ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 10, No. 4, 2015, *pp*. 311-320

Local Stability and Hopf Bifurcation Analysis for a Predator-Prey control Model with Two Delays

Hongxing Yao[1,2] Jie Zhang[2]
[1,2]School of finance & Economic,Jiangsu University,Zhenjiang 212013,China
[2]Faculty of Science,Jiangsu University,Zhenjiang 212013,China
(*Received April 08, 2015, accepted September 29, 2015)*

Abstract. Based on facts of time delay will influence dynamical behavior and control theory is more appropriate to reflect the natural rule, constructed a predator-prey control model with two delays,choose the two delays as the bifurcation parameter,according to the Routh-Hurwize discriminant method,we studied the local stability and Hopf branch of the control system.The sufficient conditions for the local stability and the existence of Hopf bifurcation are established.Finally,the effectiveness of the controller is verified by numerical simulation.

Keywords: Predator-prey model, stability analysis, Hopf bifurcation, time delay, control

1. Introduction

Time delay is often appear in biological activity,the fact that tie delay will influence dynamical behavior to reflect the nature rule.In the population ecology,since May find would destroy the stability of the logistic model's positive equilibrium[1],there has been a large number of literature [2-3],to study the effect of delay on the stability of positive equilibrium in ecological models.In biological systems,as the gestation process of species,species of digestion and transformation process as well as their mature time and so on, there are the time delay phenomenon.The existence of delays tends to evolve more complex dynamical properties in the system.If we can fully understand and master delay affects the dynamical property of ecological system,you can use the master of laws to do better a good job of prevention and control,protect the rare resources, maintaining ecological balance.

In the nonlinear biological time delay dynamical system,there is the most discussed is Hopf bifurcation.There has been some researchers already studied the Hopf bifurcation of ecosystem.Faria[4] consider hunting time delay, and regarding time delay as the bifurcation parameter study the stability of system and bifurcation.Yan hezhang[5] comprehensive two time delay,assuming two time delays are equal,the Hopf bifurcation of system is researched.

There have been research on time delay dynamical systems about Hopf bifurcation,but most of them are about 2d[6,7],about the higher time delay dimension systems's Hopf bifurcation and bifurcation control's research is relatively few.Yongli song et al [11,12,13] studied a few class with multi-delay prey system,the sufficient conditions for the local stability and the exitence of Hopf bifurcation are established.Tang C B,Chen Y Y. et al[14,15] respectively research three dimensional predator-prey systems with time delay, discussed the conditions of Hopf bifurcation near the positive equilibrium,and directive of the bifurcation is given.Fan meng et al in article [16]and[17] respectively discussed the Lotka-Volterra system with n kinds of competition and with feedback control system.Jiang Guirong and Lu Qishao [18]research the dynamics of predator-prey system behavior with state feedback control.

Considering the different biological mature time and digesting time are different,in this paper,considering the consider of two delays,in addition,on the study of ecological system's control, main consideration is state feedback control previously.For example, in order to eliminate blooms occurred,an effective way is to introduce feedback control variables in the equation (such as silver carp, bighead) to change the equilibrium of the system.In fact,intraspecific effect coefficient and interaction effect coefficient as parameters influenced by many factors such as temperature and sunshine etc,so also by changing the parameters to adjust system.In this paper, considering parameters and state feedback control,put forward the following predator-prey control model with two delays.

The remainder of this paper is organized as follows:the control model is proposed and the characteristic equation of linear system in section 2.In section 3,stability analysis and Hopf bifurcation in control model.To verify the theoretic analysis, numerical simulations are given in section 4.Finally, section 5 concludes with some discussion.

2. The characteristic equation of linear system

In article [19]，Xu research a two species Lotka-Volterra model with two delays :

$$\begin{cases} \dot{x}(t) = x(t)[r_1 - a_{11}x(t-\tau_1) - a_{12}y(t-\tau_2)], \\ \dot{y}(t) = y(t)[-r_2 + a_{21}x(t-\tau_2) - a_{22}y(t-\tau_1)]. \end{cases}$$

The dynamical property of the system is given,chen [20] on this basis research the dynamic property of the three species Lotka-Volterra model with two delays.

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1 - a_{11}x_1(t-\tau_1) - a_{13}x_3(t-\tau_2)] \\ \dot{x}_2(t) = x_2(t)[r_2 - a_{22}x_2(t-\tau_1) - a_{23}x_3(t-\tau_2)] \\ x_3(t) = x_3(t)[-r_3 + a_{31}x_1(t-\tau_2) + a_{32}x_2(t-\tau_2) - a_{33}x_3(t-\tau_1)] \end{cases} \tag{2.1}$$

In this paper,the control strategy of the design parameter and the state of feedback control is designed.The Hopf bifurcation of (2.1) system is controlled,make delayed the bifurcation behavior.Parameter perturbation of system (2.1) and increase the time delay control item $\beta x_1(t-\tau_1), \beta x_2(t-\tau_2), \beta x_3(t-\tau_3)$,the control system is:

$$\begin{cases} \dot{x}_1(t) = \alpha x_1(t)[r_1 - a_{11}x_1(t-\tau_1) - a_{13}x_3(t-\tau_2)] + \beta x_1(t-\tau_1) \\ \dot{x}_2(t) = \alpha x_2(t)[r_2 - a_{22}x_2(t-\tau_1) - a_{23}x_3(t-\tau_2)] + \beta x_2(t-\tau_1) \\ x_3(t) = \alpha x_3(t)[-r_3 + a_{31}x_1(t-\tau_2) + a_{32}x_2(t-\tau_2) - a_{33}x_3(t-\tau_1)] + \beta x_3(t-\tau_1) \end{cases} \tag{2.2}$$

The parameter α, β is non negative real number. The positive equilibrium is $E_0(x_1^*, x_2^*, x_3^*)$

$$\begin{cases} x_1^* = \dfrac{\alpha(r_1a_{22}a_{33} - r_2a_{13}a_{32} + r_3a_{22}a_{13} + r_1a_{23}a_{32}) + \beta(a_{22}a_{33} - a_{13}a_{32} + a_{23}a_{32} - a_{22}a_{13})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \\ x_2^* = \dfrac{\alpha(-r_1a_{23}a_{31} + r_2a_{11}a_{33} + r_2a_{13}a_{31} + r_3a_{11}a_{23}) + \beta(-a_{23}a_{31} + a_{11}a_{33} + a_{13}a_{31} - a_{23}a_{11})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \\ x_3^* = \dfrac{\alpha(-r_3a_{11}a_{22} + r_1a_{22}a_{31} + r_2a_{11}a_{32}) + \beta(a_{11}a_{22} + a_{22}a_{31} + a_{11}a_{32})}{\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})} \end{cases}$$

(H_1)

$$\begin{aligned} & sign(\alpha(a_{11}a_{22}a_{33} + a_{22}a_{13}a_{31} + a_{11}a_{23}a_{32})) \\ & = sign(\alpha(-r_3a_{11}a_{22} + r_1a_{22}a_{31} + r_2a_{11}a_{32}) + \beta(a_{11}a_{22} + a_{22}a_{31} + a_{11}a_{32})) \\ & \quad = sign(\alpha(-r_1a_{23}a_{31} + r_2a_{11}a_{33} + r_2a_{13}a_{31} + r_3a_{11}a_{23}) + \beta(-a_{23}a_{31} + a_{11}a_{33} + a_{13}a_{31} - a_{23}a_{11})) \\ & = sign(\alpha(r_1a_{22}a_{33} - r_2a_{13}a_{32} + r_3a_{22}a_{13} + r_1a_{23}a_{32}) + \beta(a_{22}a_{33} - a_{13}a_{32} + a_{23}a_{32} - a_{22}a_{13})) \end{aligned}$$

The system (2.1)is linearized at positive equilibrium

$$\begin{cases} \dot{x}_1(t) = m_1x_1(t) + m_2x_1(t-\tau_1) + m_3x_3(t-\tau_2) \\ \dot{x}_2(t) = n_1x_2(t) + n_2x_2(t-\tau_1) + n_3x_3(t-\tau_2) \\ \dot{x}_3(t) = k_1x_3(t) + k_2x_1(t-\tau_2) + k_3x_2(t-\tau_2) + k_4x_3(t-\tau_1) \end{cases} \tag{2.3}$$

Among them

$m_1 = \alpha(r_1 - a_{11}x_1^* - a_{13}x_3^*)$, $m_2 = \beta - \alpha a_{11}x_1^*$, $m_3 = -\alpha a_{13}x_1^*$, $m_4 = -\alpha a_{11}$,

$m_5 = -\alpha a_{13}$, $n_1 = \alpha(r_2 - a_{22}x_2^* - a_{23}x_3^*)$, $n_2 = \beta - \alpha a_{22}x_2^*$, $n_3 = -\alpha a_{23}x_2^*$,

$n_4 = -\alpha a_{22,}$ $n_5 = -\alpha a_{23}$, $k_1 = \alpha(-r_3 + a_{31}x_1^* + a_{32}x_2^* - a_{33}x_3^*)$, $k_2 = \alpha a_{31}x_3^*$,

$k_3 = \alpha a_{32}x_3^*$, $k_4 = \beta - \alpha a_{33}x_3^*$, $k_5 = \alpha a_{31}$, $k_6 = \alpha a_{32}$, $k_7 = -\alpha a_{33}$.

Equation (2.3) corresponding to the characteristic equation is

$$\det\begin{pmatrix} \lambda - m_1 - m_2e^{-\lambda\tau_1} & 0 & -m_3e^{-\lambda\tau_2} \\ 0 & \lambda - n_1 - n_2e^{-\lambda\tau_1} & -n_3e^{-\lambda\tau_2} \\ -k_2e^{-\lambda\tau_2} & -k_3e^{-\lambda\tau_2} & \lambda - k_1 - k_4e^{-\lambda\tau_1} \end{pmatrix} = 0,$$

That is

$$P_1(\lambda) + P_2(\lambda)e^{-\lambda\tau_1} + P_3(\lambda)e^{-2\lambda\tau_1} + P_4(\lambda)e^{-3\lambda\tau_1} + P_5(\lambda)e^{-2\lambda\tau_2} + P_6(\lambda)e^{-2\lambda\tau_2-\lambda\tau_1} = 0, \quad (2.4)$$

Among them

$P_1(\lambda) = \lambda^3 - (k_1 + n_1 + m_1)\lambda^2 + (n_1k_1 + m_1k_1 + m_1n_1)\lambda - m_1n_1k_1$,

$P_2(\lambda) = -(n_2 + k_4 + m_2)\lambda^2 + (n_1k_4 + n_2k_1 + m_1n_2 + m_1k_4 + m_2k_1 + m_2n_1)\lambda - (m_1n_1k_4 + m_1n_2k_1 + m_2n_1k_1)$,

$P_3(\lambda) = (n_2k_4 + m_2n_2 + m_2k_4)\lambda - (m_1n_2k_4 + m_2n_1k_4 + m_2n_2k_1)$,

$P_4(\lambda) = -m_2n_2k_4$, $P_5(\lambda) = -(n_3k_3 + k_2m_3)\lambda + m_1n_3k_3 + k_2m_3n_1$, $P_6(\lambda) = m_2n_3k_3 + k_2m_3n_2$.

3. Stability of Positive Equilibrium and Hopf Bifurcation

The sufficient conditions for the asymptotic stability and the existence of Hopf bifurcation are established.

Since the system (2.2) has two delays, we are now divided into three cases to discuss.

Case 1: $\tau_1 = \tau_2 = 0$. then the characteristic equation (2.4) becomes

$$P_1(\lambda) + P_2(\lambda) + P_3(\lambda) + P_4(\lambda) + P_5(\lambda) + P_6(\lambda) = 0, \tag{3.1}$$

That is

$$\lambda^3 + d_1\lambda^2 + d_2\lambda + d_3 = 0, \tag{3.2}$$

Among them

$d_1 = -(k_1 + n_1 + m_1 + n_2 + k_4 + m_2)$,

$d_2 = (n_1k_1 + m_1k_1 + m_1n_1 + n_1k_4 + n_2k_1 + m_1n_2 + m_1k_4 + m_2k_1 + m_2n_1 + n_2k_4 + m_2n_2 + m_2k_4) - (n_3k_3 + k_2m_3)$,

$d_3 = -(k_1 + k_4)(m_1 + m_2)(n_1 + n_2) + n_3k_3(m_1 + m_2) + k_2m_3(n_1 + n_2)$.

According to the Routh-Hurwitz discriminant method, we know,the root of (3.2) all have negative real part, the necessary and sufficient conditions is

$$(H_2) \qquad D_1 = d_1 > 0,\ D_2 = \det\begin{pmatrix} d_1 & 1 \\ d_3 & d_2 \end{pmatrix} > 0,\ D_3 = \det\begin{pmatrix} d_1 & 1 & 0 \\ d_3 & d_2 & d_1 \\ 0 & 0 & d_3 \end{pmatrix} > 0.$$

So when (H_2) was set up ,the E_0 asymptotic stability.

Case 2: $\tau_1 = 0, \tau_2 > 0$.then the characteristic equation (2.4) becomes

$$P_1(\lambda) + P_2(\lambda) + P_3(\lambda) + P_4(\lambda) + P_5(\lambda)e^{-2\lambda\tau_2} + P_6(\lambda)e^{-2\lambda\tau_2} = 0, \tag{3.3}$$

That is

$$\lambda^3 + r_1\lambda^2 + r_2\lambda + r_3 + q(\lambda)e^{-2\lambda\tau_2} = 0, \tag{3.4}$$

Among them

$$r_1 = -(k_1 + n_1 + m_1 + n_2 + m_2 + k_4),$$
$$r_2 = k_1(n_1 + m_1 + n_2 + m_2) + k_4(n_1 + m_1 + n_2 + m_2) + (m_1 + m_2)(n_1 + n_2),$$
$$r_3 = -(k_1 + k_4)(m_1 + m_2)(n_1 + n_2),\ r_4 = -(n_3k_3 + k_2m_3),$$
$$r_5 = m_1n_3k_3 + k_2m_3n_1 + m_2n_3k_3 + k_2m_3n_2,\ q(\lambda) = r_4\lambda + r_5.$$

Set characteristic equation (3.4) has a pure imaginary root, $\lambda = i\omega$ plug type equation

$$(i\omega)^3 + r_1(i\omega)^2 + r_2(i\omega) + r_3 + q(\lambda)e^{-2\tau_2(i\omega)} = 0. \tag{3.5}$$

We know

$$\mathrm{Re}\{q(i\omega)\} = r_5, \mathrm{Im}\{q(i\omega)\} = r_4.$$

To separate (3.5) imaginary part and real component

$$\begin{cases} -r_1\omega^2 + r_3 + r_5\cos 2\omega\tau_2 + r_4\omega\sin 2\omega\tau_2 = 0, \\ -\omega^3 + r_2\omega + r_4\omega\cos 2\omega\tau_2 - r_5\sin 2\omega\tau_2 = 0. \end{cases} \tag{3.6}$$

finishing

$$\omega^6 + (r_1^2 - 2r_2)\omega^4 + (r_2^2 - 2r_1r_3 - r_4^2)\omega^2 + r_3^2 - r_5^2 = 0. \tag{3.7}$$

(1)If (3.7) has no positive root, only H_1, H_2 set up, and when $\tau_2 \in [0, +\infty)$, E_0 is asymptotic stability.

(2)If (3.7) have limited positive root, $\{\omega_i\}\ i = 1,2,\dots,k,$ then by (2.9) can be solved $\{\tau_{2i}^j \mid j = 1,2,\dots\},$

Take $$\tau_{2_0} = \min\{\tau_{2i}^j \mid i = 1,2,\dots,k,\ j = 1,2,\dots\}, \tag{3.8}$$

Easy to know, when $\tau_2 = \tau_{2_0}$, the characteristic equation (3.4) has a pair of pure imaginary roots $\pm i\omega^*$;when $\tau_2 < \tau_{2_0}$, the characteristic equation (3.4) has no pure imaginary root.

In order to discuss the conditions of Hopf bifurcation, under the case (2) to calculate $\left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau_2}\right|_{\lambda = i\omega^*}$.

make

$\lambda(\tau_2) = \alpha(\tau_2) + i\omega(\tau_2)$ is nearly $\tau_2 = \tau_{2i}^j$ root of the characteristic equation,and have $\alpha(\tau_{2i}^j) = 0, \omega(\tau_{2i}^j) = \omega^*$, $\lambda(\tau_2)$ plug type equation (3.4) ,and the derivation τ_2 ,available

$$\left(\frac{d\lambda}{d\tau_2}\right)^{-1} = \frac{(3\lambda^2 + 2r_1\lambda + r_2)e^{2\lambda\tau_2} + r_4}{2\lambda(r_4\gamma + r_5)} - \frac{\tau_2}{\lambda}.$$

So

$$\left.\left(\frac{d\,\mathrm{Re}(\lambda)}{d\tau_2}\right)^{-1}\right|_{\lambda = i\omega^*} = \mathrm{Re}\left.\left\{\frac{(3\lambda^2 + 2r_1\lambda + r_2)e^{2\lambda\tau_2} + r_4}{2\lambda(r_4\lambda + r_5)}\right\}\right|_{\lambda = i\omega^*} = \frac{Q_1 + Q_2}{P}.$$

Among them

$$Q_1 = -r_4\omega^{*^2}\left[r_4 + (-3\omega^{*^2} + r_2)\cos 2\omega^*\tau_{2_0} - 2r_1\omega^*\sin 2\omega^*\tau_{2_0}\right],$$
$$Q_2 = r_5\omega^*\left[(-3\omega^{*^2} + r_2)\sin 2\omega^*\tau_{2_0} + 2r_1\omega^*\cos 2\omega^*\tau_{2_0}\right],$$
$$P = 2\omega^{*^2}(r_4^2\omega^{*^2} + r_5^2).$$

Lemma 3. 1 For system (2.2) ,when $\tau_1 = 0$, if (H_1) and (H_2) set up,then

(1)if (3.7) have no positive root, then when $\tau_2 \in [0, +\infty)$, the positive equilibrium E_0 is asymptotic stability.

(2)if（3.7）have limited positive roots $\{\omega_i\}\,(i=1,2,...,k)$， then when $\tau_2 \in \left[0, \tau_{2_0}\right), E_0$ is asymptotic stability；then when $\tau_2 \in \left[\tau_{2_0}, +\infty\right), E_0$ is unstable，and when $\frac{Q_1+Q_2}{P} \neq 0$， system（2.2）bifurcation in $\tau_2 = \tau_{2_0}$ place.

Case 3： $\tau_1 > 0$， $\tau_2 > 0$.Then fixed in the stable rang from the case 2 , τ_1 will be seen as a parameter,for the convenience of discuss，now τ_{2_0} rewritten into system （3.8）

$$\tau_{2_0} = \begin{cases} +\infty, \\ \min\left\{\tau_{2i}^{j} \middle| i=1,2,...,k,\ j=1,2,...\right\}, \end{cases}$$

The characteristic equation（2.4） can be written as follows：

$$k_0(\lambda) + k_1(\lambda)e^{-\lambda\tau_1} + k_2(\lambda)e^{-2\lambda\tau_1} + k_3(\lambda)e^{-3\lambda\tau_1} = 0, \qquad (3.9)$$

Among them

$$k_0(\lambda) = P_1(\lambda) + P_5(\lambda)e^{-2\lambda\tau_2},\ k_1(\lambda) = P_2(\lambda) + P_6(\lambda)e^{-2\lambda\tau_2},$$
$$k_2(\lambda) = P_3(\lambda), k_3(\lambda) = P_4(\lambda).$$

Set $\lambda = i\omega$（ $\omega > 0$）is the pure imaginary root of （3.9），make（3.9） multiply both side $e^{\lambda\tau_1}$， and plug $\lambda = i\omega$（ $\omega > 0$）type equation,available

$$(U_0 + iV_0)e^{i\omega\tau_1} + (U_1 + iV_1) + (U_2 + iV_2)e^{-i\omega\tau_1} + (U_3 + iV_3)e^{-2i\omega\tau_1} = 0, \qquad (3.10)$$

Among them

$$U_i = \mathrm{Re}\{k_i(i\omega)\},\ V_i = \mathrm{Im}\{k_i(i\omega)\},\ i = 0,1,2,3.$$

Separation real imaginary part,have

$$\begin{cases} (U_0+U_2)\cos\omega\tau_1 + (V_2-V_0)\sin\omega\tau_1 + U_1 = -U_3\cos 2\omega\tau_1, \\ (V_0+V_2)\cos\omega\tau_1 + (U_0-U_2)\sin\omega\tau_1 + V_1 = U_3\sin 2\omega\tau_1. \end{cases} \qquad (3.11)$$

Add (3.11) on both sides of the square, available

$$\left[(U_0+U_2)\cos\omega\tau_1 + (V_2-V_0)\sin\omega\tau_1 + U_1\right]^2 + \left[(V_0+V_2)\cos\omega\tau_1 + (U_0-U_2)\sin\omega\tau_1 + V_1\right]^2 = U_3^2. \qquad (3.12)$$

Owing to $\sin\omega\tau_1 = \pm\sqrt{1-\cos^2\omega\tau_1}$， so consider two cases respectively

(1)when $\sin\omega\tau_1 = \sqrt{1-\cos^2\omega\tau_1}$,（3.12）will become

$$\left[(U_0+U_2)\cos\omega\tau_1 + (V_2-V_0)\sqrt{1-\cos^2\omega\tau_1} + U_1\right]^2 + \left[(V_0+V_2)\cos\omega\tau_1 + (U_0-U_2)\sqrt{1-\cos^2\omega\tau_1} + V_1\right]^2 = U_3^2. \qquad (3.13)$$

The $\cos\omega\tau_1$ solution that can be obtained by (3.13) is recorded as

$$\cos\omega\tau_1 = f_1(\omega), \sin\omega\tau_1 = f_2(\omega)\ ,\ f_1^2(\omega) + f_2^2(\omega) = 1.$$

Can be solved

$$\tau_{1_1}^{(k)} = \frac{1}{\omega}\left[\arccos f_1(\omega) + 2k\pi\right], (k = 0,1,2,...),$$

the root of $f_1^2(\omega) + f_2^2(\omega) = 1$ are ω

（2） when $\sin\omega\tau_1 = -\sqrt{1-\cos^2\omega\tau_1}$,（3.12） become

$$\left[(U_0+U_2)\cos\omega\tau_1 - (V_2-V_0)\sqrt{1-\cos^2\omega\tau_1} + U_1\right]^2 + \left[(V_0+V_2)\cos\omega\tau_1 - (U_0-U_2)\sqrt{1-\cos^2\omega\tau_1} + V_1\right]^2 = U_3^2. \qquad (3.14)$$

The $\cos\omega\tau_1$ solution that can be obtained by (3.14) is recorded as

$\cos\omega\tau_1 = f_1^*(\omega), \sin\omega\tau_1 = f_2^*(\omega)$, $f_1^{*2}(\omega) + f_2^{*2}(\omega) = 1.$

Can be solved

$$\tau_{1_2}^{(k)} = \frac{1}{\omega}\left[\arccos f_1^*(\omega) + 2k\pi\right], (k = 0,1,2,...),$$

the root of $f_1^{*2}(\omega) + f_2^{*2}(\omega) = 1$ are ω

We make

$$\tau_{1_0} = \min\{\tau_{1_1}^k, \tau_{1_2}^k\}, (k = 0,1,2,...).$$

Theorem 3.2 For system（2.2），if (H_1) and (H_2) set up， so when $\tau_1 < \tau_{1_0}$ and $\tau_2 < \tau_{2_0}$,the positive equilibrium of system E_0 is asymptotic.But when $\tau_1 = \tau_{1_0}$ and $\tau_2 < \tau_{2_0}$,equation （3.9） has a pair of pure imaginary roots $\pm i\omega^*$； if the equation (3.9) have further $\left.\frac{d\,\mathrm{Re}(\lambda)}{dr_1}\right|_{\lambda = i\omega^*} \neq 0,$ the system generates a Hopf bifurcation at $\tau_1 = \tau_{1_0}$.

4. Numerical simulation

In order to verify the accuracy of the previous theory,the numerical simulation of the system (2.2) is carried out by using Matlab software

$$\begin{aligned}
&\alpha = 0.1, \beta = 0.2,\\
&\gamma_1 = 1, a_{11} = 0.2, a_{13} = 1,\\
&\gamma_2 = 1, a_{22} = 0.8, a_{23} = 1,\\
&\gamma_3 = 1, a_{31} = 0.2, a_{32} = 0.5, a_{33} = 1.
\end{aligned}$$

Then get the system

$$\begin{cases}
\dot{x}_1(t) = 0.1x_1(t)[1 - 0.2x_1(t-\tau_1) - x_3(t-\tau_2)] + 0.2x_1(t-\tau_1)\\
\dot{x}_2(t) = 0.1x_2(t)[1 - 0.8x_2(t-\tau_1) - x_3(t-\tau_2)] + 0.2x_2(t-\tau_1)\\
x_3(t) = 0.1x_3(t)[-1 + 0.2x_1(t-\tau_2) + 0.5x_2(t-\tau_2) - x_3(t-\tau_1)] + 0.2x_3(t-\tau_1)
\end{cases} \quad (4.1)$$

Easy to verify condition (H_1) was established，then a positive equilibrium point $E_0\left(\frac{10}{3}, \frac{10}{9}, \frac{19}{9}\right)$ of system (4.1) can be obtained.

When $\tau_1 = 0, \tau_2$ >0, can calculate $\omega^* \approx 1.014$， $\tau_{2_0} \approx$ 7.8045.By lemma 3.1，know when $\tau_2 < \tau_{2_0} \approx$ 7.8045, the positive equilibrium E_0 of system (4.1) is asymptotic stability,when $\tau_2 > \tau_{2_0} \approx$ 7.8045,the positive equilibrium E_0 of system (4.1) is unstable, and Hopf bifurcation occurs at $\tau_2 = \tau_{2_0}$ .Specific image shown in figure 4.1 and figure 4.2.

Now fixed $\tau_2 = 5$, can be calculated $\omega^* \approx 0.2900$， $\tau_{1_0} \approx$ 2.825.By theorem 3.2, know when $\tau_1 < \tau_{1_0} \approx$ 2.825.The positive equilibrium E_0 of system (4.1) is asymptotic stability, when $\tau_1 > \tau_{1_0} \approx$ 2.825, the positive equilibrium E_0 of system (4.1) is unstable, when $\tau_1 = \tau_{1_0} \approx$ 2.825, system (4.1) occurs Hopf bifurcation.Specific image shown in figure 4.3 and figure 4.4.

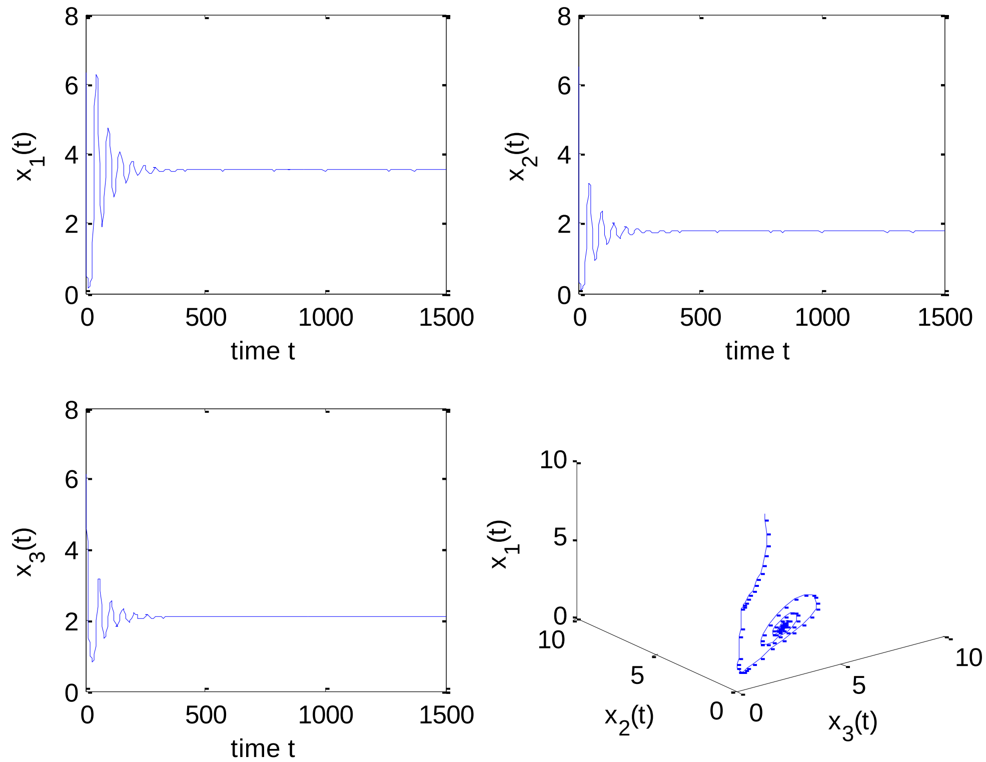

Figure 4.1 the positive equilibrium E_0 at $\tau_2 = 6.8 < \tau_{2_0} \approx 7.8045$ is local asymptotic stability.

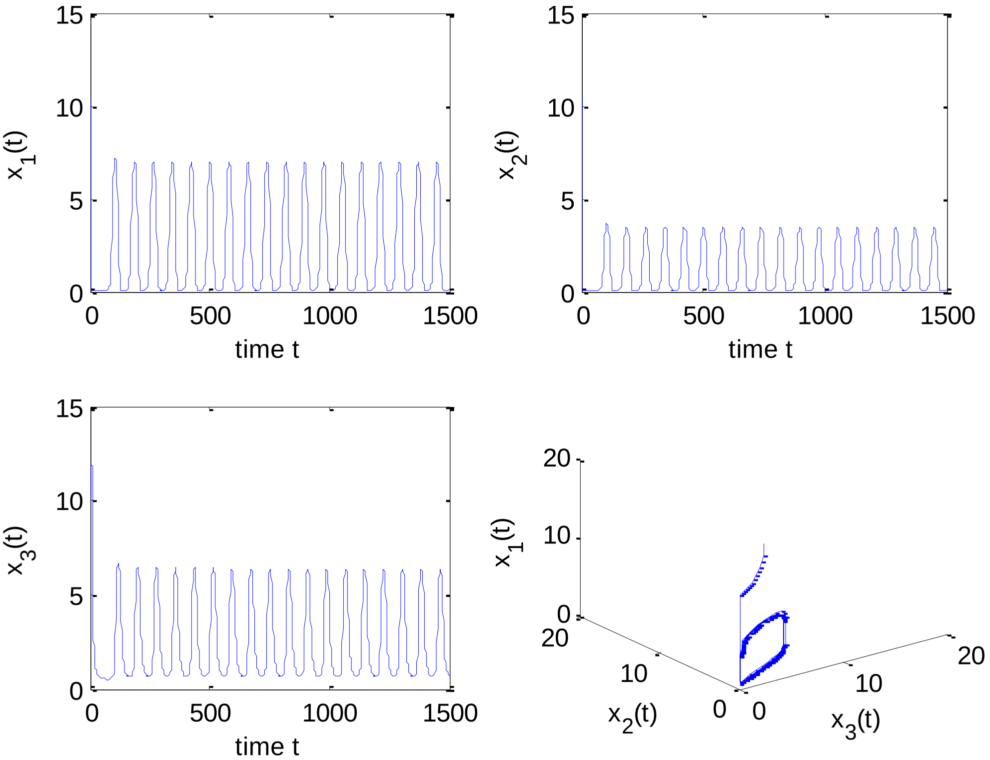

Figure 4.2 the positive equilibrium E_0 at $\tau_2 = 10.8 > \tau_{2_0} \approx 7.0845$ is unstable.

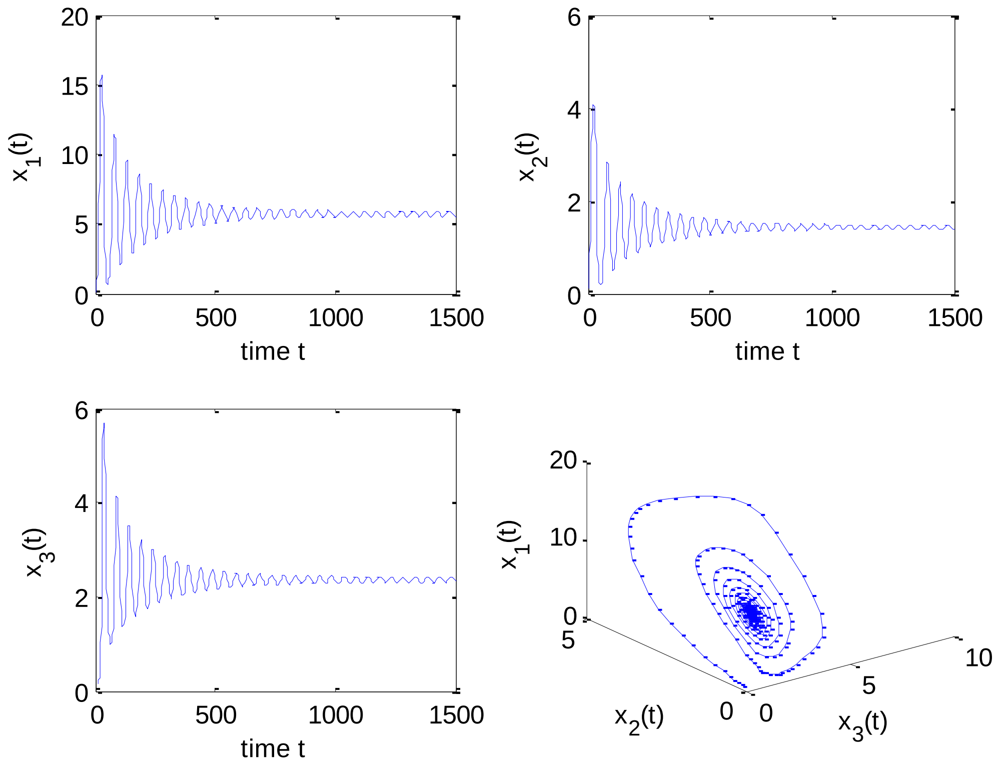

Figure 4.3 the positive equilibrium E_0 at $\tau_1 = 1.2 < \tau_{1_0} \approx 2.825$ and $\tau_2 = 5$ is local asymptotic stability.

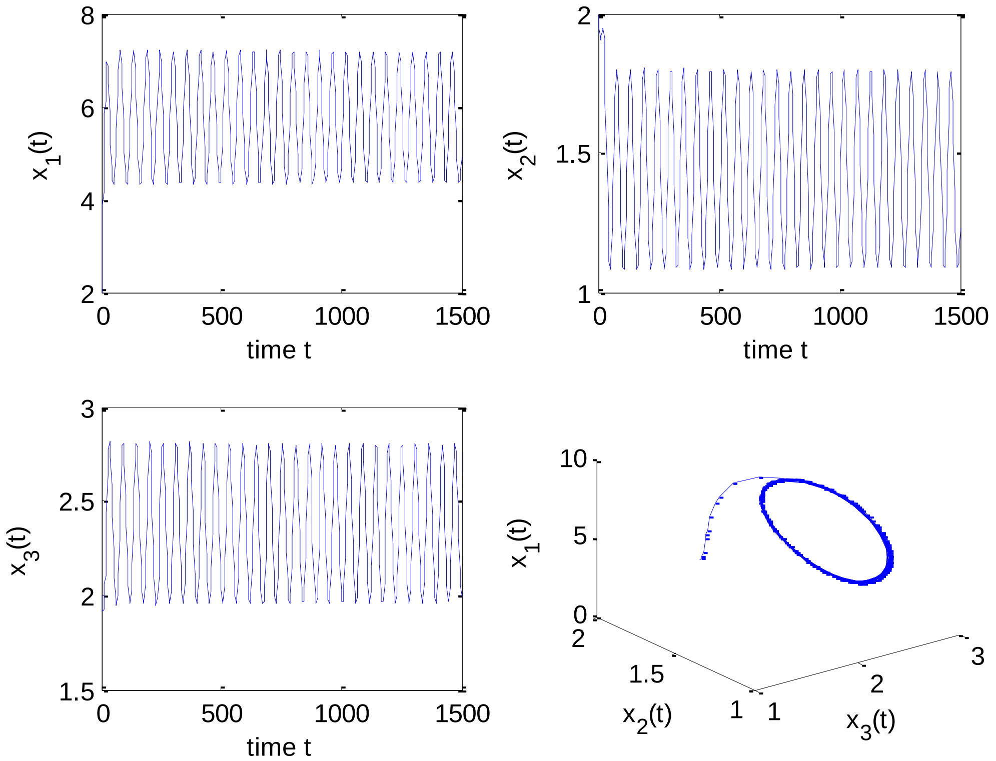

Figure 4.4 the positive equilibrium E_0 at $\tau_1 = 3.2 > \tau_{1_0} \approx 2.825$ and $\tau_2 = 5$ is unstable.

5. Conclusion

Control is closely related to the continuous survival of the population, after adjusting the parameters and the state feedback control, the unstable equilibrium points in the controlled system (4.1) become asymptotic state equilibrium points. The number of predator and prey eventually stabilized to a normal state, can coexist, never extinction. The numerical simulations verify the correctness of the theory, It has a certain ecological significance, which provides a theoretical basis for the sustainable survival of the species in nature. Can not only consider the state feedback control, but also can consider to adjust parameters, for actual control according to the specific circumstances of adjustment. The numerical simulation can find the location of the control strategy can be very good change bifurcation points, so as the bifurcation control provides a very practical method.

Just the parameters considered a_{ij} are constant, The system parameters are not related to the variables； The time delay in the model is constant, which can be considered in the future.

Acknowledge

This research is partially supported by the National Nature Science Foundation of China (71271103, 71371087, 71271107) and by the Six Talent Peaks Foundation of Jiangsu Provincial.The authors are grateful to the reviewers for their helpful comments and valuable suggestions.

6. References

[1] May. R. M.Time Delay Versus Stability in Population Models with Two or Three Tropic Levels. Ecology,. 54 (1997) 315~325.

[2] Wright. E. M． A Nonlinear Differential Equation． Journal of Reine Angewandte Mathematical,. 194(1995) 66~87.

[3] Kuang Y.Global Stability for a Class of Nonlinear Nonautonomous Delay Equations. Nonlinear Analysis.17(1991) 627~634.

[4] Faria T. Stability and bifurcation for a delayed predator–prey model and the efect of diffusion. J.Math. Anal. Appl. 254(2001)433–463.

[5] Yan.X.P, Zhang .C .H, Hopf bifurcation in a delayed Lotca-Volterra predator-prey system,Nonlinear Anal. 9(2008)114-127.

[6] Song.Y, Wei. J. Local Hopf bifurcation and global periodic solutions in a delayed predator-prey system, J. Math. Anal. Appl. 301(2005)1–21.

[7] Song.Y, Han. M, Peng. Y. Stability and Hopf bifurcation in a competitive Lotka–Volterra system with two delays. Chaos Solitons Fractals,. 22(2004) 1139–1148.

[8] Chen .G .R, Moiola. J .L ,Wang .H .O. Bifurcation control: Theories, methods and application. Int.J. Bifur. Chaos.10(3)(2000)511~548.

[9] Xiao .M, Cao .J. Stability and Hopf bifurcation in a delayed competitive web sites model. Phys.Lett. A. 353(2006)138–150.

[10] Li.J ,Ma.Z. Stability switches in a class of characteristic equations with delay-dependent parameters. Nonlinear Anal. RWA,.5(2004)389–408.

[11] Song. Y. L, Han.M.A, Xiao.D.M. Stability and Hopf bifurcations in a Competitive Lotka-Volterra System with Two Deleys[J]. Chaos, Soltions and Fractals.(2004) 1139-1148.

[12] YL Song，MA Han，JJ Wei. The stability of positive equilibrium point and global Hopf bifurcation for multiple time delay predator prey systems . Chinese annals of mathematics.25A(2004)783-790.

[13] Song .Y .L, Wei .J. J. Local Hopf Bifurcation and Global Periodic Solutions in a Delayed Predator-prey System[J]. Journal of Mathematics Analysis and Applications. 1(2005)1-21.

[14] Tang. C. B,Guan. J .B,Shen .S .W. Stability and Hopf bifurcation analysis on a differential delay system[J]. Journal of Biomathematics,. 22(2007)620-628.

[15] Chen.Y.Y, Yu. J, Sun .C.J, Stability and Hopf bifurcation analysis in a Three-level food chain system with delay[J]. Chaos, Solitons Fractals. 31(2007)683-694.

[16] Fan.M, Wang.K ,Jiang. D. Q. Existence and global attractivity of positive periodic solutions of periodic n-species Lotka-Volterra competition sysytems with several deviating arguments,Math.Biosci.160(2009)47-61.

[17] Fan. M, Wong.P. J , Agarwal. R .P. Periodicity and stability in periodic n-species

[18] Lotka-Volterra competition system with feedback controls and deviating arguments. Acta Math. Sin. Engl. Ser. 19(2013)801-822.

[19] Jiang. G. R ,Lu .Q. S. The dynamics of a prey-predator model with impulsive state feedback control. Discrete Contin.Dyn.Syst.Ser.B.6(2014)1301-1320.

[20] Xu.C.J,Liao.M.X,He.X.F.Stability and Hopf bifurcation analysis for a lotka-volterra predator-prey model with two delays.Int.J.Appl.Math.Comput.Sci.21(2011)97-107.