ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 4, 2017, *pp*.264-269

Image encryption based on fractional order 4-D system with memristor

Xuerong Shi[1], Lizhou Zhuang[2], Fanqi Meng[1], Hongsheng Cheng[3], Zuolei Wang[1]*
[1]*School of Mathematics and Statistics, Yancheng Teachers University, Yancheng 224002, China*
[2] *Department of State-owned Assets, Yancheng Teachers University, Yancheng 224002, China*
[3] *School of Information Engineering, Yancheng Teachers University, Yancheng 224002, China*
(Received March 07, 2017, accepted September 16, 2017)

Abstract. By combining fractional differential operator with the memristor-based 4-D system, a fractional order 4-D system with memristor is proposed and the chaotic behavior of it is investigated. According to the chaos characteristics of the proposed system, an image encryption method is put forward. Numerical simulations suggest that the given image encryption method can make an image effectively resist statistics attack in transferring and saving.

Keywords: Image encryption, fractional order, memristor.

1. Introduction

As a classical mathematical notion, fractional calculus is a generalization of the integer-order differentiation. When it was first proposed, due to its complexity and lack of application background, fractional order differential equation did not attract much attention of researchers. Until recent decades, it is proved that fractional order differential equations play an important role and can better describe many systems in science and engineering [1-4].

In recent years, some researchers made much progress in the mathematical construction and theoretical analysis of fractional chaotic systems. It was found that some chaotic systems still present chaos or hyperchaos when the order of these systems is a fraction, such as Lorenz system, Chua's system, Rössler system, Chen system, Lü system. Fractional order Chua's system was studied and a result was obtained that the system could produce chaos when the order of it is less 3 [5, 6]. The least order for fractional order Rössler to present chaos is 2.4 and the least order for fractional order Rössler to appear hyperchaos is 3.8 [7]. The fractional order Chen system is analyzed using fractional calculus theory [8-10]. With the development of chaos or hyperchaos, it has been applied into much fields, such as physics, biology, electrical engineering, communication theory, etc. [11-16].

Based on above, in this paper, the dynamics of a fractional order 4-D system with memristor is investigated and the condition to generate chaot is found. According to this feature, the proposed fractional order 4-D system with memristor is applied into image encryption.

2. Preliminary and system description

In this section, definition of Caputo's fractional derivative of order for a function is introduced to discuss the dynamics of fractional order 4-D system with memristor and the fractional order 4-D system to be investigated is depicted.

Definition 2.1: [1]. Caputo's fractional derivative of order q for function $f \in C^n([0,+\infty),R)$ is defined by

$$ {}_0^cD_t^q f(t) = \frac{1}{\Gamma(n-q)}\int_0^t \frac{f^n(\tau)}{(t-\tau)^{q-n+1}}d\tau , \tag{2.1}$$

where $t \geq 0$, n is a positive integer such that $n-1<q<n$. $\Gamma(\cdot)$ is gamma function satisfying $\Gamma(\tau)=\int_0^\infty t^{\tau-1}e^{-t}dt$ and $\Gamma(\tau+1)=\tau\Gamma(\tau)$.

From formula (1), we can know that, for $0<q<1$, it can be obtained

$$ {}_{0}^{c}D_{t}^{q} f\left(t\right)=\frac{1}{\Gamma\left(1-q\right)}\int_{0}^{t}\frac{f'\left(\tau\right)}{\left(t-\tau\right)^{q}}d\tau . $$

It should be noted that, for simplicity, we denote ${}_{0}^{c}D_{t}^{q} f\left(t\right)$ as $D^{q} f\left(t\right)$ in following operator.

According to the method reported in [17], a new memristive system had been constructed by utilizing memristor to substitute resistor in a three-dimensional hypogenetic chaotic jerk system proposed by [18], which is given as

$$ \begin{cases} \dot{x} = |y| - b \\ \dot{y} = W(w)z \\ \dot{z} = |x| - y - az - c \\ \dot{w} = z \end{cases}, \tag{2.2} $$

where x, y, z are state variables, w is the inner dimensionless state variable of the proposed memristor, and $W(w) = \alpha - \beta|w|$ is the memductance function used to describe an ideal and active flux-controlled memristor with an absolute value nonlinearity. The dynamical behaviors of memristive system (2) have been studied and initial condition-dependent dynamical phenomenon of extreme multistability has been found [19].

Learned from model (2), the corresponding fractional order system with memristor is described as

$$ \begin{cases} D^{q}x = |y| - b \\ D^{q}y = W(w)z \\ D^{q}z = |x| - y - az - c \\ D^{q}w = z \end{cases}. \tag{2.3} $$

When the parameters are selected as a = 0.6, b = 1.3, c = 2, α= 1, β= 0.1 and the initial conditions are set as (0, 0, 0.001, 0), the lookalike chaotic attractor can be observed in the fractional order system (3) at q=0.98, which can be seen in Fig. 1. Furthermore, time series of variables x and z are given in Fig.2 and Fig.3, respectively, which verifies that the variables can appear chaotic behavior for appropriate parameters.

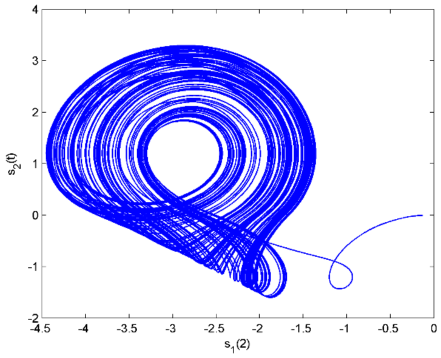

Fig-2.1:- Lookalike chaotic attractor of system (3) for q=0.98.

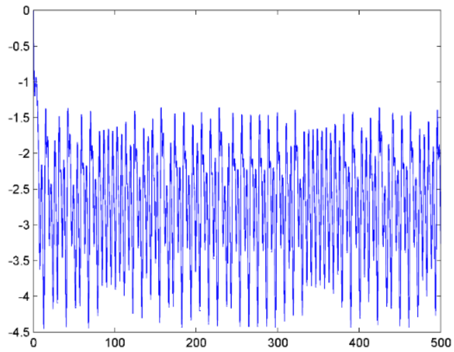

Fig-2.2:- Time series of variable x in system (3).

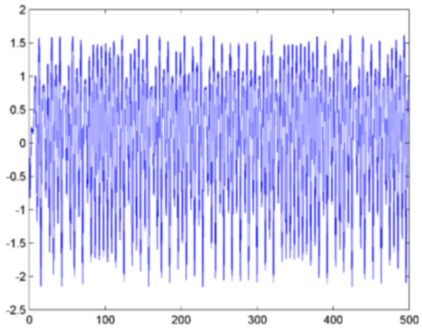

Fig-2.3:- Time series of variable z in system (3).

3. Image encryption

3.1. Encryption process

According to the sequences of state variables produced in Section 2, an image encryption method is given, which mainly includes following steps.

Step 1. Read the image and obtain its size $M \times N$.

Step 2. Generate chaotic sequence.

Choose suitable system parameters and initial values of system (3) and make it have chaotic behavior, then throw away some groups at the beginning of the sequences and take the remaining groups as

$$S = (x_j, y_j, z_j, w_j)(j = 1,2,\cdots).$$

Step 3. Pixel permutation

Image pixels are permutated according to logistic map [20]

$$x_{n+1} = \mu x_n (1 - x_n) \tag{3.1.1}$$

and formula

$$\begin{cases} x_{i+1} = (x_i + y_i + abs(fix(z_j)) + abs(fix(z_{j+k}))) \bmod M. \\ y_{i+1} = (y_i + abs(fix(z_{j+k})) + K \sin(\dfrac{2\pi x_{i+1}}{N})) \bmod N. \end{cases} \tag{3.1.2}$$

In map (4), μ $(3.5699456 < \mu \le 4)$ is the control parameter. In formula (5), (x_i, y_i) and (x_{i+1}, y_{i+1}) are the locations before and after permutation, respectively. The extraction of (x_i, y_i) is in line with the system (3) while the extraction of z_j is in light of the logistic map (4). K is a constant. $abs(x)$ is the absolute value of x. $fix(x)$ rounds the element of x toward zero resulting in an integer. f is positive integer indicating interval between r_x and r_y.

Step 3. Pixel diffusion

Pixel values are modified sequentially according to

$$v = p \oplus (c \times x_i + d \times y_i) \bmod L, \tag{3.1.3}$$

where p and v are the pixels' values before and after the modification, respectively. (x_i, y_i) means the pixel location. L is the grayscale of pixels. c and d are diffusion parameters in view of

$$\begin{cases} c = abs(10^l x_j - round(10^l x_j)) \times 10^3. \\ d = abs(10^l y_j - round(10^l y_j)) \times 10^3. \end{cases} \tag{3.1.4}$$

According to the inverse of above process, encrypted image can be decrypted.

3.2. Simulation results and security analysis

Without loss of generality, Lenna is selected as simulation example to verify the effectiveness of the proposed method. Fig.4 depicts the encryption and decryption results of Lena image based on fractional order 4-D system with memristor, which means that the proposed method not only can encrypt Lenna image effectively but also can make the encrypted image be well restored.

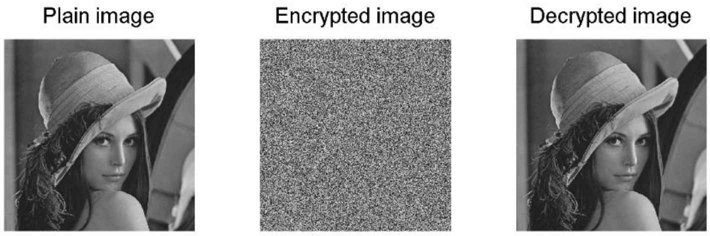

Fig-3.1:- The encryption and decryption results of Lena image based on fractional order 4-D system with memristor.

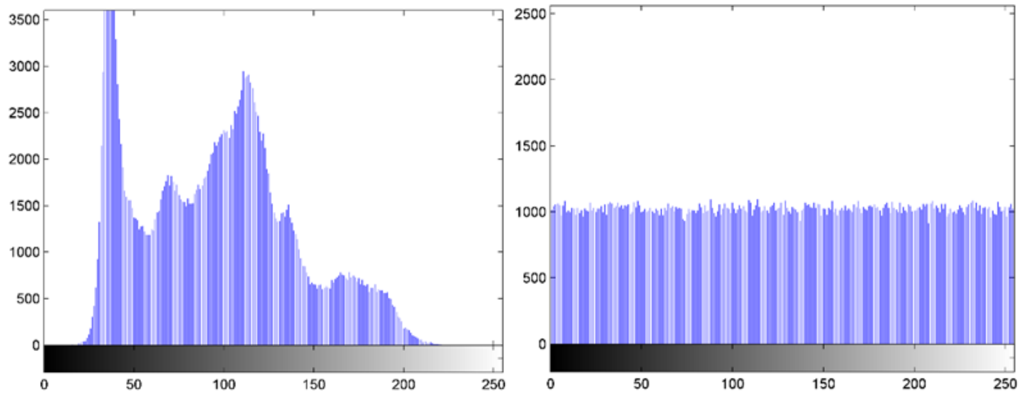

Fig-3.2:-Histogram of plain Lenna image and the encrypted one,
(a) Histogram of plain Lenna image, (b) Histogram of encrypted Lenna image.

To examine the security of the proposed encryption method, Fig.5 draws the histogram of plain Lenna image and the encrypted Lenna image, from which it obvious to see that the histogram of plain Lenna image is provided with strong statistical characteristics while the histogram of the encrypted Lenna image has a strong randomness. It means that the encryption method can better discretize the statistical features in the image. To furthermore illustrate this feature, the scatters of plain Lenna image and the encrypted Lenna image are drawn in Fig.6. From Fig.6(a)-(c), it can be seen that, adjacent pixels in the plain Lenna image have a strong correlation in three directions, such as horizontal, vertical and diagonal. But from Fig.6(d)-(f), it’s known that, in the encrypted Lenna image, correlation between adjacent pixels becomes very weak in the same three directions. This result suggests that the image encryption based on fractional order 4-D system with memristor can greatly reduce the correlation between adjacent pixels. Fig.5 and Fig.6 suggest that the proposed image encryption method can make an image resist statistical attacks effectively.

(a)

4. Conclusion

In this paper, by applying fractional differential operator, a fractional order 4-D system with memristor is proposed and chaotic behaviors of it have been explored. Results suggest that proposed system can show chaotic characteristics dependent on the system parameters and initial values of it. According to this fact, an image encryption method based on the fractional order system with memristor is put forward. Numerical simulation results suggest that this image encryption scheme not only can make an image be well encrypted but also can make the encrypted image be fully restored. Furthermore, analysis of histogram and the correlation between adjacent pixels illustrates that the proposed image encryption method has good resistance to statistical attack in image transferring and can be used in image encryption.

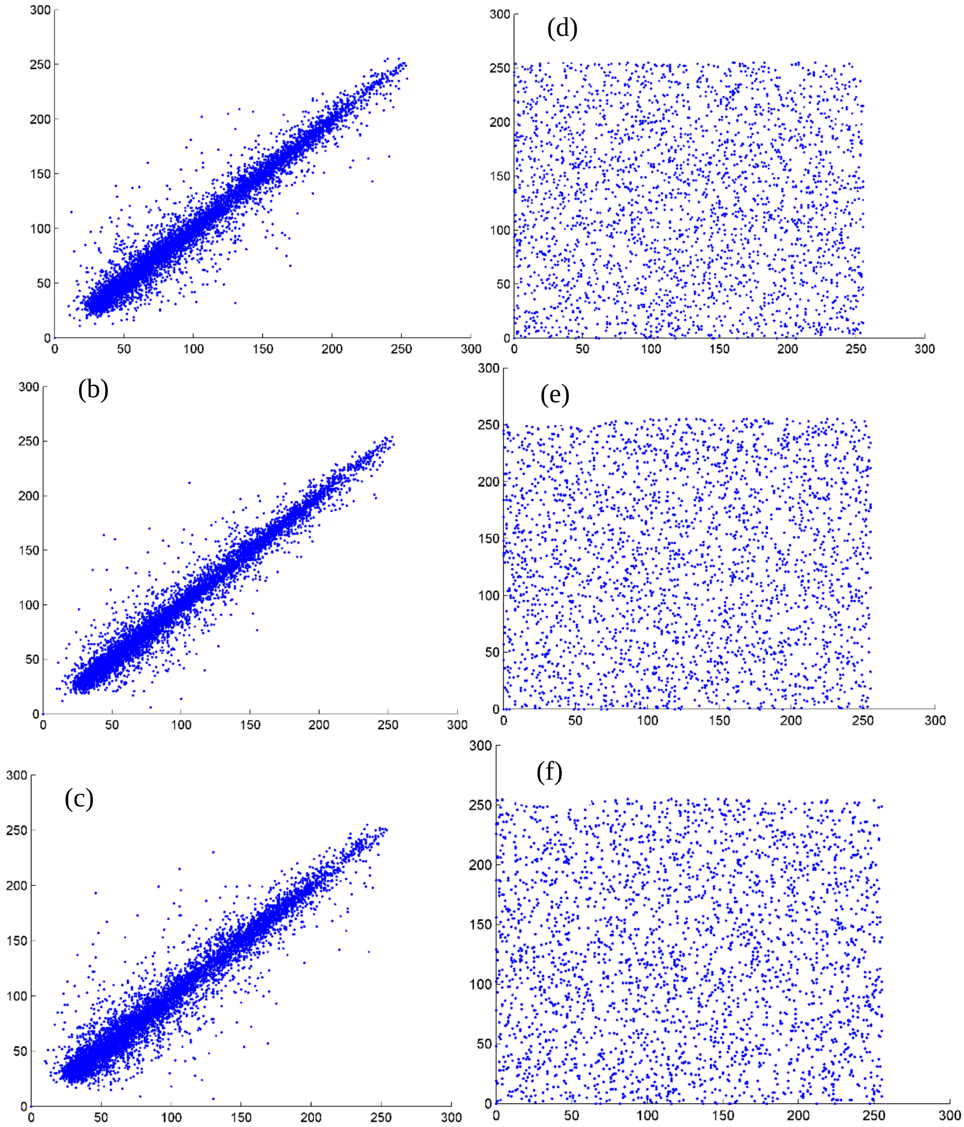

Fig-3.3:- Scatter of the correlation between adjacent pixels in plain Lenna image and the encrypted Lenna image, (a) Horizontal scatter of plain Lenna image, (b) Vertical scatter of plain Lenna image, (c) Diagonal scatter of plain Lenna image, (d) Horizontal scatter of encrypted Lenna image, (e) Vertical scatter of encrypted Lenna image, (f) Diagonal scatter of encrypted Lenna image.

5. Acknowledgement

This work is supported by National Natural Science Foundation of China (Grant Nos. 11472238 and 51777180) and the Qing Lan Project of the Jiangsu Higher Education Institutions of China.

6. References

[1] A. Kilbas, H. Srivastava, J. Trujillo, Theory and application of fractional differential equations, Elsevier, New York, 2006.

[2] E. Ahmeda, A. Elgazzar, On fractional order differential equations model for nonlocal epidemics, Physica A 2007, 379: 607-614.

[3] K. Moaddy, A. Radwan, K.Salama, et al., The fractional-order modeling and synchronization of electrically coupled neuron systems, Comput. Math. Appl. 2012,64: 3329-3339.

[4] S. Bhalekar, V. Daftardar-Gejji, Synchronization of differential fractional-order chaotic systems using active control, Commun. Nonlinear Sci. Numer. Simul. 2010, 15: 3536-3546.

[5] T.T. Hartley, C.F. Lorenzo, Qammer H. Killory, Chaos in a fractional order Chua's system, Circuits & Systems I Fundamental Theory & Applications IEEE Transactions on IEEE Trans CAS-I 1995, 42(08): 485-490.

[6] J.G. Lu, Synchronization of a class of fractional-order chaotic systems via a scalar transmitted signal, Chaos Soliton. Fract. 2006, 27(2): 519-525.

[7] C.G. Li, G.R. Chen, Chaos and hyperchaos in the fractional-order Rössler equations, Physica A 2004, 341(1): 55-61.

[8] C.P. Li, G.J. Peng, Chaos in Chen's system with a fractional-order, Chaos Soliton. Fract. 2004, 22(2): 443-450.

[9] C.G. Li, G.R. Chen, Chaos in the fractional-order Chen system and its control, Chaos Soliton. Fract. 2004, 22(3): 549-554.

[10] J.G. Lu, G.R. Chen, A note on the fractional-order Chen system, Chaos Soliton. Fract. 2006, 27(3): 685-688.

[11] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 1990, 64 : 821- 824.

[12] X.J. Wu, J.S. Liu, G.R. Chen, Chaos synchronization of Rikitake chaotic attractor using the passive control technique, Nonlinear Dyna. 53 (2007) 45-53.

[13] X.Y. Wang, C.F. Duan, Observer based chaos synchronization and its application to secure communication, J. Chin.Inst. Commun. 2005, 26:105-111.

[14] H. Wang, Z.Z. Han, W. Zhang, et al., Chaotic synchronization and secure communication based on descriptor observer, Nonlinear Dynam. 2009, 57: 69-73.

[15] F.T. Werner, B.K. Rhea, R.C. Harrison, et al., Electronic implementation of a practical matched filter for a chaos-based communication system, Chaos Soliton. Fract. 2017, 104: 461-467.

[16] D. Prasad, A. Mukherjee, V. Mukherjee, Application of chaotic krill herd algorithm for optimal power flow with direct current link placement problem, Chaos Soliton. Fract. 2017, 103: 90-100.

[17] B.C. Bao, H. Bao, N. Wang, et al., Hidden extreme multistability in memristive hyperchaotic system, Chaos Soliton. Fract., 2017, 94: 102-111.

[18] C. Li, J.C. Sprott, W. Thio, et al. A new piecewise linear hyperchaotic circuit. Circuits & Systems II Express Briefs IEEE Transactions on 2014, 61(12): 977-981.

[19] H. Bao, N. Wang, B.C. Bao, et al., Initial condition-dependent dynamics and transient period in memristor-based hypogenetic jerk system with four line equilibria, Commun. Nonlinear Sci. Numer. Simulat. 2018, 57: 264-275.

[20] N.K. Pareek, V. Patidar, K.K. Sud. Image encryption using chaotic logistic map, Image Vision Comput. 2006, 24(9):926-934.