4}U''+\frac{3}{2}U^2+3(-V^2+W)+C_1=0,$$
$$-\lambda V'+\frac{1}{2}V''+3UV'=0, \tag{3.20}$$
$$-\lambda W'+\frac{1}{2}W'''+3UW'=0,$$

where C_1 is the integration constant. Balancing the highest order derivative U'' and the nonlinear term U^2 and V^2, consequently, the solutions of Eq. (3.3) take the following form

$$U=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)+a_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2,$$
$$V=b_0+b_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)+b_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2, \tag{3.21}$$
$$W=L_0+L_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)+L_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2,$$

where $a_i, b_i, L_i (i=0,1,2)$ are constants to be determined later, and $\psi(\xi)$ is an arbitrary function to be determined later. Substituting (3.21) into (3.20) and equating all the coefficients of $\psi(\xi)$ to be zero, we obtain

$$\lambda a_0 - 3b_0^2 + \frac{3}{2}a_0^2 + 3L_0 + C_1 = 0,$$
$$-3b_2^2 + \frac{3}{2}a_2 + \frac{3}{2}a_2^2 = 0,$$
$$(3a_1 a_0 + \lambda a_1 - 6b_1 b_0 + 3L_1)\psi' + \frac{1}{4}a_1\psi''' = 0, \tag{3.22}$$
$$(-3b_1^2 + 3a_2 a_0 - 6b_2 b_0 + \frac{3}{2}a_1^2 + \lambda a_2 + 3L_2)\psi'^2 - \frac{3}{4}a_1\psi'\psi'' + \frac{1}{2}a_2\psi'\psi''' + \frac{1}{2}a_2\psi''^2 = 0,$$
$$-\frac{5}{2}a_2\psi'^2\psi'' + 3a_1 a_2\psi'^3 + \frac{1}{2}a_1\psi'^3 - 6b_2 b_1\psi'^3 = 0,$$

$$6a_2 b_2 + 12b_2 = 0,$$
$$(3b_1 a_0 - \lambda b_1)\psi'' + \frac{1}{2}b_1\psi^{(4)} = 0,$$
$$-2b_1\psi'''\psi' - \frac{3}{2}b_1\psi''^2 + 3b_2\psi''\psi''' + b_2\psi'\psi^{(4)} + (-3a_0 b_1 + \lambda b_1)\psi'^2 - (2\lambda b_2 - 6b_2 a_0 - 3a_1 b_1)\psi'\psi'' = 0,$$
$$(6b_1 + 6a_1 b_2 + 3a_2 b_1)\psi'^2\psi'' - (3b_1 a_1 + 6b_2 a_0 - 2\lambda b_2)\psi'^3 - 12b_2\psi'\psi''^2 - 7b_2\psi'^2\psi''' = 0,$$
$$-(3b_1 + 3a_2 b_1 + 6a_1 b_2)\psi^{(4)} + (27b_2 + 6a_2 b_2)\psi'^3\psi'' = 0, \tag{3.23}$$

and

$$6a_2 L_2 + 12L_2 = 0,$$
$$(3L_1 a_0 - \lambda L_1)\psi'' + \frac{1}{2}L_1\psi^{(4)} = 0,$$
$$-2L_1\psi'''\psi' - \frac{3}{2}L_1\psi''^2 + 3L_2\psi''\psi''' + L_2\psi'\psi^{(4)} + (-3a_0 L_1 + \lambda L_1)\psi'^2$$
$$-(2\lambda L_2 - 6L_2 a_0 - 3a_1 L_1)\psi'\psi'' = 0,$$
$$(6L_1 + 6a_1 L_2 + 3a_2 L_1)\psi'^2\psi'' - (3L_1 a_1 + 6L_2 a_0 - 2\lambda L_2)\psi'^3 - 12L_2\psi'\psi''^2 - 7L_2\psi'^2\psi''' = 0,$$
$$-(3L_1 + 3a_2 L_1 + 6a_1 L_2)\psi^{(4)} + (27L_2 + 6a_2 L_2)\psi'^3\psi'' = 0, \tag{3.24}$$

With the help of Maple software package, we solve Eqs. (3.22) , (3.23) and (3.24) to yield the following families:

Family 1.

$$a_2 = -1, \quad a_1 = A_1, \quad a_0 = -\frac{1}{6}A_1^2 + \frac{\lambda}{3}, \quad c_1 = \frac{1}{48}A_1^4 - \frac{1}{6}\lambda A_1^2 - \frac{\lambda^2}{2} - 3L_0,$$
$$b_1 = \pm\frac{1}{6}\sqrt{3(A_1^2 - 8\lambda)}, \quad b_0 = \mp\frac{A_1}{12}\sqrt{3(A_1^2 - 8\lambda)}, \quad \psi(\xi) = \frac{1}{A_1}e^{A_1(\xi + A_2)} + A_3 \tag{3.25}$$
$$b_2 = L_2 = 0,$$

where A_1, A_2, A_3, L_0, L_1 and λ are arbitrary constants. The solution of Eq. (3.19) takes the following form

$$U=-\frac{1}{6}A_1^2+\frac{\lambda}{3}+A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)-A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)^2,$$

$$V=\mp\frac{A_1}{12}\sqrt{3(A_1^2-8\lambda)}\pm\frac{1}{6}A_1\sqrt{3(A_1^2-8\lambda)}\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right), \tag{3.26}$$

$$W=L_0+L_1A_1\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right),$$

In the special case when $A_3=1/A_1$ the solitary wave solutions take the form:

$$U=\frac{\lambda}{3}-\frac{1}{6}A_1^2+\frac{1}{4}A_1^2\,sec\,h^2\left[\frac{1}{2}A_1(\xi+A_2)\right],$$

$$V=\pm\frac{A_1}{12}\sqrt{3(A_1^2-8\lambda)}\,tanh\left[\frac{1}{2}A_1(\xi+A_2)\right], \tag{3.27}$$

$$W=L_0+L_1A_1+L_1A_1\,tanh\left[\frac{1}{2}A_1(\xi+A_2)\right]$$

Family 2.

$$a_2=-2,\qquad a_1=2A_1,\qquad a_0=-\frac{1}{6}A_1^2+\frac{\lambda}{3},$$

$$b_2=\pm1,\qquad b_1=\mp A_1,\qquad b_0=\pm\frac{L_2}{2}\mp\frac{2\lambda}{3}\pm\frac{1}{12}A_1^2,\ L_1=-L_2A_1, \tag{3.28}$$

$$c_1=\frac{5\lambda^2}{6}-\frac{1}{48}A_1^4+\frac{A_1^2L_2}{4}-2L_2\lambda+\frac{3L_2^2}{4}-3L_0,\quad \psi(\xi)=\frac{1}{A_1}e^{A_1(\xi+A_2)}+A_3$$

where A_1, A_2, A_3, L_0, L_2 and λ are arbitrary constants. The solution of Eq. (3.19) takes the following form

$$U=-\frac{1}{6}A_1^2+\frac{\lambda}{3}+2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)-2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)^2,$$

$$V=\pm\frac{L_2}{2}\mp\frac{2\lambda}{3}\pm\frac{1}{12}A_1^2\mp A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)\pm A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)^2, \tag{3.29}$$

$$W=L_0-L_2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)+L_2A_1^2\left(\frac{e^{A_1(\xi+A_2)}}{A_3A_1+e^{A_1(\xi+A_2)}}\right)^2,$$

In the special case when $A_3=1/A_1$ the solitary wave solutions take the form:

$$U=\frac{\lambda}{3}-\frac{1}{6}A_1^2+\frac{1}{2}A_1^2\,sec\,h^2\left[\frac{1}{2}A_1(\xi+A_2)\right],$$

$$V=\pm\frac{L_2}{2}\mp\frac{2\lambda}{3}\pm\frac{1}{12}A_1^2\mp\frac{A_1^2}{4}\,sec\,h^2\left[\frac{1}{2}A_1(\xi+A_2)\right], \tag{3.30}$$

$$W=L_0-\frac{L_2A_1^2}{4}\,sec\,h^2\left[\frac{1}{2}A_1(\xi+A_2)\right]$$

In the special case when $A_2=0, A=2\sqrt{C_2}$, and $\lambda=-V$ the exact solutions (3.30) equivalent the exact solutions (3.7) which obtained in [23,24].

Family 3.

$$a_2 = -2,\ b_2 = \pm 1,\ a_0 = \frac{\lambda}{3},\quad b_0 = \mp\frac{2\lambda}{3},\quad c_1 = \frac{5\lambda^2}{6} - 3L_0,\quad \psi(\xi) = A_1\xi + A_2$$
$$a_1 = b_1 = L_1 = 0, \tag{3.31}$$

The solution of Eq. (3.19) takes the following form

$$U = \frac{\lambda}{3} - 2\left(\frac{A_1}{A_1\xi + A_2}\right)^2,$$
$$V = \mp\frac{2\lambda}{3} \pm \left(\frac{A_1}{A_1\xi + A_2}\right)^2, \tag{3.32}$$
$$V = L_0 + L_2\left(\frac{A_1}{A_1\xi + A_2}\right)^2,$$

Remark. All solutions presented in this paper have been checked with Mathematica by putting them back in the original equations.

4. Conclusions

In this paper, we have used the modified simple equation method to construct some exact solutions for nonlinear partial differential equations in mathematical physics via the coupled Hirota Satsuma equations and generalized Hirota Satsuma euations. As a result, many exact solutions and solitary wave solutions are obtained. We believe that this method should play an important role for finding exact solutions in mathematical physics because this method is much more simple than the other methods.

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