ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 1, 2017, *pp*. 020-025

Synchronization of the neurons with external disturbance via single sliding mode controller

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(Received July 23, 2016, accepted December 04, 2016)

Abstract. The synchronization of the Hindmarsh-Rose neuron system with external disturbance is investigated via sliding mode control. Based on the Lyapunov stability theory, a single sliding mode controller is derived even when the response Hindmarsh-Rose neuron system with external disturbance. The numerical simulation is presented to verify the effectiveness of the proposed control scheme.

Keywords: synchronization; hindmarsh-rose system; chaotic bursting; sliding mode control.

1. Introduction

The brain consists of about a hundred specialized modules with different functions. These modules form complex networks [1] which shows complicated dynamic behaviors, some of which are closely related to physiological phenomena of the brain. Therefore, many researchers have explored the dynamics between the neurons of the brain to explain some neurophysiologic phenomena[2-6], especially the synchronization of the neurons [7-12]. Experimental studies [2,8] have pointed out that the synchronization is significant in the information processing of large ensembles of neurons. Hence, it is necessary to employ networks to investigate the complex spatial-temporal behavior of neural systems. Some results have been obtained about it. For example, in [13], it was noted in three types of regular networks that the critical values depended on specific coupling styles when neurons achieved complete synchronization. Shi et al. studied spike synchronization and burst synchronization of two coupled Hindmarsh-Rose (HR) neurons[14]. Wang et al. investigated ordered burst synchronization and complex spatial temporal firing behavior in a ring network of MHH neurons with excitatory chemical synapses[15]. Zheng et al. investigated the effect of various network parameters on bursting dynamics in a small-world HR neuronal network in detail[16].

In this paper, the synchronization of two identical Hindmarsh-Rose (HR) neuron systems is to be investigated. Based on the Lyapunov stability theory, a single sliding mode controller is derived even when the response Hindmarsh-Rose neuron system with external disturbance. Furthermore, the sufficient conditions for synchronization of the coupled systems with chaotic bursting behavior can be obtained. Finally, numerical simulations are given to verify the effectiveness of the proposed scheme.

2. Hindmarsh-Rose neuron system

In this section, the considered Hindmarsh-Rose neuron systems is given to realize the synchronization of the neurons with external disturbance. The HR neuronal model was first proposed by Hindmarsh and Rose as a mathematical representation of the firing behavior of neurons, and it was originally introduced to give a bursting type with long inters pike intervals of real neurons [17]. The form of Hindmarsh-Rose system is given by

$$\begin{aligned} \dot{x}_1 &= ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}, \\ \dot{x}_2 &= c - dx_1^2 - x_2, \\ \dot{x}_3 &= r(S(x_1+k) - x_3), \end{aligned} \tag{1}$$

where x_1 is the membrane potential, x_2 is associated with the fast current of Na^+ or K^+ ions, and x_3 is associated with the slow current of , for example, Ca^+ ions. a, b, c, d, r, S, k, I_{ext} are real constants. The Hindmarsh-Rose system is a slow-fast system. Slow oscillations x_3 drive the fast subsystem (x_1, x_2)

through periods of oscillatory and quiescent behavior. Model (1) may describe regular bursting or chaotic bursting for certain domains of the parameters. If the parameters are taken as a = 3.0, b = 1.0, c = 1.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, system (1) is regular bursting for I_{ext} = 2.0 and chaotic bursting for I_{ext} = 3.0, respectively (see Fig.1).

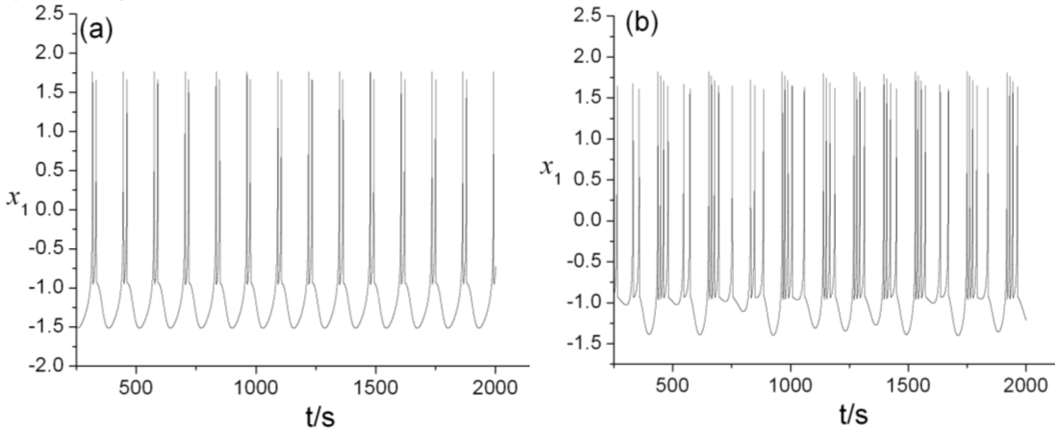

Fig.1. (a) Regular bursting of system (1) for I_{ext} =2.0,

(b) Chaotic bursting of system (1) for I_{ext} =3.0.

The response Hindmarsh-Rose system with external disturbances is described as follows:

$$\dot{y}_1 = ay_1^2 - by_1^3 + y_2 - y_3 + I_{ext} + d_1(t) + U_1,$$
$$\dot{y}_2 = c - dy_1^2 - y_2 + d_2(t),$$
$$\dot{y}_3 = r(S(y_1 + k) - y_3) + d_3(t), \tag{2}$$

where $d_1(t)$ and $d_3(t)$ are mismatched disturbances and $d_2(t)$ is a matched disturbance [18,19]. It is assumed that these disturbances are bounded, i.e. $\|d_i(t)\| \le \sigma < 1 (i = 1,2,3)$, where σ is a constant. The aim of this paper is to design a controller U_1 such that the response Hindmarsh-Rose system (2) with disturbances is synchronous with the drive Hindmarsh-Rose system (1).
For this end, let the error variables as

$$e_1 = y_1 - x_1,\ e_2 = y_2 - x_2,\ e_3 = y_3 - x_3, \tag{3}$$

in view of systems (1) and (2) , we have the following error dynamics:

$$\dot{e}_1 = a(y_1 + x_1)e_1 - b(y_1^2 + y_1x_1 + x_1^2)e_1 + e_2 - e_3 + d_1(t) + U_1,$$
$$\dot{e}_2 = -d(y_1 + x_1)e_1 - e_2 + d_2(t),$$
$$\dot{e}_3 = rSe_1 - re_3 + d_3(t). \tag{4}$$

3. Synchronization of Hindmarsh-Rose systems with disturbances

3.1. Switching surface and controller design

In this section, sliding mode control method is used to synchronize the coupled Hindmarsh-Rose systems with perturbations. This method involves two basic steps. The first step is selecting an appropriate switching surface such that the sliding motion on the sliding manifold is stable and ensures $\lim_{t\to\infty} e_i = 0\ (i = 1,2,3)$. The second step is establishing a robust control law which guarantees the existence of the sliding manifold $s_1(t)$ = 0 even in the event of external perturbation.
Let

$$U_1 = u_1 - e_1 - e_2 - a(y_1 + x_1)e_1 + b(y_1^2 + y_1x_1 + x_1^2)e_1, \tag{5}$$

and we have the following error system:

$$\dot{e}_1 = -e_1 - e_3 + d_1(t) + u_1,$$
$$\dot{e}_2 = -d(y_1 + x_1)e_1 - e_2 + d_2(t),$$
$$\dot{e}_3 = rSe_1 - re_3 + d_3(t). \tag{6}$$

By the concept of extended system [20], we construct an extended system as follows:

$$\dot{e}_1 = e_4,$$
$$\dot{e}_2 = -d(y_1 + x_1)e_1 - e_2 + d_2(t),$$
$$\dot{e}_3 = rSe_1 - re_3 + d_3(t),$$
$$\dot{e}_4 = -e_4 - rSe_1 + re_3 - d_3(t) + \dot{d}_1(t) + \dot{u}_1, \tag{7}$$

and select a suitable sliding mode surface as $s_1 = 0$, where

$$s_1 = e_4 + \int_0^t (c_1e_1 + c_3e_3 + c_4e_4)dt, \tag{8}$$

and c_1, c_3, c_4 are constants to be determined.

Theorem 1. Consider the error system (7), if this system is controlled by u_1 with

$$u_1 = \int_0^t [rSe_1 + e_4 - re_3 - c_1e_1 - c_3e_3 - c_4e_4 - k\varphi(s_1/\varepsilon)]dt, \tag{9}$$

where $\varphi(s_1/\varepsilon) = \begin{cases} s_1/\varepsilon & if\ |s_1/\varepsilon| \le 1 \\ sign(s_1/\varepsilon)\ if\ |s_1/\varepsilon| > 1 \end{cases}$, ε is a positive constant and $k \ge 2\sigma + 1$, then the system trajectory converges to the sliding surface $s_1 = 0$ in a finite time.

Proof. Consider the following Lyaponov function candidate

$$V = s_1^2, \tag{10}$$

Taking the derivative of V with respect to time, one has

$$\begin{aligned} \dot{V} &= 2s_1[\dot{e}_4 + c_1e_1 + c_3e_3 + c_4e_4] \\ &= 2s_1[-e_4 - rSe_1 + re_3 - d_3(t) + \dot{d}_1(t) + c_1e_1 + c_3e_3 + c_4e_4 + \dot{u}_1] \\ &= 2s_1[\dot{d}_1(t) - d_3(t) - k\varphi(s_1/\varepsilon)]. \end{aligned} \tag{11}$$

when $\varepsilon \to 0$, the saturation nonlinearity $\varphi(s_1/\varepsilon)$ approaches the signum nonlinearity $sign(s_1)$. In the region $|s_1| > \varepsilon$, if $k \ge 2\sigma + 1$, we have

$$\dot{V} \le 2[|\dot{d}_1| + |d_3| - k]|s_1| \le -2\sqrt{V(t)} \le 0. \tag{12}$$

It shows that when $|s_1(0)| > \varepsilon$, $|s_1|$ will be strictly decreasing until it reaches the set $|s_1| \le \varepsilon$ in a finite time and remains inside thereafter. The set $|s_1| \le \varepsilon$ is called the boundary layer. On the other hand, in the boundary layer, we have

$$\begin{aligned} \dot{V} &\le 2(|\dot{d}_1| + |d_3|)|s_1| - \frac{2k}{\varepsilon}s_1^2 \\ &\le 4\sigma|s_1| - \frac{2k}{\varepsilon}s_1^2 = 4\sigma V^{1/2} - \frac{2k}{\varepsilon}V, \end{aligned} \tag{13}$$

which implies that

$$\sqrt{V(t)} \le \sqrt{V(0)}e^{-\varepsilon t/k} + \frac{2\varepsilon}{k}\sigma. \tag{14}$$

The proof is completed.

3.2. Synchronization analysis

We are now in a position to present the synchronization analysis. Because of Theorem 1, we need to analyze the error system on the sliding mode surface. On the sliding mode surface, the error system is

$$\dot{e}_1 = e_4 ,$$
$$\dot{e}_2 = -d(y_1 + x_1)e_1 - e_2 + d_2(t) ,$$
$$\dot{e}_3 = rSe_1 - re_3 + d_3(t) ,$$
$$\dot{e}_4 = -c_1e_1 - c_3e_3 - c_4e_4 - d_3(t) + \dot{d}_1(t) . \tag{15}$$

From (15), it can be gotten that

$$\begin{pmatrix} e_1 \\ e_3 \\ e_4 \end{pmatrix} = e^{At}\left[\begin{pmatrix} e_1(0) \\ e_3(0) \\ e_4(0) \end{pmatrix} + \int_0^t e^{-At}\begin{pmatrix} 0 \\ d_3 \\ -d_3 + \dot{d} \end{pmatrix} dt\right] , \tag{16}$$

where

$$A = \begin{pmatrix} 0 & 0 & 1 \\ rS & r & 0 \\ -c_1 & -c_3 & -c_4 \end{pmatrix} . \tag{17}$$

The characteristic polynomial of matrix A is

$$f(\lambda) = \lambda^3 + (c_4 - r)\lambda^2 + (c_1 + c_4r)\lambda + r(c_3S - c_1) . \tag{18}$$

According to Routh–Hurwitz theorem, we know that the real parts of its all characteristic roots are negative if and only if

$$\Delta_1 = r(c_3S - c_1) > 0 ,$$
$$\Delta_2 = c_4 - r > 0 ,$$
$$\Delta_3 = (c_4 - r)(c_1 + c_4r) - r(c_3S - c_1) > 0 . \tag{19}$$

Obviously, there always exist such constants of c_1, c_3 and c_4 satisfying the condition in (19). Therefore, there are positive constants α and β, such that $\left|e^{At}x\right| \le \alpha e^{-\beta t}\left|x\right|$ for every $x \in R^3$ and $t \ge 0$. Thus

$$\left|e_i\right| \le \alpha e^{-\beta t}\max_{i=1,3,4}\left|e_i(0)\right| + \frac{2\alpha\sigma}{\beta} . \tag{20}$$

Additionally, the following equation can be obtained

$$e_2 = e^{-t}\{e_2(0) + \int_0^t e^t[d_2(t) - d(y_1 + x_1)e_1]dt\} . \tag{21}$$

Since a chaotic system has bounded trajectories, then there exists a positive constant M, such that $\left|x_i\right|, \left|y_i\right| \le M\,(i = 1, 2, 3)$, thus,

$$\left|e_2\right| \le e^{-t}\{\left|e_2(0)\right| + \int_0^t e^t[\sigma + 2dM\left|e_1\right|]dt\} . \tag{22}$$

It's easy to get the following theorem and corollary.

Theorem 2. If the controller is selected as

$$U_1 = u_1 - e_1 - e_2 - a(y_1 + x_1)e_1 + b(y_1^2 + y_1x_1 + x_1^2)e_1 , \tag{23}$$

where

$$u_1 = \int_0^t [rSe_1 - e_4 - re_3 - c_1e_1 - c_3e_3 - c_4e_4 - k\,\mathrm{sign}(s_1/\varepsilon)]dt , \tag{24}$$

in which c_1, c_3, c_4 are constants satisfying the inequalities (19), $k \ge 2\sigma + 1$, there exists a constant ρ such that the controlled unified chaotic system (2) with external perturbation is synchronous with the system (1) with ultimate error bound $\rho\sigma$, i.e.

$$\overline{\lim_{t\to\infty}}\left|y_i - x_i\right| \le \rho\sigma \;(i = 1, 2, 3) . \tag{25}$$

It is worth mentioning that the controller in Theorem 2 could be implemented in practice because it is continuous and there is no chatter phenomenon.

4. Numerical simulations

To verify the performance of the proposed method, numerical simulations are presented in this section. The initial values of the master system and the slave system are chosen as (1.0, 2.0, 3.0) and (2.0, 1.0, 5.0), respectively. The system parameters are taken as a = 3.0, b = 1.0, c = 1.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, with which the dynamical behaviors is seen in Fig.1. The constants in the sliding mode are selected as c_1=1, c_3=1, c_4=3, and the constant ε in the sliding mode controller defined in Eq. (9) is selected as 0.01. All characteristic roots are derived as (-2.6221, -0.3524, -0.0195). When $d_1(t)$, $d_3(t)$ and $d_2(t)$ are chosen as $0.5\sin 2t$, $0.2\cos t$ and $0.3\sin 2t$, respectively. Fig.2 depicts the synchronization error of the state variables between the master system and the slave system. Obviously, the numerical simulations verify the theoretical analysis.

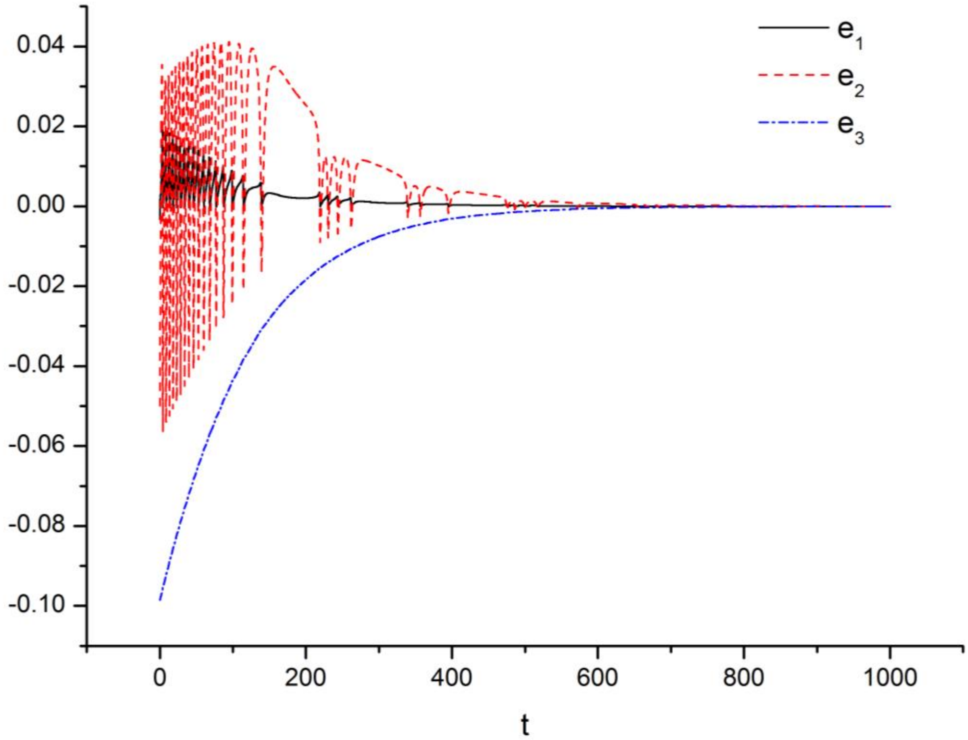

Fig.2. Dynamics of error system (6)

5. Conclusion

In this paper, an extending sliding mode control method only with a single controller is presented. By means of the sliding mode control method, synchronization between Hindmarsh-Rose neuron systems with external perturbation is investigated and sufficient conditions of synchronization are derived. Furthermore, numerical simulations are also included to visualize the effectiveness and the feasibility of the developed approach.

6. Acknowledgement

This work is supported by National Natural Science Foundation of China (Grant No. 11472238), the Natural Science Foundation of Xinjiang Uygur Autonomous Region (Grant No. 2015211C267) and the Qing Lan Project of the Jiangsu Higher Education Institutions of China.

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