ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 1, 2017, *pp*. 064-072

System of linear equations in imprecise environment

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(Received December 06, 2016, accepted December 27, 2016)

Abstract. The paper discusses linear systems in imprecise environment. In this paper we have developed the solution procedure of system of linear equations with coefficients and the right-hand side as fuzzy and intuitionistic fuzzy numbers. We have also calculated the necessary and sufficient conditions for the existence of the solutions by developing some theorems. We have solved the system by using the concept of Strong and Weak solution with numerical example.

Keywords: fuzzy system of linear equation (FSLE); intuitionistic fuzzy system of linear equation (IFSLE), trapezoidal fuzzy number (TrFN); trapezoidal intuitionisic fuzzy number (TrIFN); strong and weak solutions.

1. Introduction

There are so many applications of systems of linear equations in various areas of mathematical, physical and engineering sciences such as traffic flow, circuit analysis, heat transport, structural mechanics, fluid flow etc. In most of the applications, the system's parameters and measurements are vague or imprecise. In that situation we can represent the systems with given data as fuzzy and more generally Intuitionistic fuzzy numbers rather than crisp numbers.

Fuzzy linear systems are the linear systems whose parameters are all or partially represented by fuzzy numbers. A general model for solving a Fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number was first proposed by Friedman et al. [12]. They have used the parametric form of fuzzy numbers and replace the original $n \times n$ fuzzy system by a $2n \times 2n$ crisp system. Fuzzy linear system has been studied by several authors [1,2,4,13,14,15] but there are no such papers on intuitionistic fuzzy linear systems.

In this paper we have developed an approach to solve system of linear equations in imprecise environment i.e. fuzzy and intuitionistic fuzzy environment following Friedman et al. [12]. In Section-3 we have discussed the detailed solution procedure of system of linear equations by using the concept of Strong and Weak solution. We have also illustrated the method by considering a numerical example where coefficients are taken as TrIFNs.

2. Preliminaries

Definition 2.1: Fuzzy Set: Let X be a universal set. The fuzzy set $\tilde{A} \subseteq X$ is defined by the set of tuples as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \colon \mu_{\tilde{A}}: X \to [0,1]\}$.

The membership function $\mu_{\tilde{A}}(x)$ of a fuzzy set $\tilde{A}$ is a function with mapping $\mu_{\tilde{A}}: X \to [0,1]$. So every element x in X has membership degree $\mu_{\tilde{A}}(x)$ in [0,1] which is a real number.

Definition 2.2:**$\propto$-Level or $\propto$-cut of a fuzzy set:** Let X be an universal set. Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}(\subseteq X)$ be a fuzzy set. $\propto$ -cut of the fuzzy set $\tilde{A}$ is a crisp set. It is denoted by $A_\propto$. It is defined as

$$A_\propto = \{x : \mu_{\tilde{A}}(x) \geq \propto \ \forall x \in X\}$$

Definition-2.3: Intuitionistic Fuzzy Sets: Let $U = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An Intuitionistic Fuzzy Set $\tilde{A}^i$ in a given universal set U is an object having the form

$$\tilde{A}^i = \{\langle x_i, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle : x_i \in U\}$$

Where the functions

$\mu_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$

and $\upsilon_{\tilde{A}^i}: U \to [0,1]$; i.e. , $x_i \in U \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions:

$$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1, \forall x_i \in U$$

which is known as Intuitionistic Condition. The degree of acceptance $\mu_{\tilde{A}^i}(x_i)$and of non-acceptance $\upsilon_{\tilde{A}^i}(x_i)$ can be arbitrary.

Definition-2.4: $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cuts: A set of (α, β)-cut, generated by IFS $\tilde{A}^i$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \begin{cases} \left(x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\right); \quad x \in U \\ \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta; \quad \alpha, \beta \in [0,1] \end{cases},$$

where (α, β)-cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ at least to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.5: Fuzzy Number: $\tilde{A} \in \mathcal{F}(R)$ is called a fuzzy number where R denotes the set of whole real numbers if

i. $\tilde{A}$ is normal i.e. $x_0 \in R$ exists such that $\mu_{\tilde{A}}(x_0) = 1$.

ii. $\forall \propto \in (0,1]$ $A_\propto$ is a closed interval.

If $\tilde{A}$ is a fuzzy number then $\tilde{A}$ is a convex fuzzy set and if $\mu_{\tilde{A}}(x_0) = 1$ then $\mu_{\tilde{A}}(x)$ is non decreasing for $x \le x_0$ and non increasing for $x \ge x_0$.

Definition-2.6: Intuitionistic Fuzzy Number (IFN): An Intuitionistic Fuuzy Number $\tilde{A}^i$ is

i. An Intuitionistic Fuuzy Subset on the real line

ii. Normal i.e. there exists at least one $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\upsilon_{\tilde{A}^i}(x_0) = 0$)

iii. Convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \ge \min\left\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

iv. Concave for the non-membership function $\upsilon_{\tilde{A}^i}$ i.e.

$$\upsilon_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \le \max\left\{\upsilon_{\tilde{A}^i}(x_1), \upsilon_{\tilde{A}^i}(x_2)\right\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

Definition-2.7: Trapezoidal Intuitionistic Fuzzy Number: A Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is denoted by $\tilde{A}^i = \langle (a_1, a_2, a_3, a_4), (a_1', a_2, a_3, a_4') \rangle$ is a special Intuitionistic Fuzzy Set on a real number set $\mathbb{R}$, whose membership function and non-membership function are defined as

$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1\ , & a_2 \le x \le a_3\ , \\ \frac{a_4 - x}{a_4 - a_3}\ , & a_3 \le x \le a_4 \\ 0\ , & a_4\ \le x \end{cases}$$

$$\upsilon_{\tilde{A}^i}(x) = \begin{cases} 1 & x \le a_1' \\ 1 - \dfrac{x - a_1'}{a_2 - a_1'}, & a_1' \le x \le a_2 \\ 0\ , & a_2 \le x \le a_3 \\ 1 - \dfrac{a_4' - x}{a_4' - a_3}\ , & a_3 \le x \le a_4' \\ 1\ , & a_4'\ \le x \end{cases}$$

and $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$

Note:-

1. If $a_2 = a_3$ then Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is transformed into Triangular Intuitionistic Fuzzy Number (TIFN) $(\langle (a_1, a_2, a_4), (a_1', a_2, a_4') \rangle)$.

2. If $a_1' = a_1 \le a_2 \le a_3 \le a_4 = a_4'$ and then Trapezoidal Intuitionistic Fuzzy Number (TrIFN) is transformed into Trapezoidal Fuzzy Number (TrFN) (a_1, a_2, a_3, a_4).

Definition-2.8: The $m \times n$ linear system

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = y_n \end{gathered} \tag{2.1}$$

where the co-efficient matrix $\tilde{A}^i = \left(\widetilde{a_{rs}}^i\right)$, $1 \le r \le$ m, $1 \le$ s $\le$ n is an inuitionistic fuzzy $m \times n$ matrix and $y_i, 1 \le r \le m$ and $m \le n$ are intuitionistic fuzzy numbers is called a fully intuitionisic fuzzy linear system(FIFLS).

Note:- If the co-efficient matrix $\tilde{A} = (\widetilde{a_{rs}})$, $1 \le r \le$ m, $1 \le$ s $\le$ n is an fuzzy $m \times n$ matrix and $y_i, 1 \le r \le m$ and $m \le n$ are fuzzy numbers then the system is called a fully fuzzy linear system(FFLS).

Theorem-2.1: An intuitionistic fuzzy number vector $\left(\widetilde{x_1}^i, \widetilde{x_2}^i, \dots, \widetilde{x_n}^i\right)^T$ given by

$(x_s)_{\alpha,\beta} = x_s = \langle\left[\underline{x_s}(\alpha), \overline{x_s}(\alpha)\right], \left[\underline{x'_s}(\beta), \overline{x'_s}(\beta)\right]\rangle, 1 \le s \le n, 0 \le \alpha, \beta \le 1$ is called a solution of the intuitionistic fuzzy linear system (2.1) if

$$\begin{cases} \underline{\sum_{s=1}^{n} a_{rs} x_s} = \sum_{s=1}^{n} \underline{a_{rs} x_s} = \underline{y_r} \\ \overline{\sum_{s=1}^{n} a_{rs} x_s} = \sum_{s=1}^{n} \overline{a_{rs} x_s} = \overline{y_r} \end{cases} \tag{2.2}$$

and

$$\begin{cases} \underline{\sum_{s=1}^{n} a'_{rs} x'_s} = \sum_{s=1}^{n} \underline{a'_{rs} x'_s} = \underline{y'_r} \\ \overline{\sum_{s=1}^{n} a'_{rs} x'_s} = \sum_{s=1}^{n} \overline{a'_{rs} x'_s} = \overline{y'_r} \end{cases} \tag{2.3}$$

If for a particular r, $\widetilde{a_{rs}}^i > 0, 1 \le s \le n$, we simply get

$$\begin{cases} \sum_{s=1}^{n} \underline{a_{rs}}\ \underline{x_s} = \underline{y_r}, \sum_{s=1}^{n} \overline{a_{rs}}\ \overline{x_s} = \overline{y_r} \\ \sum_{s=1}^{n} \underline{a'_{rs}}\ \underline{x'_s} = \underline{y'_r}, \sum_{s=1}^{n} \overline{a'_{rs}}\ \overline{x'_s} = \overline{y'_r} \end{cases} \tag{2.4}$$

Note:- An fuzzy number vector $(\widetilde{x_1}, \widetilde{x_2}, \dots, \widetilde{x_n})^T$ given by

$(x_s)_\alpha = x_s = \left[\underline{x_s}(\alpha), \overline{x_s}(\alpha)\right], 1 \le s \le n, 0 \le \alpha \le 1$ is called a solution of the fuzzy linear system (2.1) if

$$\begin{cases} \underline{\sum_{s=1}^{n} a_{rs} x_s} = \sum_{s=1}^{n} \underline{a_{rs} x_s} = \underline{y_r} \\ \overline{\sum_{s=1}^{n} a_{rs} x_s} = \sum_{s=1}^{n} \overline{a_{rs} x_s} = \overline{y_r} \end{cases} \tag{2.5}$$

If for a particular r, $\widetilde{a_{rs}} > 0, 1 \le s \le n$, we simply get

$$\sum_{s=1}^{n} \underline{a_{rs}}\ \underline{x_s} = \underline{y_r}, \sum_{s=1}^{n} \overline{a_{rs}}\ \overline{x_s} = \overline{y_r} \tag{2.6}$$

3. Solution procedure for $m \times n$ fully intuitionistic fuzzy linear system

Consider the $m \times n$ linear system

$$\begin{aligned} \widetilde{a_{11}}^i \widetilde{x_1}^i + \widetilde{a_{12}}^i \widetilde{x_2}^i + \dots + \widetilde{a_{1n}}^i \widetilde{x_n}^i &= \widetilde{b_1}^i \\ \widetilde{a_{21}}^i \widetilde{x_1}^i + \widetilde{a_{22}}^i \widetilde{x_2}^i + \dots + \widetilde{a_{2n}}^i \widetilde{x_n}^i &= \widetilde{b_2}^i \\ &\vdots \\ \widetilde{a_{m1}}^i \widetilde{x_1}^i + \widetilde{a_{m2}}^i \widetilde{x_2}^i + \dots + \widetilde{a_{mn}}^i \widetilde{x_n}^i &= \widetilde{b_m}^i \end{aligned} \tag{3.1}$$

where the co-efficient matrix $\tilde{A}^i = \left(\widetilde{a_{rs}}^i\right)$, $1 \le r \le$ m, $1 \le$ s $\le$ n is a intuitionistic fuzzy $m \times n$ matrix, $\widetilde{b_r}^i, 1 \le r \le m$ are known Intuitionistic fuzzy numbers and $\widetilde{x_s}^i, 1 \le s \le n$ are unknown Intuitionistic fuzzy numbers.

In order to solve the above fully intuitionistic fuzzy linear system we must solve two $(2m) \times (2n)$ crisp linear systems where the right hand side columns are the function vector $\left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_m}, \overline{b_1}, \overline{b_2}, \dots, \overline{b_m}\right)^T$ and $\left(\underline{b'_1}, \underline{b'_2}, \dots, \underline{b'_m}, \overline{b'_1}, \overline{b'_2}, \dots, \overline{b'_m}\right)^T$.

Let us now rearrange the linear system so that the unknowns are $\underline{x_s}, (-\overline{x_s}), \underline{x'_s}$ and $\left(-\overline{x'_s}\right), 1 \le s \le n$, and the right hand side columns are

$b = \left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_m}, -\overline{b_1}, -\overline{b_2}, \dots, -\overline{b_m}\right)^T$ and $b' = \left(\underline{b'_1}, \underline{b'_2}, \dots, \underline{b'_m}, -\overline{b'_1}, -\overline{b'_2}, \dots, -\overline{b'_m}\right)^T$.

We get the 1st $(2m) \times (2n)$ crisp linear system

$$\begin{aligned} S_{1,1}\ \underline{x_1} + S_{1,2}\ \underline{x_2} + \dots + S_{1,n}\ \underline{x_n} + S_{1,n+1}(-\overline{x_1}) \\ + S_{1,n+2}(-\overline{x_2}) + \dots + S_{1,2n}(-\overline{x_n}) = \underline{b_1} \\ \vdots \\ S_{m,1}\underline{x_1} + S_{m,2}\underline{x_2} + \dots + S_{m,n}\underline{x_n} + S_{m,n+1}(-\overline{x_1}) \\ + S_{m,n+2}(-\overline{x_2}) + \dots + S_{m,2n}(-\overline{x_n}) = \underline{b_m} \end{aligned}$$

$$S_{m+1,1}\underline{x_1} + S_{m+1,2}\underline{x_2} + \cdots + S_{m+1,n}\underline{x_n} + S_{m+1,n+1}(-\overline{x_1})$$
$$+S_{m+1,n+2}(-\overline{x_2}) + \cdots + S_{m+1,2n}(-\overline{x_n}) = -\overline{b_1}$$
$$\vdots$$
$$S_{2m,1}\underline{x_1} + S_{2m,2}\underline{x_2} + \cdots + S_{2m,n}\underline{x_n} + S_{2m,n+1}(-\overline{x_1})$$
$$+S_{2m,n+2}(-\overline{x_2}) + \cdots + S_{2m,2n}(-\overline{x_n}) = -\overline{b_m}$$

Where $S_{r,s}$ are determined as follows:

$$\widetilde{a_{rs}} \geq 0 \Rightarrow S_{r,s} = \underline{a_{rs}} \ , S_{r,s+n} = 0\ , S_{r+m,s} = 0, S_{r+m,s+n} = -\overline{a_{rs}}\ ,$$
$$\widetilde{a_{rs}} < 0 \Rightarrow S_{r,s} = 0, \quad S_{r,s+n} = -\underline{a_{rs}}, \quad S_{r+m,s} = \overline{a_{rs}}, \quad S_{r+m,s+n} = 0,$$
$$1 \leq r \leq m, 1 \leq s \leq n$$

Using matrix notation we get

$$Sx = b \tag{3.2}$$

where $= (S_{r,s}),\ \ 1 \leq r \leq 2m, 1 \leq s \leq 2\text{n}$,

$$x = \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}, -\overline{x_1}, -\overline{x_2}, \dots, -\overline{x_n}\right)^T$$

and $b = \left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_m}, -\overline{b_1}, -\overline{b_2}, \dots, -\overline{b_m}\right)^T$.

Similarly we get the 2nd $(2m) \times (2n)$ crisp linear system as $S'x' = b'$ (3.3)

where $S' = (S'_{r,s}),\ \ 1 \leq r \leq 2m, 1 \leq s \leq 2\text{n}$,

$$x' = \left(\underline{x'_1}, \underline{x'_2}, \dots, \underline{x'_n}, -\overline{x'_1}, -\overline{x'_2}, \dots, -\overline{x'_n}\right)^T$$

and $b' = \left(\underline{b'_1}, \underline{b'_2}, \dots, \underline{b'_m}, -\overline{b'_1}, -\overline{b'_2}, \dots, -\overline{b'_m}\right)^T$

Case1: when m=n then we see that both S and S' are square matrices.

i.e. $S = (S_{r,s})$, and $S' = (S'_{r,s})\ \ 1 \leq r, s \leq 2\text{n}$ and $x = \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}, -\overline{x_1}, -\overline{x_2}, \dots, -\overline{x_n}\right)^T$,

$x' = \left(\underline{x'_1}, \underline{x'_2}, \dots, \underline{x'_n}, -\overline{x'_1}, -\overline{x'_2}, \dots, -\overline{x'_n}\right)^T$ and $b = \left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_n}, -\overline{b_1}, -\overline{b_2}, \dots, -\overline{b_n}\right)^T$,

$b' = \left(\underline{b'_1}, \underline{b'_2}, \dots, \underline{b'_n}, -\overline{b'_1}, -\overline{b'_2}, \dots, -\overline{b'_n}\right)^T$

So we can write (3.2) as $\begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}\begin{pmatrix} \underline{x} \\ \overline{x} \end{pmatrix} = \begin{pmatrix} \underline{b} \\ \overline{b} \end{pmatrix}$ (3.4)

and (3.3) as

$$\begin{pmatrix} S'_1 & S'_2 \\ S'_3 & S'_4 \end{pmatrix}\begin{pmatrix} \underline{x'} \\ \overline{x'} \end{pmatrix} = \begin{pmatrix} \underline{b'} \\ \overline{b'} \end{pmatrix} \tag{3.5}$$

Theorem3.1: (following Friedman et al 1998): The matrix $S = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}$ is non singular if and only if

i. the matrix $S_1 - S_2{S_4}^{-1}S_3$ and S_4 both are nonsingular.
ii. the matrix $S_4 - S_3{S_1}^{-1}S_2$ and S_1 both are nonsingular
iii. the matrix $S_1S_4 - S_2S_3$ is non singular assuming $S_3S_4 = S_4S_3$

Proof: Given $S = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}$

i. Assuming S_4 nonsingular we can write

$$S = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}\begin{pmatrix} \text{I} & 0 \\ -{S_4}^{-1}S_3 & \text{I} \end{pmatrix} = \begin{pmatrix} S_1 - S_2{S_4}^{-1}S_3 & S_2 \\ 0 & S_4 \end{pmatrix}$$

Clearly $|S| = \left|(S_1 - S_2{S_4}^{-1}S_3)S_4\right| = \left|S_1 - S_2{S_4}^{-1}S_3\right||S_4|$

Therefore $|S| \neq 0$ if and only if $|S_4| \neq 0$ and $\left|S_1 - S_2{S_4}^{-1}S_3\right| \neq 0$

ii. Again assuming S_1 nonsingular we can write

$$S = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}\begin{pmatrix} \text{I} & -{S_1}^{-1}S_2 \\ 0 & \text{I} \end{pmatrix} = \begin{pmatrix} S_1 & 0 \\ S_3 & S_4 - S_3{S_1}^{-1}S_2 \end{pmatrix}$$

Clearly $|S| = \left|S_1(S_4 - S_3{S_1}^{-1}S_2)\right| = |S_1|\left|S_4 - S_3{S_1}^{-1}S_2\right|$

Therefore $|S| \neq 0$ if and only if $|S_1| \neq 0$ and $\left|S_4 - S_3{S_1}^{-1}S_2\right| \neq 0$

iii. Let $S_3S_4 = S_4S_3$

$$\begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}\begin{pmatrix} S_4 & 0 \\ -S_3 & \text{I} \end{pmatrix} = \begin{pmatrix} S_1S_4 - S_2S_3 & S_2 \\ S_3S_4 - S_4S_3 & S_4 \end{pmatrix} = \begin{pmatrix} S_1S_4 - S_2S_3 & S_2 \\ 0 & S_4 \end{pmatrix}$$

Clearly $|S||S_4| = |S_4(S_1S_4 - S_2S_3)| = |S_4||S_1S_4 - S_2S_3| \Rightarrow |S| = |S_1S_4 - S_2S_3|$
Therefore $|S| \neq 0$ if and only if $|S_1S_4 - S_2S_3| \neq 0$
which concludes the proof.

Similarly for S'.

Definition3.1: If $X = \langle \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}, \overline{x_1}, \overline{x_2}, \dots, \overline{x_n}\right)^T, \left(\underline{x'_1}, \underline{x'_2}, \dots, \underline{x'_n}, \overline{x'_1}, \overline{x'_2}, \dots, \overline{x'_n}\right)^T \rangle$ is a solution of (3.1) and for each $1 \leq s \leq n$ the inequalities $\underline{x_s} \leq \overline{x_s}$, $\underline{x'_s} \leq \overline{x'_s}$ and hold, then the solution $\widetilde{x_s}^i$ is called a strong solution of the system (3.1).

Definition3.2: If $X = \langle \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}, \overline{x_1}, \overline{x_2}, \dots, \overline{x_n}\right)^T, \left(\underline{x'_1}, \underline{x'_2}, \dots, \underline{x'_n}, \overline{x'_1}, \overline{x'_2}, \dots, \overline{x'_n}\right)^T \rangle$ is a solution of (3.1) and for some $s \in [1, n]$ the inequality $\underline{x_s} > \overline{x_s}$ or, $\underline{x'_s} > \overline{x'_s}$ hold, then the solution $\widetilde{x_s}^i$ is called a weak solution of the system (3.1).

Theorem3.2: The necessary and sufficient conditions for the existence of a strong solution:
Let $S = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}$ be a nonsingular matrix. The system (3.2) has a strong solution if and only if

i. $S_4^{-1}\left(S_1 - S_2S_4^{-1}S_3\right)^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\} \leq 0$ provided $|S_4| \neq 0$
ii. $\left(S_4 - S_3S_1^{-1}S_2\right)^{-1}S_1^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\} \leq 0$ provided $|S_1| \neq 0$
iii. $(S_1S_4 - S_2S_3)^{-1}\left((S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\right) \leq 0$ provided $S_3S_4 = S_4S_3$

Proof: Let us define $\underline{x} = \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}\right)^T$, $\overline{x} = (\overline{x_1}, \overline{x_2}, \dots, \overline{x_n})^T$ and

$$\underline{b} = \left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_n}\right)^T, \overline{b} = \left(\overline{b_1}, \overline{b_2}, \dots, \overline{b_n}\right)^T$$

from the system (3.2) we obtain

$$\begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}\begin{pmatrix} \underline{x} \\ -\overline{x} \end{pmatrix} = \begin{pmatrix} \underline{b} \\ -\overline{b} \end{pmatrix}$$

Hence $S_1\underline{x} - S_2\overline{x} = \underline{b}$ and $S_3\underline{x} - S_4\overline{x} = -\overline{b}$
Solving the above two equations we get
$(S_1S_4 - S_2S_3)\left(\underline{x} - \overline{x}\right) = (S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}$
Here given that S is non singular

i. If S_4 is nonsingular then we can write $S_1S_4 - S_2S_3 = \left(S_1 - S_2S_4^{-1}S_3\right)S_4$

Then $\left(\underline{x} - \overline{x}\right) = S_4^{-1}\left(S_1 - S_2S_4^{-1}S_3\right)^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\}$
By the theorem1 the matrices S_4 and $S_1 - S_2S_4^{-1}S_3$ as S is nonsingular.
Now If the system (3.2) has a strong solution then by the definition3, we have $\underline{x} - \overline{x} \leq 0$.
Hence $S_4^{-1}\left(S_1 - S_2S_4^{-1}S_3\right)^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\} \leq 0$ holds.

ii. If S_1 is nonsingular then we can write $S_1S_4 - S_2S_3 = S_1\left(S_4 - S_3S_1^{-1}S_2\right)$

Then $\left(\underline{x} - \overline{x}\right) = \left(S_4 - S_3S_1^{-1}S_2\right)^{-1}S_1^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\}$
By the theorem1 the matrices S_1 and $S_4 - S_3S_1^{-1}S_2$ as S is nonsingular.
Now If the system (3.2) has a strong solution then by the definition3, we have $\underline{x} - \overline{x} \leq 0$.
Hence $\left(S_4 - S_3S_1^{-1}S_2\right)^{-1}S_1^{-1}\{(S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\} \leq 0$ holds.

iii. Again we can write $\underline{x} - \overline{x} = (S_1S_4 - S_2S_3)^{-1}\left((S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\right)$

If $S_3S_4 = S_4S_3$ then by the theorem1 the matrix $S_1S_4 - S_2S_3$ as S is nonsingular.
If the system (3.2) has a strong solution then by the definition3, we have $\underline{x} - \overline{x} \leq 0$
Hence $(S_1S_4 - S_2S_3)^{-1}\left((S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}\right) \leq 0$ holds.
Conversely if the above inequalities hold then from the relation

$(S_1S_4 - S_2S_3)\left(\underline{x} - \overline{x}\right) = (S_4 - S_3)\underline{b} - (S_2 - S_1)\overline{b}$ we get $\underline{x} - \overline{x} \leq 0$.

Hence the proof.
Similarly we can apply the above theorem for the system (3.3)
In order to solve the above two linear systems we must now calculate $S^{-1}$and S'^{-1} (if exists).
Assuming that S is nonsingular we get
$x = S^{-1}b$

Now

$$S^{-1} = \begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}^{-1}$$

Let

$$\begin{pmatrix} S_1 & S_2 \\ S_3 & S_4 \end{pmatrix}^{-1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

So

$$\begin{pmatrix} S_1 & S_2 \\ S_2 & S_1 \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \mathrm{I} & \mathbf{0} \\ \mathbf{0} & \mathrm{I} \end{pmatrix}$$

where **0** is the null matrix and I is the unit matrix.
or,

$$S_1A + S_2C = \mathrm{I} \quad , \qquad S_1B + S_2D = \mathbf{0}$$
$$S_3A + S_4C = \mathbf{0} \quad , \qquad S_3B + S_4D = \mathrm{I}$$

Solving the above equations we get

$$A = S_4(S_1S_4 - S_2S_3)^{-1}, B = -S_2(S_1S_4 - S_2S_3)^{-1},$$
$$C = -S_3(S_1S_4 - S_2S_3)^{-1}, D = S_1(S_1S_4 - S_2S_3)^{-1}$$

Similarly assuming S' as nonsingular we get $x' = S'^{-1}b'$

Note: further if all $\widetilde{a_{rs}}^i, 1 \le r, s \le n$ are positive then we get

$$S = \begin{pmatrix} S_1 & 0 \\ 0 & S_3 \end{pmatrix}$$
$$S^{-1} = \begin{pmatrix} S_1 & 0 \\ 0 & S_3 \end{pmatrix}^{-1} = \begin{pmatrix} {S_1}^{-1} & 0 \\ 0 & {S_3}^{-1} \end{pmatrix}$$

Therefore the solution is

$$x = S^{-1}b$$

i.e. $\begin{pmatrix} \underline{x} \\ \overline{x} \end{pmatrix} = \begin{pmatrix} {S_1}^{-1} & O \\ O & {S_3}^{-1} \end{pmatrix}\begin{pmatrix} \underline{b} \\ \overline{b} \end{pmatrix}$

where $S = \begin{pmatrix} S_1 & 0 \\ 0 & S_3 \end{pmatrix}, x = \begin{pmatrix} \underline{x} \\ \overline{x} \end{pmatrix}, b = \begin{pmatrix} \underline{b} \\ \overline{b} \end{pmatrix}$ and $\underline{x} = \left(\underline{x_1}, \underline{x_2}, \dots, \underline{x_n}\right)^T, \overline{x} = (-\overline{x_1}, -\overline{x_2}, \dots, -\overline{x_n})^T$ and $\underline{b} = \left(\underline{b_1}, \underline{b_2}, \dots, \underline{b_n}\right)^T, \overline{b} = \left(-\overline{b_1}, -\overline{b_2}, \dots, -\overline{b_n}\right)^T$ or, $\underline{x} = {S_1}^{-1}\underline{b}$ and $\overline{x} = {S_3}^{-1}\overline{b}$.

Similarly we get $\underline{x'} = {S'_1}^{-1}\underline{b'}$ and $\overline{x'} = {S'_3}^{-1}\overline{b'}$

Case2: When $m \ne n$ then both S and S' are not square matrices. These are rectangular matrix so we calculate inverse of S and S' in terms of generalized inverse.

Let the generalized inverse of S be S^+

Where $S^+ = \begin{cases} (S^TS)^{-1}S^T & \text{if S has full column rank i.e. rank}(S) = n < m \\ S^T(SS^T)^{-1} & \text{if S has full row rank i.e. rank}(S) = m < n \end{cases}$

Similarly for the matrix S'

Therefore the solution is

$$x = S^+b \text{ and } x' = S'^+b'$$

Note:- If we only solve the system (3.2) then that is solution procedure for fully fuzzy linear system.

Example3.1: Consider the 2×2 fully intuitionistic fuzzy linear system

$\langle(3,4,5,6)(2,4,5,7)\rangle\widetilde{x_1}^i + \langle(4,5,6,8)(3,5,6,9)\rangle\widetilde{x_2}^i = \langle(25,35,50,67)(20,35,50,73)\rangle$

$\langle(5,6,7,7)(4,6,7,8)\rangle\widetilde{x_1}^i + \langle(3,3,4,5)(2,3,4,6)\rangle\widetilde{x_2}^i = \langle(27,,32,48,55)(22,32,48,62)\rangle$

$\left(\widetilde{a_{11}}^i\right)_{\alpha,\beta} = \langle\left[\underline{a_{11}}, \overline{a_{11}}\right], \left[\underline{a'_{11}}, \overline{a'_{11}}\right]\rangle = \langle[3+\alpha, 6-\alpha], [2+2\beta, 7-2\beta]\rangle$,

$\left(\widetilde{a_{12}}^i\right)_{\alpha,\beta} = \langle\left[\underline{a_{12}}, \overline{a_{12}}\right], \left[\underline{a'_{12}}, \overline{a'_{12}}\right]\rangle = \langle[4+\alpha, 8-2\alpha], [3+2\beta, 9-3\beta]\rangle$,

$\left(\widetilde{a_{21}}^i\right)_{\alpha,\beta} = \langle\left[\underline{a_{21}}, \overline{a_{21}}\right], \left[\underline{a'_{21}}, \overline{a'_{21}}\right]\rangle = \langle[5+\alpha, 7], [4+2\beta, 8-\beta]\rangle$,

$\left(\widetilde{a_{22}}^i\right)_{\alpha,\beta} = \langle\left[\underline{a_{22}}, \overline{a_{22}}\right], \left[\underline{a'_{22}}, \overline{a'_{22}}\right]\rangle = \langle[3, 5-\alpha], [2+\beta, 6-2\beta]\rangle$

$\left(\widetilde{b_1}^i\right)_{\alpha,\beta} = \langle\left[\underline{b_1}, \overline{b_1}\right], \left[\underline{b'_1}, \overline{b'_1}\right]\rangle = \langle[25+10\alpha, 67-17\alpha], [20+15\beta, 73-23\beta]\rangle$,

$\left(\widetilde{b_2}^i\right)_{\alpha,\beta} = \langle\left[\underline{b_2}, \overline{b_2}\right], \left[\underline{b'_2}, \overline{b'_2}\right]\rangle = \langle[27+5\alpha, 55-7\alpha], [22+10\beta, 62-14\beta]\rangle$

Let the (α,β)-cut of the solution be

$(\widetilde{x_1}^i)_{\alpha,\beta} = \langle[\underline{x_1}, \overline{x_1}], [\underline{x'_1}, \overline{x'_1}]\rangle$, $(\widetilde{x_2}^i)_{\alpha,\beta} = \langle[\underline{x_2}, \overline{x_2}], [\underline{x'_2}, \overline{x'_2}]\rangle$

Then the 1st 4×4 system is

$(3+\alpha).\underline{x_1} + (4+\alpha).\underline{x_2} + 0.(-\overline{x_1}) + 0.(-\overline{x_2}) = 25 + 10\alpha$

$(5+\alpha).\underline{x_1} + 3.\underline{x_2} + 0.(-\overline{x_1}) + 0.(-\overline{x_2}) = 27 + 5\alpha$

$0.\underline{x_1} + 0.\underline{x_2} + (6-\alpha).(-\overline{x_1}) + (8-2\alpha).(-\overline{x_2}) = -(67 - 17\alpha)$

$0.\underline{x_1} + 0.\underline{x_2} + 7.(-\overline{x_1}) + (5-\alpha).(-\overline{x_2}) = -(55 - 7\alpha)$

or, $\begin{pmatrix} 3+\alpha & 4+\alpha & 0 & 0 \\ 5+\alpha & 3 & 0 & 0 \\ 0 & 0 & 6-\alpha & 8-2\alpha \\ 0 & 0 & 7 & 5-\alpha \end{pmatrix}\begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ -\overline{x_1} \\ -\overline{x_2} \end{pmatrix} = \begin{pmatrix} 25+10\alpha \\ 27+5\alpha \\ 17\alpha-67 \\ 7\alpha-55 \end{pmatrix}$

here $S = \begin{pmatrix} 3+\alpha & 4+\alpha & 0 & 0 \\ 5+\alpha & 3 & 0 & 0 \\ 0 & 0 & 6-\alpha & 8-2\alpha \\ 0 & 0 & 7 & 5-\alpha \end{pmatrix} = \begin{pmatrix} S_1 & S_2 \\ S_2 & S_3 \end{pmatrix}$

where $S_1 = \begin{pmatrix} 3+\alpha & 4+\alpha \\ 5+\alpha & 3 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0}, S_3 = \begin{pmatrix} 6-\alpha & 8-2\alpha \\ 7 & 5-\alpha \end{pmatrix}$

Now $S^{-1} = \begin{pmatrix} 3+\alpha & 4+\alpha & 0 & 0 \\ 5+\alpha & 3 & 0 & 0 \\ 0 & 0 & 6-\alpha & 8-2\alpha \\ 0 & 0 & 7 & 5-\alpha \end{pmatrix}^{-1} = \begin{pmatrix} S_1 & 0 \\ 0 & S_3 \end{pmatrix}^{-1} = \begin{pmatrix} S_1^{-1} & 0 \\ 0 & S_3^{-1} \end{pmatrix}$

Now $S_1^{-1} = \begin{pmatrix} 3+\alpha & 4+\alpha \\ 5+\alpha & 3 \end{pmatrix}^{-1} = \frac{1}{(-\alpha^2-6\alpha-11)}\begin{pmatrix} 3 & -4-\alpha \\ -5-\alpha & 3+\alpha \end{pmatrix}$

$S_3^{-1} = \begin{pmatrix} 6-\alpha & 8-2\alpha \\ 7 & 5-\alpha \end{pmatrix}^{-1} = \frac{1}{(\alpha^2+3\alpha-26)}\begin{pmatrix} 5-\alpha & -8+2\alpha \\ -7 & 6-\alpha \end{pmatrix}$

So $S^{-1} = \begin{pmatrix} \frac{3}{-\alpha^2-6\alpha-11} & \frac{-4-\alpha}{-\alpha^2-6\alpha-11} & 0 & 0 \\ \frac{-5-\alpha}{-\alpha^2-6\alpha-11} & \frac{3+\alpha}{-\alpha^2-6\alpha-11} & 0 & 0 \\ 0 & 0 & \frac{5-\alpha}{\alpha^2+3\alpha-26} & \frac{-8+2\alpha}{\alpha^2+3\alpha-26} \\ 0 & 0 & \frac{-7}{\alpha^2+3\alpha-26} & \frac{6-\alpha}{\alpha^2+3\alpha-26} \end{pmatrix}$

Similarly the 2nd 4×4 system is

$(2+2\beta).\underline{x'_1} + (3+2\beta).\underline{x'_2} + 0.(-\overline{x'_1}) + 0.(-\overline{x'_2}) = 20 + 15\beta$

$(4+2\beta).\underline{x'_1} + (2+\beta).\underline{x'_2} + 0.(-\overline{x'_1}) + 0.(-\overline{x'_2}) = 22 + 10\beta$

$0.\underline{x'_1} + 0.\underline{x'_2} + (7-2\beta).(-\overline{x'_1}) + (9-3\beta).(-\overline{x'_2}) = -(73 - 23\beta)$

$0.\underline{x'_1} + 0.\underline{x'_2} + (8-\beta).(-\overline{x'_1}) + (6-2\beta).(-\overline{x'_2}) = -(62 - 14\beta)$

or, $\begin{pmatrix} 2+2\beta & 3+2\beta & 0 & 0 \\ 4+2\beta & 2+\beta & 0 & 0 \\ 0 & 0 & 7-2\beta & 9-3\beta \\ 0 & 0 & 8-\beta & 6-2\beta \end{pmatrix}\begin{pmatrix} \underline{x'_1} \\ \underline{x'_2} \\ -\overline{x'_1} \\ -\overline{x'_2} \end{pmatrix} = \begin{pmatrix} 20+15\beta \\ 22+10\beta \\ 23\beta-73 \\ 14\beta-62 \end{pmatrix}$

here $S' = \begin{pmatrix} 2+2\beta & 3+2\beta & 0 & 0 \\ 4+2\beta & 2+\beta & 0 & 0 \\ 0 & 0 & 7-2\beta & 9-3\beta \\ 0 & 0 & 8-\beta & 6-2\beta \end{pmatrix} = \begin{pmatrix} S'_1 & S'_2 \\ S'_2 & S'_3 \end{pmatrix}$

where $S'_1 = \begin{pmatrix} 2+2\beta & 3+2\beta \\ 4+2\beta & 2+\beta \end{pmatrix}, S'_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0}, S'_3 = \begin{pmatrix} 7-2\beta & 9-3\beta \\ 8-\beta & 6-2\beta \end{pmatrix}$

Now $S'^{-1} = \begin{pmatrix} 2+2\beta & 3+2\beta & 0 & 0 \\ 4+2\beta & 2+\beta & 0 & 0 \\ 0 & 0 & 7-2\beta & 9-3\beta \\ 0 & 0 & 8-\beta & 6-2\beta \end{pmatrix}^{-1} = \begin{pmatrix} S'_1 & 0 \\ 0 & S'_3 \end{pmatrix}^{-1} = \begin{pmatrix} {S'_1}^{-1} & 0 \\ 0 & {S'_3}^{-1} \end{pmatrix}$

Now $S'_1{}^{-1} = \begin{pmatrix} 2+2\beta & 3+2\beta \\ 4+2\beta & 2+\beta \end{pmatrix}^{-1} = \frac{1}{(-2\beta^2-8\beta-8)}\begin{pmatrix} 2+\beta & -3-2\beta \\ -4-2\beta & 2+2\beta \end{pmatrix}$

$S'_3{}^{-1} = \begin{pmatrix} 7-2\beta & 9-3\beta \\ 8-\beta & 6-2\beta \end{pmatrix}^{-1} = \frac{1}{(\beta^2+7\beta-30)}\begin{pmatrix} 6-2\beta & -9+3\beta \\ -8+\beta & 7-2\beta \end{pmatrix}$

So $S'^{-1} = \begin{pmatrix} \frac{2+\beta}{-2\beta^2-8\beta-8} & \frac{-3-2\beta}{-2\beta^2-8\beta-8} & 0 & 0 \\ \frac{-4-2\beta}{-2\beta^2-8\beta-8} & \frac{2+2\beta}{-2\beta^2-9\beta-10} & 0 & 0 \\ 0 & 0 & \frac{6-2\beta}{\beta^2+7\beta-30} & \frac{-9+3\beta}{\beta^2+7\beta-30} \\ 0 & 0 & \frac{-8+\beta}{\beta^2+7\beta-30} & \frac{7-2\beta}{\beta^2+7\beta-30} \end{pmatrix}$

Therefore we get the solution from

$x = S^{-1}b$ and $x' = S'^{-1}b'$

Now, $x = S^{-1}b \Longrightarrow \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ -\overline{x_1} \\ -\overline{x_2} \end{pmatrix}$

$$= \begin{pmatrix} \frac{3}{-\alpha^2-6\alpha-11} & \frac{-4-\alpha}{-\alpha^2-6\alpha-11} & 0 & 0 \\ \frac{-5-\alpha}{-\alpha^2-6\alpha-11} & \frac{3+\alpha}{-\alpha^2-6\alpha-11} & 0 & 0 \\ 0 & 0 & \frac{5-\alpha}{\alpha^2+3\alpha-26} & \frac{-8+2\alpha}{\alpha^2+3\alpha-26} \\ 0 & 0 & \frac{-7}{\alpha^2+3\alpha-26} & \frac{6-\alpha}{\alpha^2+3\alpha-26} \end{pmatrix}\begin{pmatrix} 25+10\alpha \\ 27+5\alpha \\ 17\alpha-67 \\ 7\alpha-55 \end{pmatrix}$$

and $x' = S'^{-1}b' \Longrightarrow \begin{pmatrix} \underline{x'_1} \\ \underline{x'_2} \\ -\overline{x'_1} \\ -\overline{x'_2} \end{pmatrix}$

$$= \begin{pmatrix} \frac{2+\beta}{-2\beta^2-8\beta-8} & \frac{-3-2\beta}{-2\beta^2-8\beta-8} & 0 & 0 \\ \frac{-4-2\beta}{-2\beta^2-8\beta-8} & \frac{2+2\beta}{-2\beta^2-9\beta-10} & 0 & 0 \\ 0 & 0 & \frac{6-2\beta}{\beta^2+7\beta-30} & \frac{-9+3\beta}{\beta^2+7\beta-30} \\ 0 & 0 & \frac{-8+\beta}{\beta^2+7\beta-30} & \frac{7-2\beta}{\beta^2+7\beta-30} \end{pmatrix}\begin{pmatrix} 20+15\beta \\ 22+10\beta \\ 23\beta-73 \\ 14\beta-62 \end{pmatrix}$$

$\therefore \underline{x_1} = \left(\frac{3}{-\alpha^2-6\alpha-11}\right)(25+10\alpha) + \left(\frac{-4-\alpha}{-\alpha^2-6\alpha-11}\right)(27+5\alpha) = \frac{5\alpha^2+20\alpha+33}{\alpha^2+6\alpha+11}$

$\underline{x_2} = \left(\frac{-5-\alpha}{-\alpha^2-6\alpha-11}\right)(25+10\alpha) + \left(\frac{3+\alpha}{-\alpha^2-6\alpha-11}\right)(27+5\alpha) = \frac{5\alpha^2+33\alpha+44}{\alpha^2+6\alpha+11}$

$\overline{x_1} = -\left(\frac{5-\alpha}{\alpha^2+3\alpha-26}\right)(17\alpha-67) - \left(\frac{-8+2\alpha}{\alpha^2+3\alpha-26}\right)(7\alpha-55) = \frac{3\alpha^2+14\alpha-105}{\alpha^2+3\alpha-26}$

$\overline{x_2} = -\left(\frac{-7}{\alpha^2+3\alpha-26}\right)(17\alpha-67) - \left(\frac{6-\alpha}{\alpha^2+3\alpha-26}\right)(7\alpha-55) = \frac{7\alpha^2+22\alpha-139}{\alpha^2+3\alpha-26}$

and $\underline{x'_1} = \left(\frac{2+\beta}{-2\beta^2-8\beta-8}\right)(20+15\beta) + \left(\frac{-3-2\beta}{-2\beta^2-8\beta-8}\right)(22+10\beta) = \frac{3\beta^2+22\beta+26}{2\beta^2+8\beta+8}$

$\underline{x'_2} = \left(\frac{-4-2\beta}{-2\beta^2-8\beta-8}\right)(20+15\beta) + \left(\frac{2+2\beta}{-2\beta^2-9\beta-10}\right)(22+10\beta) = \frac{10\beta^2+36\beta+36}{2\beta^2+8\beta+8}$

$\overline{x'_1} = -\left(\frac{6-2\beta}{\beta^2+7\beta-30}\right)(23\beta-73) - \left(\frac{-9+3\beta}{\beta^2+7\beta-30}\right)(14\beta-62) = \frac{4\beta^2+28\beta-120}{\beta^2+7\beta-30}$

$\overline{x'_2} = -\left(\frac{-8+\beta}{\beta^2+7\beta-30}\right)(23\beta-73) - \left(\frac{7-2\beta}{\beta^2+7\beta-30}\right)(14\beta-62) = \frac{5\beta^2+35\beta-150}{\beta^2+7\beta-30}$

Here we see that the solutions

$\widetilde{x_1}^i \approx \langle(3,3.22,4,4),(2.83,3.25,4,4)\rangle$ and $\widetilde{x_2}^i \approx \langle(4,4.56,5,5.35),(4.5,4.5,5,5)\rangle$ are Trapezoidal Shaped Intuitionistic Fuzzy Numbers and both are strong solutions.

4. Conclusion

Various approaches have been used by the other authors to solve fully fuzzy and intuitionistic fuzzy system of linear equations. In this paper we have used a new approach to solve both fuzzy and intuitionistic fuzzy linear systems. We have also considered a numerical example and solved by using this approach. There are so many areas in Engineering Sciences such as electrical, civil, mechanical and chemical engineering where we can handle the problems involving linear systems by this approach.

5. References

[1] Amit Kumar, Neetu, Abhinav Bansal, A new approach for solving fully fuzzy linear systems, Hindawi Publishing Corporation, Advances in Fuzzy Systems, vol.2011, Article ID 943161, 8 pages.

[2] Amit Kumar, Neetu, Abhinav Bansal, A new method to solve fully fuzzy linear system with trapezoidal fuzzy numbers, Canadian Journal on Science and Engineering Mathematics, vol.1,no.3, August 2010.

[3] Didier Dubois **,** Henri Prade, Systems of fuzzy linear constraints(1978)

[4] T. Allahviranloo, N. Mikaeilvand, F. Hoseinzadeh Lotfi, M. Fallah Jelodar, Fully Fuzzy Linear Systems, International Journal of Applied Operaional Research, vol.1, no.1, pp.35-48, summer 2011.

[5] J. J. Buckley, Y. Qu, Solving systems of linear fuzzy equations, Fuzzy Sets and Systems 43:33-43(1991).

[6] J. J. Buckley, Fuzzy input-output analysis , European journal of operational research39:54-60(1989).

[7] J. J. Buckley , Y. Qu, Solving linear and quadratic fuzzy equations, Fuzzy Sets and Systems 38:43-59(1990).

[8] J. J. Buckley, Y. Qu, On using ∝-cuts to evaluate fuzzy equations, Fuzzy Sets and Systems 38: 309-312(1990).

[9] J.J. Buckley, Y. Qu, Solving fuzzy equations: a new solution concept,Fuzzy Sets and Systems 39:291–301(1991).

[10] L. A. Zadeh. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1:3–28, 1978.

[11] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, I Information Sciences. 8: 199-249(1975).

[12] M. Friedman, Ma. Ming, A. Kandel, Fuzzy linear systems, Fuzzy Sets and Systems 96: 201–209(1998).

[13] Sahin Emrah Amrahov, Iman N. Askerzade, Strong Solutions of the Fuzzy Linear Systems.

[14] S. H. Nasseri, M. Gholami, Linear system of equations with trapezoidal fuzzy numbers , The Journal of Mathematics and Computer Science, vol.3, no.1: 71-79(2011).

[15] Taher Rafgooy , Hadi Sadghi Yazhi , Reza Monsefi, Fuzzy Complex System of Linear Equations Applied to Circuit Analysis (International Journal of Computer and Electrical Engineering,vol.1, no.5, December2009.

[16] S.H. Nasseri and F. Zahmatkesh,Huang method for solving fully fuzzy linear system of equations, The Journal of Mathematics and Computer Science, Vol .1 No.1 : 1-5(2010).

[17] Sukanta Nayak , S. Chakraverty, A New Approach to Solve Fuzzy System of Linear Equations, Journal of Mathematics and Computer Science,vol 7: 205-212(2013).

[18] S.H.Nasseri, M.Sohrabi, Gram-Schmidt Approach for Linear System of Equations with Fuzzy Parameters, The Journal of Mathematics and Computer Science, Vol .1 No.2 : 80-89(2010).