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Adaptive synchronization of fractional-order Lorenz systems with memristor

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Abstract. In this paper, adaptive synchronization of fractional-order Lorenz systems with memristor is investigated. Based on the Lyapunov stability theories, several novel criteria are adopted to realize the synchronization of fractional-order Lorenz systems with memristor. Finally, some numerical examples are exploited to demonstrate the theoretical results.

Keywords: Fractional-order system; Memristor; Synchronization; Lorenz system.

1. Introduction

In 1971, the existence of memristor, as the fourth fundamental circuit element included along with the resistor, capacitor and inductor, was theoretically postulated by Chua [1] and experimentally confirmed by the researchers of Hewlett-Packard (HP) who reported on the first memristor device in 2008 [2,3]. The main advantage of the memristor is that the value of resistance would depend on the polarity e and magnitude of the voltage, and also remember the current resistance when the voltage is turned off. On the other hand, fractional calculus, which mainly deals with derivatives and integrals of arbitrary order, was firstly introduced 300 years ago. However, it is only in recent decades that fractional calculus is applied to physics and engineering [4,5]. The major merit of fractional calculus, different from integer calculus, lies in the fact that it has memory and has proven to be a very suitable tool for describing memory and hereditary properties of various materials and processes, it has been widely used in many research fields such as fluid mechanics [6], physics [7] and control processing [8]. Meanwhile, the chaos synchronization is applied in many fields such as cryptography [9], electromagnetic field [10,11], secure communication [12] and bioengineering [13]. Therefore, many scholars investigated the chaos synchronization problems [14-23].

Based on the above analysis of motivation, this paper will study the adaptive synchronization strategy of fractional-order Lorenz systems with memristor, it has a great value in practical applications. Finally, some examples are exploited to demonstrate the theoretical results via numerical simulations.

The rest of this paper is organized as follows. Section 2 describes the model formulation and some basic definitions. In Sect. 3, the adaptive synchronization of fractional-order Lorenz systems with memristor is realized. The numerical examples are provided to show the effectiveness of our theoretical results in Sect. 4. In Sect. 5, some conclusions are proposed.

2. Preliminaries

In this section, some basics of fractional calculus, definitions and lemmas assumptions are recalled. In addition, we introduce the mathematic model of fractional-order chaotic systems with memristor. For convience, $\mathbb{R}^n$will be the n-dimensional Euclidean space with norm $\|\cdot\|$in the all following.

2.1. Definitions and Lemmas

Definition 2.1.1: [4]. Caputo's fractional derivative of order q for a function $f(t):\ [0,+\infty)\rightarrow R$ is defined by

$$ {}_0^cD_t^q f(t)=\frac{1}{\Gamma(n-q)}\int_0^t\frac{f^n(\tau)}{(t-\tau)^{q-n+1}}d\tau\ ,$$

where $t \geq 0$, n is a positive integer such that $n-1<q<n$, and $\Gamma(\cdot)$ is the gamma function, that is $\Gamma(\tau)=\int_0^\infty t^{\tau-1}e^{-t}dt$ and $\Gamma(\tau+1)=\tau\Gamma(\tau)$.

Moreover, when $0<q<1$,

$$ {}_0^cD_t^q f(t)=\frac{1}{\Gamma(1-q)}\int_0^t \frac{f'(\tau)}{(t-\tau)^q}d\tau . $$

For simplicity, we denote $D^q f(t)$ as the ${}_0^cD_t^q f(t)$, and describe all of the following Caputo operators.

Definition 2.1.2: [4]. Mittag-Leffler function is defined by

$$ E_q(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(kq+1)}, $$

where $q>0$ and $z \in C$.

Lemma 2.1.1: [24] For $q\in(0,1)$, suppose $x(t)\in R^n$ is continuous function, the following inequality holds

$$ \frac{1}{2}D^q x^T(t)x(t) \leq x^T(t)D^q x(t). $$

2.2. Assumptions

Assumption 2.2.1: It is assumed that there is positive constant η such that for any $x, y \in R^n$,

$$ f(y)-f(x) \leq \eta(y-x). $$

Assumption 2.2.2: It is assumed that system (5) is a chaotic system with bounders, that is to say, there are positive constants M_1, M_2 such that

$$ |x_1(t)| \leq M_1, \qquad |x_4(t)| \leq M_2 $$

2.3. Model description

In [25], ϕ is flux, $W(\phi)$ is the memductance, the flux-controlled memristor is considered by

$$ \begin{cases} w(\phi)=a\phi+b\phi^3, \\ W(\phi)=\dfrac{dw(\phi)}{d\phi}=a+3b\phi^2, \end{cases} \tag{2.3.1} $$

Similar to [25], a modified Lorenz system with a memristor can be described by

$$ \begin{cases} \dot{x}_1=-\alpha x_1+\beta x_2-W(x_4)x_1, \\ \dot{x}_2=\gamma x_1-x_2-x_1x_3, \\ \dot{x}_3=x_1x_2-\xi x_3, \\ \dot{x}_4=-x_1. \end{cases} \tag{2.3.2} $$

Refer to the above model, the new fractional-order Lorenz system with memristor according to (2) is described as

$$ \begin{cases} D^q x_1=-\alpha x_1+\beta x_2-W(x_4)x_1, \\ D^q x_2=\gamma x_1-x_2-x_1x_3, \\ D^q x_3=x_1x_2-\xi x_3, \\ D^q x_4=-x_1. \end{cases} \tag{2.3.3} $$

Usually, in order to obtain the chaos generation, we set $q=0.97$, $a=0.1\times10^{-3}$, $b=0.1\times10^{-3}$, $\alpha=8$, $\beta=12$, $\gamma=30$, $\xi=2$, than the simulation is done with the initial value (1, 2, -3, -5) to system (3), the simulation results are shown in Fig. 2.3.1.

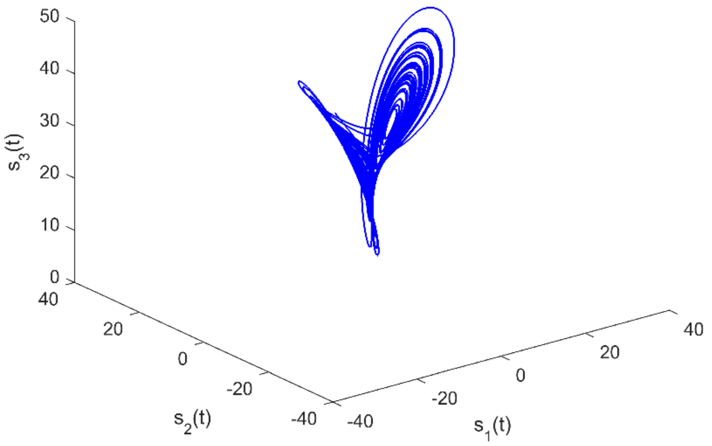

Fig-2.3.1: The chaotic attractor of the fractional-order Lorenz system with memristor

Let $x=[x_1,x_2,x_3,x_4]^T$, the system (5) can be described as

$$D^q x = Ax + f(x) + \phi(x), \tag{2.3.4}$$

where

$$A=\begin{bmatrix} -\alpha & \beta & 0 & 0 \\ \gamma & -1 & 0 & 0 \\ 0 & 0 & -\xi & 0 \\ -\varphi & 0 & 0 & 0 \end{bmatrix},\quad f(x)=\begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \\ 0 \end{bmatrix},\quad \phi(x)=\begin{bmatrix} -\alpha W(x_4)x_1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

and $a, b, \gamma, \xi, \alpha, \beta$ are positive constants.

To investigate the synchronization of the fractional-order Lorenz systems with memristor, the drive system can be rewritten as

$$D^q x(t) = Ax(t) + f(x(t)) + \phi(x(t)). \tag{2.3.5}$$

Similarly, the response system can be described as

$$D^q y(t) = Ay(t) + f(y(t)) + \phi(y(t)) + u(t) \tag{2.3.6}$$

where $y \in R^4$ is the state vector of the system, the controller $u(t)$ is defined as

$$u(t) = -k(y(t) - x(t)),\ k \in R^+.$$

Let $e(t) = y(t) - x(t)$ is the synchronization error between the states of system (5) and system (6), the error system between them can be obtained as

$$D^q e(t) = Ae(t) + f(y(t)) - f(x(t)) + \phi(e(t)) + u(t), \tag{2.3.7}$$

where

$$\begin{aligned}\phi(e(t)) &= \phi(y(t)) - \phi(x(t)) \\ &= [W(x_4(t))x_1(t) - W(y_4(t))y_1(t), 0, 0, 0]^T.\end{aligned} \tag{2.3.8}$$

3. Main result

In this section, the adaptive synchronization problem of fractional-order Lorenz systems with memristor is investigated.

Theorem 3.1: Suppose Assumption 1 and Assumption 2holds. The systems (6) is synchronized with systems (5) and following conditions hold

$$\lambda_A + \eta + 3bM_1M_2 - k < 0, \tag{3.1}$$

where λ_A is the largest eigenvalue of A.

Proof.: Construct the following Lyapunov function

$$V(t)=\frac{1}{2}e^T(t)e(t)\,. \tag{3.2}$$

Take the time derivative of $V(t)$ along the solution $e(t)$ of the system (10). via Lemma 1, and we have

$$\begin{aligned}
D^qV(t) &\le e^T(t)D^qe(t)\\
&= e^T(t)\big(Ae(t)+f(y(t))-f(x(t))+\phi(e(t))+u(t)\big)\\
&= e^T(t)Ae(t)+e^T(t)\big(f(y(t))-f(x(t))\big)+e^T(t)\phi(e(t))-e^T(t)ke(t)\\
&= e^T(\mathrm{t})Ae(\mathrm{t})+e^T(t)\big(f(y(t))-f(x(t))\big)+\Big(\big(a+by_4^2(\mathrm{t})\big)y_1(\mathrm{t})\\
&\qquad -\big(a+bx_4^2(\mathrm{t})\big)x_1(\mathrm{t})\Big)-ke^T(t)e(t)\\
&\le e^T(t)Ae(t)+e^T(t)\eta e(t)+\Big(-ae_1^2(t)+3b\big(y_4^2(t)y_1(t)-y_4^2(t)x_1(t)\\
&\qquad +y_4^2(t)x_1(t)-x_4^2(t)x_1(t)\big)e_1(t)\Big)-ke^T(t)e(t)\\
&\le \lambda_A e^T(t)e(t)+\eta e^T(t)e(t)+3bM_1M_2e^T(t)e(t)-ke^T(t)e(t)\\
&= (\lambda_A+\eta+3bM_1M_2-k)e^T(t)e(t)
\end{aligned} \tag{3.3}$$

Thus from the condition, we get the derivative of the Lyapunov function $V(t)$ as $D^qV(t)<0$.

Hence we may conclude $\lim_{t\to\infty}\|e(t)\|=0$, and the function adaptive synchronization between the master and response systems is achieved.This completes the proof.

4. Numerical simulations

In this section, we give some numerical simulations to illustrate the theoretical results obtained in the above subsection by MATLAB program.

We set $q=0.97$ and choose the initial values of the drive and response systems are (1, 2, -3, -5) and (0, -10, 0, -10). It follows from Theorem 1 that the response system (6) is globally synchronized with the drive system (5) under the controller. Figure 2 shows the time evolution curves of the drive system (5) and the response system (6) without controller, from which it couled be seen that two systems have different state trajectories from each other over time. To obtain our main point, the error dynamics of systems (5) and (6) with controller are to be simulated in several conditions. Figure 3 gives the state trajectories of the drive system (5) and the response system (6) with controller. Figure 4 depicts the error dynamics of systems (5) and (6) with controller, which suggests that impulsive synchronization can be realized in finite time.

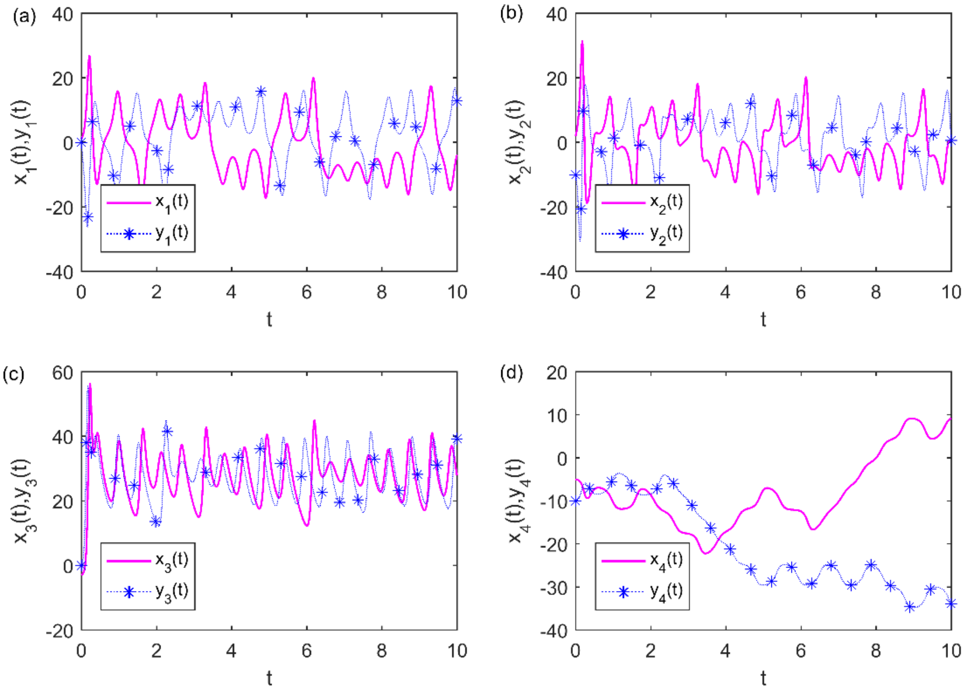

Fig-4.1: Dynamical behaviors of the drive system (5) and the response system (6) without controller, (a) $x_1(t)$, $y_1(t)$; (b) $x_2(t)$, $y_2(t)$; (c) $x_3(t)$, $y_3(t)$; (d) $x_4(t)$, $y_4(t)$

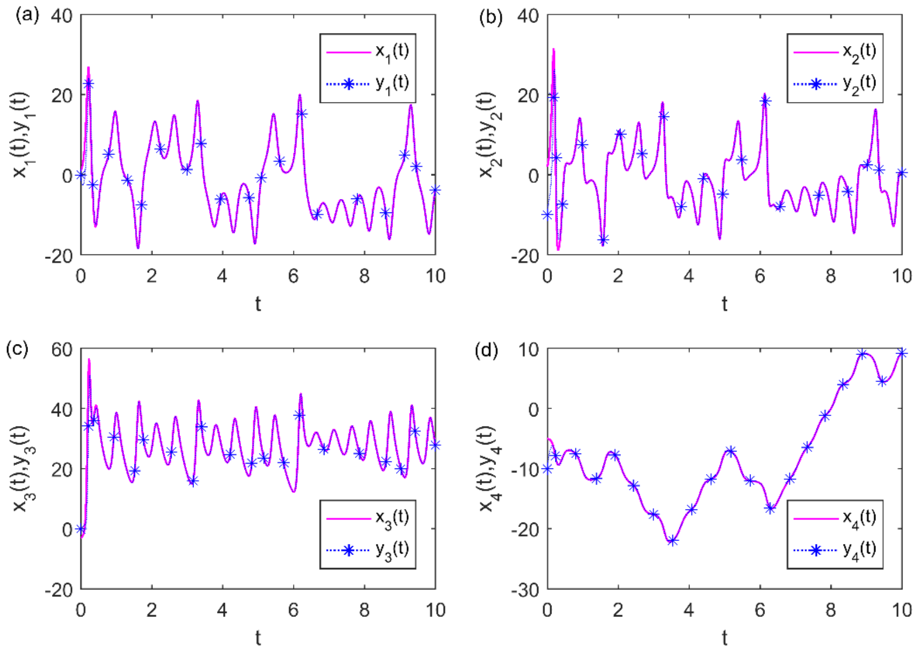

Fig-4.2: Dynamical behaviors of the drive system (5) and the response system (6) with controller, (a) $x_1(t)$, $y_1(t)$; (b) $x_2(t)$, $y_2(t)$; (c) $x_3(t)$, $y_3(t)$; (d) $x_4(t)$, $y_4(t)$

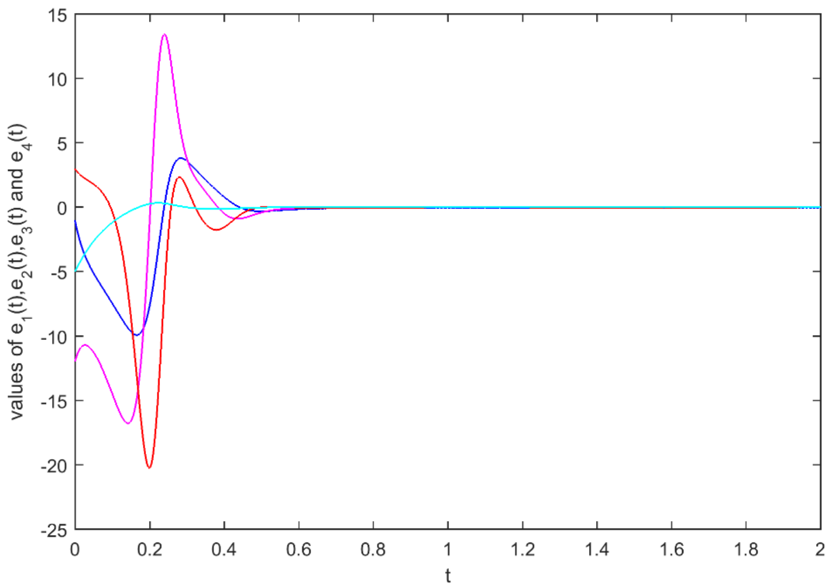

Fig-4.2: Synchronization errors between systems (5) and (6) with Impulsive control matrix

5. Conclusions and future works

In this paper, based on Lyapunov stability theories and some novel criteria, the adaptive synchronization of fractional-order Lorenz systems with memristor have been achieved in finite time. The results are given not only by theoretical analysis, but also by numerical simulations. Because the discussed the Lorenz systems with memristor take into fractional-order, it is practical and attractive in understanding the Lorenz systems with memristor. It will have potential applications to image encryption, cryptography, and chaotic radar. Our future research is to investigate the adaptive synchronization of fractional-order memristive Lorenz systems with time delay.

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