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Control and Stability of the Time-delay Linear Systems

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Abstract. This paper presents a method for control discrete-time system with time-delay. The main idea is a convert the discrete-time delay linear controllable system into the linear systems without delay. Then by using similarity transformations a state feedback matrix was obtained, so that time-delay has no effect on the system. The proposed technique is illustrated by means of numerical examples.

Keywords: stability, Discrete-time, Time-delay, State feedback matrix, Pole Assignment.

1. Introduction

The problem of investigation of time delay systems has been exploited over many years. Time-delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time-delay, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability.

During the last four decades, the problem of stability analysis of time delay systems has received considerable attention and many papers dealing with this problem have appeared [1]. In the literature, various stability analysis techniques have been utilized to derive stability criteria for asymptotic stability of the time delay systems by many researchers [2]-[6]. The developed stability criteria are classified often into two categories according to their dependence on the size of the delay: delay-dependent and delay-independent stability criteria [7]. It has been shown that delay dependent stability conditions that take into account the size of delays, are generally less conservative than delay-independent ones which do not include any information on the size of delays.

Further, the delay-dependent stability conditions can be classified into two classes: frequency-domain (which are suitable for systems with a small number of heterogeneous delays) and time-domain approaches (for systems with a many heterogeneous delays).

In the first approach, we can include the two or several variable polynomials [8],[9] or the small gain theorem based approach [10].

In the second approach, we have the comparison principle based techniques [11] for functional differential equations [12]-[14] and respectively the Lyapunov stability approach with the Krasovskii and Razumikhin based methods [1],[15] The stability problem is thus reduced to one of finding solutions to Lyapunov [5] or Riccati equations [12], solving linear matrix inequalities (LMIs) [16],[17] or analyzing eigenvalue distribution of appropriate finite-dimensional matrices [18] or matrix pencils [10] . For further remarks on the methods see also the guided tours proposed by [13], [19]-[22].

In this paper we used of results [23],[24] for discrete-time delay systems. This is mainly due to the fact that such systems can be transformed into augmented high dimensional systems (equivalent systems) without delay.

The remainder of this paper is organized as follows. In Section 2, the problem statement and some necessary preliminaries are given. In Section 3, we proposed method for stability this systems. Numerical simulations are provided in Section 4. Finally, some concluding remarks are given in Section 5.

2. Problem statement

Consider a controllable linear time-invariant system with time-delay defined by the state equation

$$x(k+1) = \sum_{i=0}^{p} A_i\, x(k-i) + \sum_{i=0}^{q} B_i\, u(k-i), \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input and the matrices A_i and B_i are real constant matrices of dimensions $n \times n$ and $n \times m$, respectively, with $rank(B_i) = m$.

By definition state vector such as

$$X_1(k+1) = \begin{pmatrix} x(k+1) \\ x(k) \\ \vdots \\ x(k-(p-1)) \\ u(k) \\ u(k-1) \\ \vdots \\ u(k-(q-1)) \end{pmatrix} \tag{2}$$

The system (1) with p delays in state and q delays in input vector can be rewritten as a standard system

$X_1(k+1) = AX(k) + Bu(k)$ (3)

Where

$$A = \begin{pmatrix} A_0 & A_1 & \cdots & A_{p-1} & A_p & B_1 & B_2 & \cdots & B_{q-1} & B_q \\ I_n & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_n & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & I_m & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & I_m & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & I_m & 0 \end{pmatrix}, B = \begin{pmatrix} B_0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ I_m \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

and $X_1 \in R^{\tilde{n}}, u \in R^m$, $\tilde{n} = n(p+1) + mq$.

We define control law as

$$u(k) = FX_1(k), \tag{5}$$

Where F is a feedback gain. Therefore, the system (1) changes to a standard closed-loop system

$$X_1(k+1) = (A + BF)X_1(k). \tag{6}$$

In this paper we determined the state feedback matrix F such that the eigenvalues of the closed-loop system $\Gamma = A + BF$ lie in the self-conjugate eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, \cdots \lambda_{\tilde{n}}\}$.

Karbassi and Bell [25]-[26] have introduced an algorithm obtaining an explicit parametric controller matrix F by performing three successive transformations T,S and R which transforms the controllable pair (A,B) into standard echelon form, primary vector companion form and parametric vector companion form, respectively. Let F represent the primary feedback matrix which assigns the desired set of eigenvalues to the closed-loop system.

3. Main results

Consider the state transformation

$$X_1(k) = T\tilde{X}_1(k), \tag{7}$$

where T can be obtained by elementary similarity operations as described in [26]. By replace (7) in equation (3) we have

$$\tilde{X}_1(k+1) = T^{-1}AT\,\tilde{X}_1(k) + T^{-1}B\tilde{u}(k), \tag{8}$$

In this way, $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$ are in a compact canonical form known as vector companion form:

$$\tilde{A} = \begin{bmatrix} G_0 \\ I_{\tilde{n}-m} \quad , \quad 0_{\tilde{n}-m,m} \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} B_0 \\ 0_{\tilde{n}-m,m} \end{bmatrix}. \tag{9}$$

Here G_0 is a $m \times \tilde{n}$ matrix and B_0 is an $m \times m$ upper triangular matrix. Note that if the Kroneckerinvariants of the pair (A, B) are regular, then $\tilde{A}$ and $\tilde{B}$ are always in the above form [25]. In the case of irregular Kronecker invariants, some rows of $I_{\tilde{n}-m}$ in $\tilde{A}$ are displaced [26]. It may also be concluded that if the vector companion form of $\tilde{A}$ obtained from similarity operations has the above structure, then the Kronecker invariants associated with the pair (A, B) are regular [26].

The transformed closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{F}_p$ assumes a compact Jordan form with zero eigenvalues

$$\tilde{\Gamma} = \begin{bmatrix} & 0_{m,\tilde{n}} & \\ I_{\tilde{n}-m} & , & 0_{\tilde{n}-m,m} \end{bmatrix} \tag{10}$$

It is from this form that the state feedback matrix, which assigns a set of arbitrary eigenvalues to the system and also the location of parameters, can be obtained. The controller matrix $\tilde{F}_p$ is then modified by adding a diagonal matrix $D = diag\{\lambda_1, \lambda_2, ..., \lambda_{\tilde{n}}\}$ for an arbitrary set of self-conjugate eigenvalues to $\tilde{\Gamma}$ as defined in [8]. Then the closed-loop system matrix $\tilde{A} + \tilde{B}\tilde{F}_p$ becomes

$\tilde{V}_p = \left(\tilde{A} + \tilde{B}\tilde{F}_p\right) + D$ (11)

Simple elementary similarity operations can be used to obtain the matrix $\tilde{A}_\lambda$ from $\tilde{V}_p$ such that

$$\tilde{A}_\lambda = \begin{bmatrix} & G_\lambda & \\ I_{n-m} & , & 0_{n-m,m} \end{bmatrix} \tag{12}$$

where G_λ is the first $m \times \tilde{n}$ sub-matrix of $\tilde{A}_\lambda$. Obviously, $\tilde{A}_\lambda$ possesses the desired set of eigenvalues and is in the same canonical form as $\tilde{A}$.

Thus, the primary feedback matrix which gives rise to the assignment of eigenvalues becomes

$$\tilde{F} = \tilde{F}_p + B_0^{-1} G_\lambda = B_0^{-1}(-G_0 + G_\lambda) \tag{13}$$

Clearly, disturbance is rejected and the eigenvalues of the closed-loop system $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{F}$ are in the spectrum Λ, then eigenvalues of the closed-loop $\Gamma = A + BF$ are in the spectrum Λ, where $F = \tilde{F}T^{-1}$.

4. Numerical Examples

In this section, we give three examples to show the success of the proposed method.

Example 1. Consider a discrete-time linear system with delay in state vector

$$x(k+1) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix} x(k) + \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 0 & 3 & 4 \end{pmatrix} x(k-1) + \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 2 \end{pmatrix} u(k) \tag{14}$$

We obtion

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -1 & 2 & -3 \\ 0 & 0 & 2 & 0 & 3 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{15}$$

The system (14) is practically unstable (eigenvalue ofA is $\Lambda = \{3.198, 2.194, 1.136, -1.76+0.88i, -1.76-0.88i, -1\}$, then , system open-loop is unstable). stabilization of this system is our next aim, we find feedback matrix $ F$ which the eigenvalues of the closed-loop system assign in spectrum $\Lambda = \{-0.1, -0.3, 0, 0.1, 0.3, 0.5\}$. Since, we obtain

$$F = \begin{pmatrix} 1.3333 & 1.0000 & -1.0000 & 3.5467 & -3.9100 & 1.7267 \\ -2.4900 & 1.5000 & -0.0050 & -1.0050 & -0.7500 & -1.4975 \end{pmatrix} \tag{16}$$

With norm 6.8477. Also, The system response to a unit step input is shown in Fig . 1. As you can see in Fig . 2, system of this example from initial value $X(0) = [-1, 5, 3, 1, -0.5, -3]^T$ is practically stable.

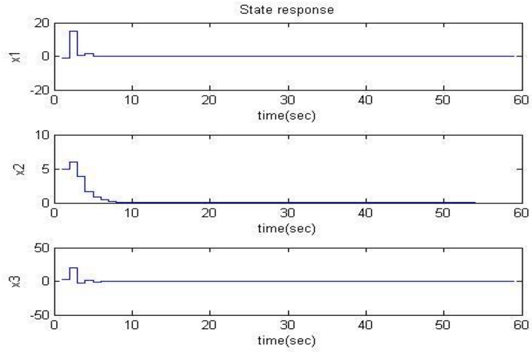

Fig. 1: State response in example1

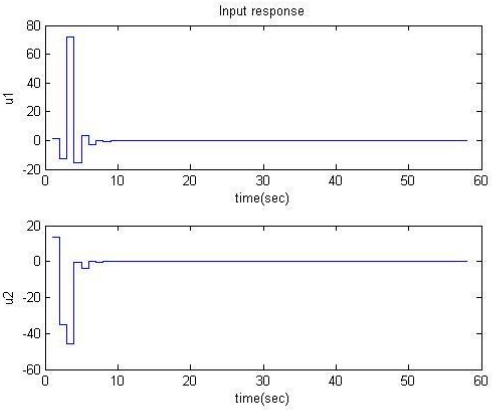

Fig. 1: Input response in example1

Example 2. Consider a discrete-time linear system with delay in state vector [27]

$$x(k+1) = \begin{pmatrix} 0.9512 & 0 \\ 0 & 0.9048 \end{pmatrix} x(k) + \begin{pmatrix} 4.8770 & 4.8770 \\ 0 & 0 \end{pmatrix} u(k) + \begin{pmatrix} 0 & 0 \\ -1.1895 & 3.5890 \end{pmatrix} u(k-1) \tag{17}$$

We find feedback matrix F which the eigenvalues of the closed-loop system assign in spectrum $\Lambda = \{-0.1, 0.1, -0.5, 0.5\}$. therefore, we obtain

$$F = \begin{pmatrix} -0.0049 & 1.1688 & -1.4814 & -0.1104 \\ -0.0005 & -0.0017 & 0.0023 & -0.5480 \end{pmatrix} \tag{18}$$

As you can see in Fig. 3, system of this example from initial value x(0) =[-1,1,5,-3]^T is practiclly stable. Fig. 4 shown Control input.

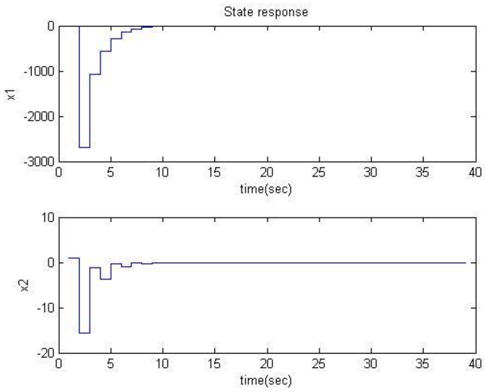

Fig. 3: State response in example2

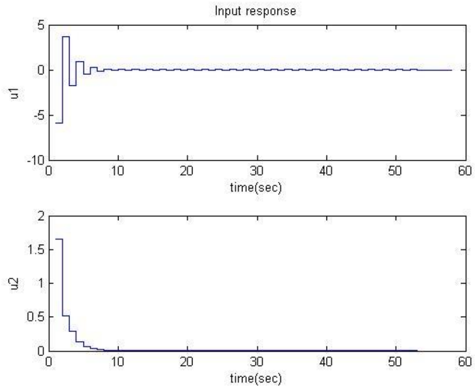

Fig. 4: Input response in example2

Example 3. Consider a discrete-time linear system with delay in state and input vector

$$x\left(k+1\right)=\begin{pmatrix}1 & 1\\ 0 & 2\end{pmatrix}x(k)+\begin{pmatrix}2 & 0\\ -1 & 2\end{pmatrix}x(k-1)+\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}u(k)+\begin{pmatrix}3 & 4\\ 2 & 1\end{pmatrix}u(k-1)+\begin{pmatrix}-2 & 3\\ 0 & 1\end{pmatrix}u(k-2) \qquad (19)$$

We find feedback matrix F which the eigenvalues of the closed-loop system assign in spectrum $\Lambda=\{-0.1,-0.2,-0.3,-0.4,0.1,0.2,0.3,0.4\}$. therefore, we obtain

$$A=\begin{pmatrix}1 & 1 & 2 & 0 & 3 & 4 & -2 & 3\\ 0 & 2 & -1 & 2 & 2 & 1 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\end{pmatrix}, B=\begin{pmatrix}1 & 0\\ 1 & 1\\ 0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{pmatrix} \qquad (20)$$

this system unstable. For stability system, state feedback matrix which locates all the eigenvalues of the closed-loop system is found to be:

$$F=\begin{pmatrix}3.1645 & -3.4568 & 4.6940 & -3.4696 & 0.0169 & 4.8777 & -2.9585 & 2.7040\\ -3.3480 & 2.6617 & -4.0619 & 2.6294 & -0.2156 & -5.3864 & 2.7439 & -2.8044\end{pmatrix} \qquad (21)$$

Also, The system response to a unit step input is shown in Fig . 5. As you can see in Fig . 6, system of this example from initial value x(0) =[-2,2,-1,1,3,-3,4,-4 $]^T$ is practically stable

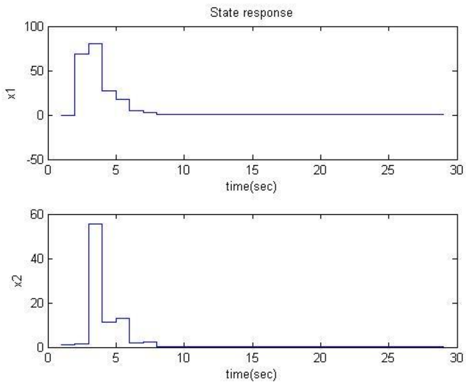

Fig. 5: State response in example3

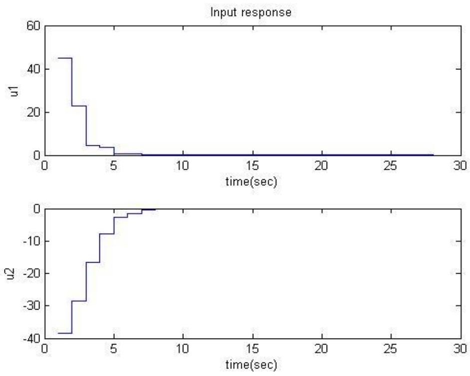

Fig. 6: Input response in example3

5. Conclusion

The control problem for linear discrete systems with time-delay is investigated in this paper. We used of augmented vector that system with time-delay convert to system without delay. Then by using similarity transformations, computed optimal controller. Also, the stability analysis of time-discrete systems with time varying delay and fractional-order system appear to be an interesting problem and open for future investigation.

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