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Partial eigenvalue assignment in descriptor systems via derivative and propositional state feedback

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Abstract. A method for solving the descriptor continuous-time linear system is focused. For easily, it is converted to two standard continuous-time linear systems by the definition of a derivative and propositional state feedback. Then partial eigenvalue assignment is used for obtaining the derivative and propositional state feedbacks and solving the standard systems. In partial eigenvalue assignment, just a part of the open loop spectrums of two standard linear systems are reassigned, while leaving the rest of the spectrum invariant and for reassigning, similarity transformation is used. Using partial eigenvalue assignment is easier than using eigenvalue assignment. Because by partial eigenvalue assignment, size of matrices and state and input vectors are decreased and stability is kept, too. It is worthy to mention that eigenvalues of closed-loop matrix of original system, i.e., descriptor and second converted system are inverse of each other. Also concluding remarks and an algorithm are proposed to the descriptions be obvious. At the end, convergence of state and input vectors in the descriptor system to balance point (zero) are showed by figures in a numerical example.

Keywords: descriptor system, derivative and propositional state feedback, partial eigenvalue assignment, converge to balance point.

1. Introduction

Descriptor systems that are also called singular systems are more general and precise than a normal model to depict a dynamical physical. Applications of descriptor systems can be found in various fields such as artificial neuron networks, circuits systems, chemical processes, economics, biologic, power, modeling of mechanical multi body systems and etc. [6, 10, 11, 20, 25, 28]

Some of the first fundamental works on eigenstructure assignment in descriptor linear systems were established in the 1980s by a number of researchers, such as Cobb (1981) [5], Armentano (1984) [1], Fletcher (1986) [9], Ozcaldiran and Lewis (1987) [21].

In recent years there are many subjects are related to these problems like switched descriptor systems and eigenvalue assignment in state feedback control for uncertain systems [22, 27]. Also Karbassi et al. worked on non-linear state feedback controllers like in [19].

In the available literature on descriptor systems, there are two kinds of stabilization problems for singular systems. One consists in designing a state or output feedback controller in such a way that the closed-loop system is regular, impulse-free, and stable or equivalently admissible. The other is to design a state or output feedback controller in order to make the closed-loop system regular and stable. Concerning the stability analysis and the stabilization problem, a number of approaches assuming or not assuming the regularity of the descriptor system have been proposed in the literature let us quote for instance [2, 6, 26] among those assuming the regularity and [6, 26] without assuming the regularity. Also positivity and stability of linear descriptor systems have been investigated in [13, 15] for systems with regular pencils.

Many practical applications such as the design of large and sparse structures, electrical networks, power systems, computer networks, etc., give rise to very large and sparse problems and the conventional numerical methods for EVA problem do not work well. Furthermore, in the most of these applications only a small number of eigenvalues, which are responsible for instability and other undesirable eigenvalues, need to be reassigned. Clearly, a complete EVA, in case when only a few eigenvalues are bad, does not make sense. This consideration gives rise to the partial eigenvalue assignment (PEVA) problem for the linear control system such that undesirable eigenvalues are reassigned and other eigenvalues unaltered. An explicit solution to the partial eigenvalue problem by using one of orthogonality relations between eigenvectors for matrix polynomial is considered in [23]. The conditions for existence and uniqueness of the solution for the single-input problem were given in [24] and for multi-input were presented in [8].

In this paper, the stability of descriptor continuous-time linear systems will be investigated. Our method is mixed of PEVA, EVA by similarity transformation and a useful method for converting the descriptor linear system to the standard linear systems. First, the descriptor continuous-time linear system (1) is converted to the standard continuous-time linear systems (8) and (10) by the definition of the derivative and propositional state feedback (5) that are calculated by the PEVA method (section 3). In other hand, we need to reassign undesired eigenvalues of open-loop spectrums in standard systems with smaller sizes of matrices such that other eigenvalues unchanged. Also a theorem for existence and uniqueness solution for PEVA in multi-input is represented. Then feedbacks in descriptor system are obtained by an easy relationship between these feedbacks and gained feedbacks by PEVA from (21). It is important to say for reassigning undesired eigenvalues, similarity transformation (section 4) is used that is a simple method with high accuracy.

As mentioned it is clear that our method has some advantages that solving the descriptor continuous-time linear systems will be more and more easier. The first advantage is, converting descriptor continuous-time linear system to standard continuous-time linear systems, because working on standard systems is easier than descriptor systems. Also we do not need the assumption of being full rank open-loop matrix in standard systems because of using derivative and propositional state feedback [3]. EVA have been an applicable method for finding the solution in standard systems and their stability, but by PEVA just by reassigning a part of open-loop matrix spectrum in standard systems while keeping other eigenvalues unvariant, their stability are kept. In PEVA we decrease the size of matrices and state and input vectors, it is obvious that calculating is more easily than EVA and obtaining state feedback is so comfortable by state feedback governed in PEVA and they are other advantages of our method. Therefore the state and input vectors in the original system, i.e., descriptor continuous-time linear system, converge to balance point and we show this by figures in our example. It is worthy to mention that we do not need some assumptions like no having eigenvalues near zero and some criteria on some vectors and being distinct eigenvalues by orthogonality relations for PEVA in [23] or dealing with full row rank matrices in every performed algorithm and finding index of Shuffle and Drazin for descriptor systems in [4, 12, 14, 16] and they are other excellence of method in this paper. Also this method can be used for discrete-time descriptor linear systems by defining a suitable state feedback.

This paper is organized as follows. Next section, presents converting the descriptor continuous-time linear system to the standard continuous-time linear systems that the closed-loop matrix of the second standard system, i.e., (10) has inverse of eigenvalues of closed-loop matrix in original system, i.e., descriptor system (1). The PEVA problem for obtaining the derivative and propositional state feedbacks is displayed in section 3. Section 4 proposes the similarity transformation for reassigning eigenvalues in PEVA. An algorithm and numerical results are presented in section 5 by an algorithm with all proposed details in its previous sections and numerical examples with the results of all steps of algorithm in it. Also convergence of state and input vectors to balance point, i.e. zero, by their figures are showed. At final section, conclusion is given.

The following notation will be used: $\Re$ - the set of real numbers, $\mathbb{C}$ - the set of complex numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices and $\Re^m = \Re^{m\times 1}$, A^T - the transposed matrix of A, $\Omega(A)$ - spectrum of eigenvalues of the matrix A, I_n - the unit matrix of size n.

2. Statement of the problem

Consider the descriptor linear time-invariant controllable system of the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \qquad (1)$$

where $E \in R^{n\times n}$ with $rank\,(E) \le n$, $x(t) \in R^n$ is state vector and $u(t) \in R^m$ is input vector. It is assumed that $1 \le m \le n$, $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are open-loop and input matrices respectively. Also $x(0) = x_0$ is a nonzero definite vector.

The aim is the eigenvalue assignment to design a derivative and propositional state feedback controller matrix which produce a closed-loop system of (1) with a satisfactory response by shifting controllable poles $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$ from undesirable to desirable locations where $\lambda_i \in \mathbb{C}$ and $\lambda_i \ne 0$ and are self-conjugate complex numbers for $i = 1,2,...,n$ and by using the method in section 3 means PEVA we reassign p eigenvalues which $p \le n$ while other eigenvalue of open-loop matrix unchanged.

As a brief displaying, we discuss the advantage of using the derivative and propositional state feedback controller instead of the derivative state feedback.

Derivative state feedback. Consider system (1) by derivative state feedback (2).

$$u(t) = F'_{de}\dot{x}(t) \tag{2}$$

To establish the proposed results, consider the following assumptions:

$$I) rank(E|B) = n; \quad II) rank(A) = n; \quad III) rank(B) = m$$

It is clear that if assumption (I) holds, then there exists F'_{de} such that [3]:

$$rank(E - BF'_{de}) = n. \tag{3}$$

For F'_{de} such that (3) holds, then from (2) it follows that (1) can be rewrite such as a standard linear system, given by:

$E\dot{x}(t) = Ax(t) + BF'_{de}\dot{x}(t) \Rightarrow (E - BF'_{de})\dot{x}(t) = Ax(t)$

therefore

$$\dot{x}(t) = (E - BF'_{de})^{-1} Ax(t) \tag{4}$$

which is well defined by (3).

It is clear that assumption (II) includes special systems.

Derivative and propositional state feedback. Consider system (1) by the derivative and propositional state feedback (5)

$$u(t) = F_{de}\dot{x}(t) + F_{pr}x(t) \tag{5}$$

To establish the proposed results, consider following assumptions:

$$I) rank(E|B) = n; \quad II) rank(B) = m$$

It is clear that if assumption (I) holds, then there exists F_{de} such that [3]:

$$rank(E - BF_{de}) = n. \tag{6}$$

For F_{de} such that (6) holds, then from (5) it follows that (1) can be rewrite such as a standard linear system, given by:

$$E\dot{x}(t) = Ax(t) + BF_{de}\dot{x}(t) + BF_{pr}x(t) \Rightarrow (E - BF_{de})\dot{x}(t) = (A + BF_{pr})x(t)$$

therefore

$$\dot{x}(t) = (E - BF_{de})^{-1}(A + BF_{pr})x(t) \tag{7}$$

which is well defined by (6).

Remark 1. When we use the derivative and propositional state feedback instead of the derivative state feedback, we do not need the condition of being full rank of matrix A in system (1). It is an excellence for using derivative and propositional state feedback. In continuation, we focus on derivative and propositional state feedback (5).

As displayed for obtaining F_{pr} and F_{de} in (5), first we obtain the propositional state feedback F_{pr} by using the method of partial eigenvalue assignment (in section 3) on system (8) which by using PEVA just we need to reassign p eigenvalues and $p \le n$ and other eigenvalues unaltered.

$$\begin{cases} \dot{g}(t) = Ag(t) + Bv(t) \\ v(t) = F_{pr}g(t) \end{cases} \tag{8}$$

It means we assign non-zero desired eigenvalues $\Omega = \{\mu_1, \mu_2, ..., \mu_n\}$ to the closed-loop of the system (8).

Then we obtain the derivative state feedback F_{de}, using the method in section 3 on system (10) by assigning $L^{-1} = \{\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_n^{-1}\}$, where $\lambda_i \in \mathbb{C}$ and $\lambda_i \ne 0$, $i = 1,2,...,n$ and by using PEVA just we need to reassign p eigenvalues which $p \le n$. For calculating F_{de} see Theorem 1.

Lemma 1 Consider a matrix $M \in \Re^{n \times n}$ with rank(M) = n and the eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$. Then, the eigenvalues of M^{-1} are the following: $\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_n^{-1}$. [18, 19]

Remark 2 Consider that $\lambda = a + bi$ is an eigenvalue of M, then from Lemma 1

$$\lambda^{-1} = (\mathrm{a}+\mathrm{bi})^{-1} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i$$

is also an eigenvalue of M^{-1}.

Theorem 1. Define the matrices N and M as (9), desired set of Ω and matrix F_{pr} such that the pair of (N,M) be controllable,

$$N = (A+BF_{pr})^{-1}E, M = -(A+BF_{pr})^{-1}B \quad (9)$$

Also let F_{de} be state feedback matrix, such that $L^{-1} = \{\lambda_1^{-1}, \lambda_2^{-1}, \ldots \lambda_n^{-1}\}$ are the eigenvalues of the closed-loop system

$$\begin{cases} \dot{z}(t) = Nz(t) + Mw(t) \\ w(t) = F_{de}z(t \end{cases} \quad (10)$$

where $\lambda_i \in \mathbb{C}$ and $\lambda_i \neq 0$, $i = 1,2,...,n$ are arbitrarily assigned. Then for this gained F_{de} , the desired spectrum $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$ is the eigenvalues of the controlled system (1) with derivative feedback (5) and also, the condition (6) holds.

Proof. Considering that (N,M) is controlled, then we can find a state feedback matrix F_{de} such that the controlled system with control law (10) given by

$$\dot{z}(t) = (N + MF_{de})z(t) \quad (11)$$

has poles equals to $L^{-1} = \{\lambda_1^{-1}, \lambda_2^{-1}, \ldots \lambda_n^{-1}\}$. Now by (9) note that:

$$(N + MF_{de})^{-1} = ((A+BF_{pr})^{-1}(E - BF_{de}))^{-1},$$

So

$$(N + MF_{de})^{-1} = (E - BF_{de})^{-1}(A + BF_{pr})$$

and from (11) and Lemma 1, the spectrum $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$ is the eigenvalues of closed-loop matrix $(E - BF_{de})^{-1}(A + BF_{pr})$ which is closed-loop matrix of system (1) with feedback (5) as mentioned in (4). Therefore (6) holds and the eigenvalue of closed-loop system (1) and feedback (5) are equal to $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$.

Remark 3. Consider system (1) via derivative and propositional state feedback (5) and system (10). As displayed in proof of theorem 1 and (4), the closed-loop matrices of these two systems are $(E - BF_{de})^{-1}(A + BF_{pr})$ and $(A + BF_{pr})^{-1}(E - BF_{de})$ respectively and it is clear that closed-loop matrices are inverse of each other. So by assigning inverse of undesired eigenvalue, i.e., $L^{-1} = \{\lambda_1^{-1}, \lambda_2^{-1}, \ldots \lambda_n^{-1}\}$ on standard system (10) eigenvalues of descriptor system (1) via feedback (5) will been obtained as $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$.

3. Partial eigenvalue assignment (PEVA)

In this section we will describe a method for finding the feedback F_{pr} and F_{de} in system (8) and (10). At first, some definitions and theorems that we need them for the existence and uniqueness theorem for multi-input and single-input PEVA problem are proposed. Next we bring the Existence and Uniqueness Theorem and its proof and by the description of its proof, the PEVA method is displayed for obtaining the propositional feedback F_{pr} in system (8). In similar to the description for finding F_{pr} may be used for F_{de} , too. Also the theorem 5 for finding the feedback F_{de} in system (10) is proposed. Notice that in practically, first we have to obtain the propositional feedback F_{pr} in system (8) which we need it for finding the derivative feedback F_{de} in system (10).

Theorem 2. [8] (Eigenvector Criterion of controllability). The standard system (8) or, equivalently, the matrix pair (A,B) is controllable with respect to the eigenvalue λ of A if $y^H B \neq 0$ for all $y \neq 0$ such that $y^H B = \lambda y^H$.

Definition 1. The standard system (8) or the matrix pair (A,B) is partially controllable with respect to the subset $\lambda_1, \lambda_2, ..., \lambda_p$ of the spectrum of A if it is controllable with respect to each of the eigenvalues $\lambda_j, j = 1, ..., p$.

Definition 2. The standard system (8) or the matrix pair (A,B) is completely controllable if it is controllable with respect to every eigenvalue of A.

Theorem 3. [7] (Existence and Uniqueness for Eigenvalue Assignment Problem). The eigenvalue assignment problem for the pair (A,B) is solvable for any arbitrary set $\{\mu_1, ..., \mu_n\}$ if and only if (A,B) is completely controllable. The solution is unique if and only if the system is a single-input system (that is, if B is a vector). In the multi-input case, there are infinitely many solutions, whenever a solution exists.

Theorem 4. (Existence and Uniqueness for partial eigenvalue assignment Problem). Let $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_p, \lambda_{p+1}, ...\lambda_n\}$ be the diagonal matrix containing the eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ of $A \in \mathbb{C}^{n \times n}$. Assume that the sets $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ and $\{\lambda_{p+1}, \lambda_{p+2}, ..., \lambda_n\}$ are disjoint. Let the eigenvalues $\lambda_1, \lambda_2, ..., \lambda_p$ to be changed to $\mu_1, \mu_2, ..., \mu_p$ and the remaining eigenvalues stay invariant. Then the partial eigenvalue assignment problem for the pair (A,B) is solvable for any choice of the closed-loop eigenvalues $\mu_1, \mu_2, ..., \mu_p$ if and only if the pair (A,B) is partially controllable with respect to the set $\{\lambda_1, \lambda_2, ..., \lambda_p\}$. The solution is unique if and only if the system is a completely controllable single-input system. In the multi-input case and in the single-input case when the system is not completely controllable, there are infinitely many solutions, whenever a solution exists.

Proof. We first prove the necessity. Suppose the pair (A,B) is not controllable with respect to some $\lambda_j, j = 1, ..., p$. Then there exists a vector $y \neq 0$ such that $y^H (A - \lambda_j I) = 0$ and $y^H M = 0$. This means that for any F, we have $y^H (A + BF - \lambda_j I) = 0$, which implies that λ_j is an eigenvalue of $A + BF$ for every F, and thus λ_j cannot be reassigned.

Next we prove the sufficiently. We need to prove that there exists a feedback matrix F which assigns the eigenvalues in $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ arbitrarily while keeping all the other eigenvalues unaltered.

Let $X = \{x_1, x_2, ...x_n\}$ and $Y = \{y_1, y_2, ...y_n\}$ be, respectively, the right and left eigenvector matrix of A, and let $Y_1 = \{y_1, y_2, ...y_p\}$. Since $Y^H X = I$ and $Y^H AX = \Lambda$, then the partial controllability of the pair (A,B) with respect to eigenvalues in $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ implies the partial controllability of the pair $(\Lambda, Y^H B)$ with respect to same eigenvalues. Therefore, the pair $(\Lambda_1, Y_1{}^H B)$ is completely controllable because $\{\lambda_1, \lambda_2, ..., \lambda_p\} \cap \{\lambda_{p+1}, ..., \lambda_n\} = \emptyset$ which $\Lambda_1 = diag(\lambda_1, \lambda_2, ..., \lambda_p)$.

By Theorem 3, there exists a feedback matrix K such that the closed-loop matrix $\Lambda_1 + Y_1{}^H BK$ has the desired eigenvalues $\mu_1, ..., \mu_p$. Denote

$$F = KY_1{}^H \tag{12}$$

Then the eigenvalues of closed-loop matrix are exactly as required. This is seen as follow:

$$\{\mu_1, ..., \mu_p, \lambda_{p+1}, ...\lambda_n\} = \Omega(\Lambda + Y^H B(K,0)) = \Omega(Y^H (A + B((K,0)Y^H)X) = \Omega(A + B(KY_1{}^H)) \tag{13}$$

Uniqueness of the solution in the single-input case that is completely controllable and the existence of infinitely many solutions in the multi-input case follow directly from Theorem 3.

To complete the proof we need to show that infinitely many solutions to the PEVA problem are possible when B is a vector (single-input case) and there exists an uncontrollable eigenvalue λ_k for some $k > p$ (that is, the associated k^{th} right eigenvector y_k is such that $y_k{}^H A = \lambda_k y_k{}^H$ and $y_k{}^H B = 0$).

Let F be a solution to the partial eigenvalue assignment problem. Denote the left and right eigenvectors of the closed-loop matrix $A_c = A + BF$ by Y_c and X_c. Clearly $y_k{}^H A_c = y_k{}^H (A+BF) = \lambda_k y_k{}^H$ and thus y_k is also the k^{th} column of Y_c. Let $F_\beta = \beta y_k{}^H$, where β is an arbitrary scalar. As in (13) we can show that the eigenvalues $\mu_1,...,\mu_p,\lambda_{p+1},....\lambda_{k-1},\lambda_{k+1},...,\lambda_n$ of A_c remain unchanged by the application of feedback F_β. Furthermore, the eigenvalue λ_k of A_c also remains unchanged by the feedback F_β, since the pair (A_c, B) is not controllable with respect to λ_k by the necessity part of this theorem. Thus

$$\Omega(A+BF) = \Omega(A_c) = \Omega(A_c + BF_\beta) = \Omega(A + B(F + \beta y_k{}^H)), \tag{14}$$

showing that if F is a solution, so is $F + \beta y_k{}^H$ for an arbitrary β.

Remark 4. The theorem 4 for both of standard systems (8) and (10) may be used, so for easily F instead of F_{pr} in system (8) is considered. Also we can use this theorem for finding F_{de} in system (10) by putting the pair of (N, M) and F_{de} instead of (A, B) and F respectively in theorem and the proof of 4. In continuous the description of obtaining F_{pr} in system (8) is proposed and for finding F_{de} in system (10) the same description is proposed in theorem 5.

Suppose that

$$\Omega(A) = \{\lambda_1,...,\lambda_p,\lambda_{p+1},....\lambda_n\},$$

which p is the number of undesired eigenvalues of $\Omega(A)$ for the pair of (A, B) in system (8) and assume the set $S = \{\mu_1,...,\mu_p\}$ be closed under complex conjugation which $rank(B) = m \le p$. The aim of PEVA problem is, looking for the derivative state feedback F_{pr} such that

$$\Omega(A + BF_{pr}) = \{\mu_1,\mu_2...,\mu_p,\lambda_{p+1},....\lambda_n\} \tag{15}$$

and also the sets $\{\lambda_1,....\lambda_p\}$ and $\{\lambda_{p+1},....\lambda_n\}$ be disjoint.

This means, finding F_{pr} which reassigns eigenvalues $\{\lambda_1,....\lambda_p\}$ arbitrarily while keeping all the other eigenvalues, $\{\lambda_{p+1},....\lambda_n\}$, unaltered.

First we need to obtain left eigenvector of matrix A as follow:

$$Y = (y_1, y_2, ... y_n) \tag{16}$$

Then we put columns $y_1, y_2,...y_p$ of Y in Y_1 that are associated columns by eigenvalues $\lambda_1,....\lambda_p$. So

$$Y_1 = (y_1, y_2,...y_n) = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{bmatrix}_{n\times p} \tag{17}$$

Now consider the pair of $(\Lambda_1, Y_1{}^H B)$ as follow:

$$\Lambda_1 = diag(\lambda_1,\lambda_2,...,\lambda_p) = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_p \end{bmatrix}_{p\times p} \tag{18}$$

$$Y_1{}^H B = \begin{bmatrix} y_{11} & y_{21} & \cdots & y_{n1} \\ y_{12} & y_{22} & \cdots & y_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ y_{1p} & y_{2p} & \cdots & y_{np} \end{bmatrix}_{p\times n} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}_{n\times m} \tag{19}$$

So the aim is, finding feedback K_{pr} in system

$$\begin{cases} \dot{G}(t) = \Lambda_1 G(t) + (Y_1{}^H B)V(t) \\ V(t) = K_{pr} G(t) \end{cases} \tag{20}$$

such that the eigenvalues of the closed-loop of the system (20) be $\{\mu_1,\ldots,\mu_p\}$. At the end, for finding F_{pr} in system (8), we have:

$$F_{pr} = K_{pr} Y_1{}^H . \tag{21}$$

Theorem 5. Consider system (10) and define the matrices N, M as (9), desired set of Ω and matrix F_{pr} such that the pair of (N,M) be controllable. Also assume $\Omega(N) = \{\lambda_1{}^{-1}, \lambda_q{}^{-1}, \lambda_{q+1}{}^{-1}, \ldots \lambda_n{}^{-1}\}$, $\{\lambda_1{}^{-1},\ldots,\lambda_q{}^{-1}\}$ be the sets of eigenvalues and undesired eigenvalues for open-loop matrix of this system and the sets $\{\mu_1{}^{-1},\ldots,\mu_q{}^{-1}\}$, $\{\lambda_1{}^{-1},\ldots,\lambda_q{}^{-1}\}$ and $\{\lambda_{q+1}{}^{-1},\ldots,\lambda_n{}^{-1}\}$ be disjoint. If the pair $(\Lambda'_1,(Y_1)'^H M)$ as (22) and (23),

$$(Y_1)''^H M = \begin{bmatrix} y'_{11} & y'_{21} & \cdots & y'_{n1} \\ y'_{12} & y'_{22} & \cdots & y'_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ y'_{1q} & y'_{2q} & \cdots & y'_{nq} \end{bmatrix}_{q\times n} \times \begin{bmatrix} b'_{11} & b'_{12} & \cdots & b'_{1m} \\ b'_{21} & b'_{22} & \cdots & b'_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ b'_{n1} & b'_{n2} & \cdots & b'_{nm} \end{bmatrix}_{n\times m} , \tag{22}$$

$$\Lambda'_1 = diag(\lambda_1{}^{-1},\lambda_2{}^{-1},\ldots,\lambda_q{}^{-1}) = \begin{bmatrix} \lambda_1{}^{-1} & 0 & \cdots & 0 \\ 0 & \lambda_2{}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_q{}^{-1} \end{bmatrix}_{q\times q} \tag{23}$$

and the state feedback K_{de} in system (24)

$$\begin{cases} \dot{Z}(t) = \Lambda'_1 Z(t) + ((Y_1)'^H M)W(t) \\ W(t) = K_{de} Z(t) \end{cases} \tag{24}$$

be associated by the pair (N,M) in system (10) such that

$$\Omega((Y_1)'^H M, \Lambda'_1) = \{\mu_1{}^{-1},\ldots,\mu_q{}^{-1}\},$$

$$\Omega(N + MF_{de}) = \{\mu_1{}^{-1},\ldots,\mu_q{}^{-1},\lambda_{q+1}{}^{-1},\ldots,\lambda_n{}^{-1}\}$$

which

$$F_{de} = K_{de}(Y_1)'^H , \tag{25}$$

then the eigenvalues of the descriptor system (1) via derivative and propositional state feedback (5) are:

$$\{\mu_1,\ldots,\mu_q,\lambda_{q+1},\ldots,\lambda_n\}.$$

Proof. The proof is clear by the theorems 1, 4 and lemma 1.

4. Similarity transformation of the state space

In this section we describe a method for finding feedbacks K_{pr} and K_{de} in system (20) and (24). At the first, we assign zero eigenvalues to these systems by Φ_{pr} and Φ_{de} respectively. Because of obtaining the feedbacks K_{pr} and K_{de} are similar, so just the method for finding K_{pr} in system (20) is explained.

Consider the system (20) by defining $A_1 = \Lambda_1 \in \Re^{p\times p}$ and $B_1 = Y_1{}^H B \in \Re^{p\times m}$ as follow,

$$\begin{cases} \dot{G}(t) = \Lambda_1 G(t) + B_1 V(t) \\ V(t) = K_{pr} G(t) \end{cases} \tag{26}$$

instead of system (20) and display similarity transformation on it easier.

To obtain the derivative feedback matrix K_{pr} in system (20) or equivalently system (26), consider the state transformation

$$G(t) = T\widetilde{G}(t) \tag{27}$$

where T can be obtained by elementary similarity operations as described in [18, 19]. Substituting (27) into first relationship of (26) yields

$$\dot{\widetilde{G}}(t) = T^{-1}A_1 T\widetilde{G}(t) + T^{-1}B_1 V(t)$$

It is noted that the transformation matrix T is invertible. In this way,

$$\widetilde{A}_1 = T^{-1}A_1 T \,,\, \widetilde{B}_1 = T^{-1}B_1 \tag{28}$$

are in a compact canonical form know as vector companion form [18, 19]:

$$\widetilde{A}_1 = \begin{bmatrix} & R_0 & \\ I_{p-m} & , & 0_{p-m,m} \end{bmatrix}_{p\times p}, \widetilde{B}_1 = \begin{bmatrix} S_0 \\ 0_{p-m,m} \end{bmatrix}_{p\times m} \tag{29}$$

Here R_0 is a $m \times p$ matrix and S_0 is a $m \times m$ upper triangular matrix. Note that the Kronecker invariants of the pair (A_1, B_1) are regular if the difference between any of them is not greater than one. If Kronecker invariants of the pair of (A_1, B_1) are regular, then $\widetilde{A}_1$ and $\widetilde{B}_1$ are always in the above form [18]. In the case of irregular Kronecker invariants, some rows of I_{p-m} in $\widetilde{A}_1$ are displaced [19]. (For more details about Kronecker invariants, see [17])

The state feedback matrix which assigns all the eigenvalues to zero for the transformed pair $(\widetilde{A}_1, \widetilde{B}_1)$ is then chosen as

$$\widetilde{\Phi}_{pr} = -S_0^{-1}R_0 \tag{30}$$

which results in the primary state feedback matrix for the pair (A_1, B_1) defined as

$$\Phi_{pr} = \widetilde{\Phi}_{pr} T^{-1} \tag{31}$$

The transformed closed-loop matrix

$$\widetilde{\Gamma}_0 = \widetilde{A}_1 + \widetilde{B}_1 \widetilde{\Phi}_{pr} \tag{32}$$

assumes a compact Jordan form with zero eigenvalues

$$\widetilde{\Gamma}_0 = \begin{bmatrix} & 0_{m,p} & \\ I_{p-m} & , & 0_{p-m,m} \end{bmatrix}_{p\times p} \tag{33}$$

Theorem 6. Let D be a block diagonal matrix in the form

$$D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_k \end{bmatrix}_{p\times p}$$

where each D_j, $j = 1,2,...,k$ is either of the form

$$D_j = \begin{bmatrix} \alpha_j & \beta_j \\ -\beta_j & \alpha_j \end{bmatrix}$$

(to designate the complex conjugate eigenvalues $\alpha_j + i\beta_j$)
or in case of real eigenvalues

$$D_j = \lfloor d_j \rfloor$$

If such that diagonal matrix D with self conjugate eigenvalue spectrum is added to the transformed closed-loop matrix, $\tilde{\Gamma}_0$, then the eigenvalues of the resulting matrix is the eigenvalues in the spectrum.

Proof. The primary compact Jordan form in the case of regular Kronecker invariants is in the form (33). The sum of $\tilde{\Gamma}_0$ with D has the form:

$$\tilde{H} = \tilde{\Gamma}_0 + D$$

$$= \begin{bmatrix} & 0_{m,p} \\ I_{p-m} & , \; 0_{p-m,m} \end{bmatrix}_{p\times p} + \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_k \end{bmatrix}_{p\times p}$$

$$= \begin{bmatrix} D_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & D_l & 0 & \cdots & 0 \\ - & - & - & - & - & - & - \\ I_1 & 0 & \cdots & 0 & D_{l+1} & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 & 0 & \ddots & \vdots \\ 0 & \cdots & I_r & 0 & 0 & \cdots & D_k \end{bmatrix}_{p\times p} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} m \\ \\ \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} p-m \end{matrix} \tag{34}$$

And also $I_s, s=1,2,...,r$ is the unit matrix of size 2 in case $p-m$ is even. In case $p-m$ is odd only one I_s takes the form of unit matrix of size one.

By expanding $\det(\tilde{H} - \lambda I)$ along the first row it is obvious that the eigenvalues of $\tilde{H}$ are the same as the eigenvalues of D. For the case of irregular Kronecker invariants [19] only some of the unit columns of I_{p-m} are displaced, since the unit elements are always below the main diagonal, the proof applies in the same manner.

Therefore the closed-loop system matrix (34) becomes (35). Simple elementary similarity operations can be used to obtain the matrix $\tilde{H}_\lambda$ from $\tilde{H}$ such that

$$\tilde{H}_\lambda = \begin{bmatrix} & R_\lambda \\ I_{p-m} & , \; 0_{p-m,m} \end{bmatrix} \tag{35}$$

Thus the primary feedback matrix K_{pr} which gives rise to the assignment of eigenvalues $\{\lambda_1,....\lambda_p\}$ to the system (20) becomes

$$K_{pr} = \Phi_{pr} + S_0^{-1} R_\lambda T^{-1} \tag{36}$$

5. Algorithm and numerical experiment

In this section, we present an algorithm to obtain the solution of system (1) by the use of partial eigenvalue problem in section 3 based on assigning eigenvalue problem in section 4. Then by an example, we show the simplicity of our method.

Object. Assign desired eigenvalues $\{\mu_1^{-1},...,\mu_p^{-1}\}$ and $\{\lambda_1,...,\lambda_q\}$ to the systems (20) and (24) and find the matrices K_{pr} and K_{de} respectively such that the spectrum of closed-loop system (1) via derivative and propositional state feedback (5), i.e., $(E - BF_{de})^{-1}(A + BF_{pr})$ in (7) be $\{\mu_1,...,\mu_p,\lambda_{p+1},....\lambda_n\}$

Input. The matrices A, B and E .

Main steps:

Step 1. Obtain F_{pr} in system (8).

Step 1.1. Calculate the eigenvalues and the left eigenvector of *A* and then put associated columns $\{\lambda_1,\ldots,\lambda_p\}$ of left eigenvector *A* in Y_1 by (16) and (17).

Step 1.2. Obtain the pair $(\Lambda_1, Y_1{}^H B)$ from (18) and (19).

Step 1.3. Compute Φ_{pr} and K_{pr} by similarity transformation similar to (27) until (36).

Step 1.4. Compute F_{pr} from (21) and the results of step 1.3.

Step 2. Obtain F_{de} in system (10).

Step 2.1. Calculate the matrices *N* and *M* from (9).

Step 2.2. Repeat steps 1.1 until 1.3 and obtain K_{de} and F_{de} by (27) until (36) by using *N*,*M*, Φ_{de} ,and K_{de} instead of *A*, *B*, Φ_{pr} , and K_{pr} .

Step 2.3. Compute F_{de} from (25) and the results of step 2.2.

Step 3. Input gained F_{pr} and F_{de} in steps 1, 2 in (5) and obtain $u(t)$ of system (1).

Step 4. Obtain $x(t)$ by putting $u(t)$, F_{pr} , and F_{de} are gained from steps 1, 2 and 3 in (7).

Example 1. Consider the descriptor system (1) with following matrices which $rank(E)=9<10$.

$$A=\begin{bmatrix} 0 & 5 & 4 & -6 & 3 & 3 & -6 & 2 & 6 & 5 \\ 0 & -7 & -6 & 8 & 5 & -9 & 0 & 8 & 1 & -4 \\ 0 & -6 & 2 & -9 & 1 & 3 & 5 & -9 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & 7 & 8 & 2 & 7 \\ 0 & 5 & 5 & 4 & -9 & 1 & 2 & 5 & 8 & 8 \\ 0 & 3 & 2 & 5 & 2 & 3 & 4 & 2 & -9 & 7 \\ 0 & 0 & -9 & 6 & 5 & 3 & 5 & 7 & 6 & 2 \\ 0 & 1 & -9 & 7 & -9 & 1 & 0 & 2 & 7 & 6 \\ 0 & 1 & 3 & 6 & 1 & 4 & 4 & -9 & 2 & 8 \\ 0 & -6 & 0 & -7 & 1 & 0 & 6 & 6 & 5 & 1 \end{bmatrix}_{10\times10}, \quad B=\begin{bmatrix} 8 \\ -9 \\ 1 \\ -9 \\ 6 \\ 0 \\ 2 \\ 5 \\ -9 \\ -9 \end{bmatrix}_{10\times1},$$

$$E=\begin{bmatrix} -1 & 7 & 4 & 6 & 3 & 3 & 6 & 2 & -6 & 0 \\ 1 & -7 & 4 & 6 & 3 & 3 & 6 & 2 & 6 & 0 \\ 2 & 6 & 2 & 3 & 1 & 3 & -7 & 3 & 0 & 3 \\ -2 & 1 & 0 & 0 & 0 & 2 & 7 & 0 & 2 & 0 \\ 3 & -7 & -7 & 4 & 3 & 1 & 2 & -7 & 0 & 3 \\ -2 & 3 & 2 & -7 & 2 & 3 & 4 & 2 & 3 & -7 \\ 1 & 0 & 3 & 6 & -7 & 3 & 7 & 7 & 6 & 0 \\ 4 & 1 & 3 & 7 & 3 & 1 & 0 & 2 & 7 & 7 \\ -4 & 1 & 3 & 6 & 1 & 4 & 4 & 3 & 2 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{10\times10}.$$

Step 1:

$$\Omega(A)=\{-13.06,-6.46\pm10.22i,-5.47\pm3.11i,0,2.35,6.38\pm3.4i,20.81\}$$

Therefore *p* = 5 and by considering

$$\Lambda_1=\begin{bmatrix} 20.81 & 0 & 0 & 0 & 0 \\ 0 & 6.38+3.4i & 0 & 0 & 0 \\ 0 & 0 & 6.38-3.4i & 0 & 0 \\ 0 & 0 & 0 & 2.35 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad Y_1{}^H B=\begin{bmatrix} 8.91 \\ 7.99-3.19i \\ 7.99+3.19i \\ 8.56 \\ 10.05 \end{bmatrix}$$

the matrix Φ_{pr} which assign zero eigenvalue to the system (20) is obtained as:

$$\Phi_{pr} = \begin{bmatrix} -5.19 & 0.57+0.17i & 0.57-0.17i & 0 & 0 \end{bmatrix}$$

and by reassigning $\{-20,-10\pm 5i,-10,-5\}$ instead of $\{0,2.35,6.38\pm 3.4i,20.81\}$, the matrix feedback K_{pr} is obtained as:

$$K_{pr} = \begin{bmatrix} -42.02 & 19.77-44.03i & 19.77+44.03i & 34.73 & 0 \end{bmatrix}$$

The state feedback matrix F_{pr} for the system (8) is calculated by,

$$F_{pr} = K_{pr} Y_1^H$$
$$= \begin{bmatrix} -0.59 & -1.86 & 19.56 & 1.3 & 21.16 & 16.06 & 37.44 & -11.86 & 14.42 & 13.72 \end{bmatrix}$$

Now we have:

$$\Omega(A+BF_{pr}) = \{-20,-13.06,-10\pm 5i,-10,-6.46\pm 10.22i,-5.47\pm 3.11i,-5\}$$

Step 2:

$$N = \begin{bmatrix} 1.17 & -16.23 & -20.28 & 47.75 & -32.08 & 39.5 & 24.16 & 10.37 & 6.4 & -7.73 \\ -0.13 & 0.3 & -0.31 & 0.98 & -0.39 & 0.31 & 0.97 & -0.22 & -0.16 & 1.66 \\ -0.12 & -0.84 & -0.67 & -0.71 & 0 & -0.27 & -0.29 & -0.59 & -0.48 & -1.15 \\ -0.07 & -0.84 & -0.24 & -0.43 & -0.25 & -0.23 & -0.08 & -0.06 & 0.21 & -1.13 \\ -0.15 & 0.37 & 0.52 & 0.76 & -0.1 & 0.59 & 0.79 & 0.57 & 0 & -0.04 \\ -0.49 & -0.24 & -0.3 & -2.35 & -1.06 & -1.27 & -0.99 & 0.15 & -0.38 & -2.74 \\ 0.31 & -0.2 & -0.5 & 0.77 & -0.36 & 0.52 & 0.23 & -0.13 & 0.4 & 1.3 \\ -0.05 & -0.36 & -0.24 & -0.94 & -0.15 & -0.4 & 0.02 & -0.2 & -0.1 & -0.72 \\ 0.08 & -0.33 & -0.15 & 0.95 & -0.21 & 0.1 & 0.23 & -0.04 & -0.1 & 0.29 \\ 0.05 & 0.92 & 0.93 & 0.93 & 1.09 & 0.86 & 0.66 & 0.44 & 0.32 & 0.46 \end{bmatrix},$$

$$M = \begin{bmatrix} 1.67 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Omega(N) = \{-1.28,-1.09\pm 3.78i,-0.78,-0.43,0,0.35\pm 0.35i,0.54,2.81\}$$

Therefore $q = 5$ and by considering

$$\Lambda_1' = \begin{bmatrix} 2.81 & 0 & 0 & 0 & 0 \\ 0 & 0.35-0.35i & 0 & 0 & 0 \\ 0 & 0 & 0.54 & 0 & 0 \\ 0 & 0 & 0 & 0.35-0.35i & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (Y_1)'^H M = \begin{bmatrix} -0.06 \\ -0.02 \\ -0.02 \\ -0.02 \\ -0.01 \end{bmatrix}$$

the matrix Φ_{de} which assign zero eigenvalue to the system (24) is obtained as:

$$\Phi_{de} = \begin{bmatrix} 72.62 & -3.01-2.68i & -11.61 & -3.01+2.68i & 0 \end{bmatrix}$$

and by reassigning $\{-0.25,-0.2,-0.16,-0.12\pm 0.09i\}$ instead of $\{0,0.35\pm 0.35i,0.54,2.81\}$, the matrix feedback K_{de} is obtained as:

$$K_{de} = \begin{bmatrix} 97.76 & 1.65-12.96i & -43.36 & 1.65+12.96i & 0.02 \end{bmatrix}$$

The state feedback matrix F_{de} for the system (10) is calculated by,

$$F_{de} = K_{de}(Y_1)'^H$$
$$= [-2.94 \quad 30.03 \quad 28.57 \quad -29.82 \quad 42.25 \quad -23.98 \quad -18.54 \quad 7.48 \quad -22.55 \quad 21.1]$$

Now we have:

$$\Omega((E - BF_{de})^{-1}(A + BF_{pr})) = \Omega((N + MF_{de})^{-1})$$
$$= \{-6, -5 \pm 4i, -5, -4, -2.3, -1.27, -0.77, -0.07 \pm 0.24i\}$$

Steps 3 and 4:

Figures (1) and (2) show simulation result when

$$x_0 = [0.001 \quad -0.001 \quad 0.001 \quad -0.001 \quad 0.001 \quad -0.001 \quad 0.001 \quad -0.001 \quad 0.001 \quad -0.001]^T.$$

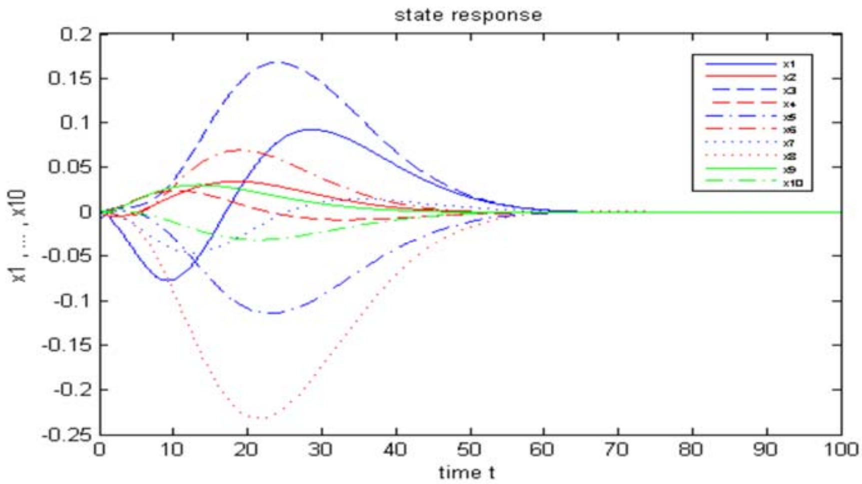

Fig. 1: State vector converge to zero in example 1

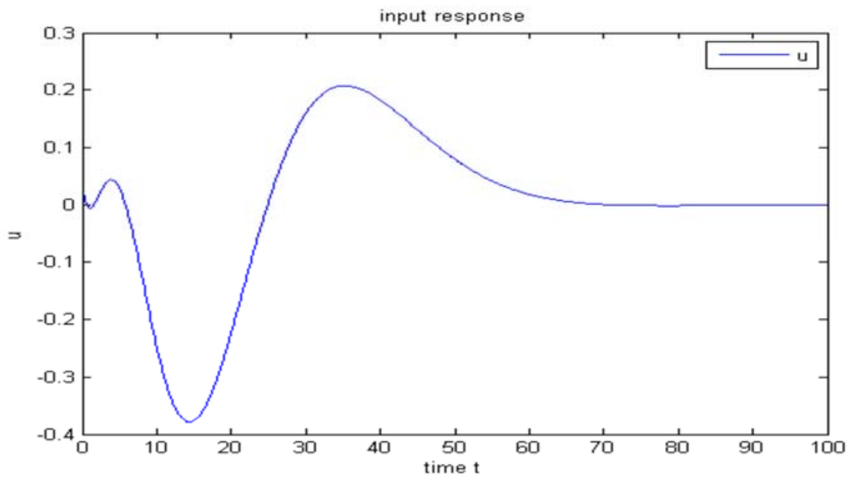

Fig. 2: Input vector converge to zero in example 1

6. Conclusion

A method for finding the solution of descriptor continuous-time linear system in form of (1) has been considered. First by the use of the derivative and propositional state feedback (5) displayed system has been converted to two standard continuous-time linear systems (8) and (10) and it explains the advantages of this method, because working with the standard systems is much easier than the descriptor mode and also because of using the derivative and propositional state feedback we do not need the assumption of being full rank of open-loop matrix in standard systems. Second the PEVA method based on similarity transformation on standard systems has been used to obtain feedbacks and in systems (8) and (10) respectively. Third the state and input vectors in (1) has been obtained by (5) and (7) and illustrated by a numerical example and showed the input and state vectors converge to balance point (zero).

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