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Wavelet based numerical solution of linear and non-linear parabolic partial differential equations using Lifting scheme

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Abstract. Partial differential equations are fundamental in modeling several natural phenomena. The present work is designed for the Wavelet based numerical solution of linear and non-linear parabolic partial differential equations using lifting scheme. To demonstrate the efficiency and competence of the proposed scheme, we used both orthogonal and biorthogonal wavelets. This scheme speeds up convergence in lesser computational time as compared with existing schemes. Some test problems are presented for the validity and applicability of the scheme.

Keywords: Parabolic partial differential equations: Lifting scheme; Orthogonal and Biorthogonal wavelets.

1. Introduction

In recent years there has been much attention for finding the numerical solution of partial differential equations (PDEs). In general parabolic PDEs with or without reaction terms are used as a fundamental tool to model a wide class of phenomena occurring in physical and biological sciences, such as Heat equation, Wave equation, Fisher's equation, Cahn-Allen equation, Burgers equation and many other important equations. These equations are usually difficult to solve analytically, so these are required to obtain efficient approximate solutions. The importance of obtaining the exact or approximate solutions of nonlinear partial differential equations in physics and mathematics is still a significant problem that needs new methods to discover exact or approximate solutions. Due to this, there is a demand on the development of accurate and efficient analytical or numerical methods able to deal with these equations.

Recently, some of the iterative methods are used for the numerical and analytical solutions of Linear and Nonlinear partial differential equations. For example, He's variational iteration technique [1], the homotopy perturbation method [2], Adomian decomposition method [3] etc. Mathematical models of basic flow equations which describe unsteady transport problems are governed by a single or a system of nonlinear PDEs.
Analytical solution of certain parabolic PDEs either does not exist or is hard to find. Due to this fact, in the last decades, there have been great advances in the development of finite difference, finite element, spectral techniques and finite volume methods for the solution of parabolic PDEs. The parabolic PDEs of the forms [4],

$$u_t = u_{xx} + f(u), \qquad 0 \le x \le 1,\ t > 0 \tag{1.1}$$

$$u_t = u_{xx} + g(x,t), \qquad 0 \le x \le 1,\ t > 0 \tag{1.2}$$

subject to initial condition (IC) and boundary conditions (BCs). Where and are the functions of dependent and independent variables.

The purpose of this paper is to give a numerical solution for a class of linear and non-linear parabolic partial differential equations by Lifting scheme using different wavelets.

Wavelet theory had been developed independently on several fronts. Different signal processing techniques, developed for signal and image processing applications, had significant contribution in this development. Some of the major contributors to this theory are: multiresolution signal processing used in computer vision; sub band coding, developed for speech and image compression; and wavelet series expansion, developed in applied mathematics. Using different wavelets, various numerical methods have been applied for solving PDEs from beginning of the early 1990s. In the last two decades this problem has attracted great attention and numerous papers about these topics have been published. Wavelets permit the perfect representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithm [5]. Wavelet based numerical methods are used for solving the system of equations with faster convergence and lower computational cost.

Some of the earlier works on wavelet based methods can be found in [6]. A collection of the discrete wavelet transforms (DWT) and the FAS were introduced recently in [7-9]. The wavelet based full approximation scheme (WFAS) has exposed to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [10]. These methods can be either used as an iterative solver or as a preconditioning technique, offering in many cases a better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, work build that is orthogonal/biorthogonal discrete wavelet transform using lifting scheme [11]. Wavelet based lifting technique is introduced by Sweldens [12], which permits some improvements on the properties of existing wavelet transforms. The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently all attempts to simplify the wavelet solutions for PDE are welcome. In PDE, matrices arising from system are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using wavelet transform and it leads to design the effective wavelets based lifting scheme.

Lifting scheme is a new approach to construct the so-called second generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as a first generation wavelets or classical methods. The lifting scheme has some additional advantages in comparison with the classical wavelets. This transform works for signals of an arbitrary size with correct treatment of boundaries. Another feature of the lifting scheme is that all constructions are derived in the spatial domain. This is in contrast to the traditional approach, which relies heavily on the frequency domain. Staying in the spatial domain leads to two major advantageous: i) It does not require the machinery of Fourier analysis as a prerequisite, this leads to a more intuitively appealing treatment better suited to those in interested in applications than mathematical foundations. ii) The algorithms that can easily be generalized to complex geometric situations, this leads to second generation wavelets. In addition, the lifting scheme makes a computational time optimal and sometimes increasing the speed of calculations.

The lifting scheme starts with a set of well-known filters, thereafter lifting steps are used an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some mathematical benefits as a reduced number of operations which are essential in the context of the iterative solvers. In addition to this, the present paper illustrates that the application of the lifting technique to the real world problems.

The present paper is organized as follows: Section 2 deals with Preliminaries of wavelet filter coefficients and Lifting scheme. Method of solution is discussed in Section 3. Section 4 provides Numerical results of the test problems and finally, in section 5 conclusions of the proposed work are discussed

2. Related Work

The important feature of the lifting scheme is that every filter bank based on lifting automatically satisfies perfect reconstruction properties. The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in attempt to improve the properties of corresponding wavelet decomposition.

Now, we have discussed about different wavelet filters as follows:

a) Haar wavelet filter coefficients

We know that low pass filter coefficients $[a_0,\ a_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ b_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition. Thus it is natural to wonder that it possible to model the decomposition in terms of linear transformations i.e. matrices. Moreover, since digital signals and images are composed of discrete data, we need a discrete analog of the decomposition algorithm so that we can process signal and image data.

b) Daubechies wavelet filter coefficients

Daubechies introduced scaling functions having the shortest possible support. The scaling function ϕ_N has support $[0,\ N-1]$, while the corresponding wavelet ψ_N has support in the interval $[1-N/2,\ N/2]$.

We have low pass filter coefficients $[a_0,\ a_1,\ a_2,\ a_3]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ b_1,\ b_2,\ b_3]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}},\ -\frac{3-\sqrt{3}}{4\sqrt{2}},\ \frac{3+\sqrt{3}}{4\sqrt{2}},\ -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$

c) Biorthogonal (CDF (2,2)) wavelets

Let's consider the (5, 3) biorthogonal spline wavelet filter pair, the low pass filter pair are $(\tilde{a}_{-1},\tilde{a}_0,\tilde{a}_1)=\left(\frac{1}{2\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{2\sqrt{2}}\right)$ and $(a_{-2},a_{-1},a_0,a_1,a_2)=\left(\frac{-1}{4\sqrt{2}},\frac{1}{2\sqrt{2}},\frac{3}{2\sqrt{2}},\frac{1}{2\sqrt{2}},\frac{-1}{4\sqrt{2}}\right)$.

But, we have $b_k=(-1)^k\tilde{a}_{1-k}$ and $\tilde{b}_k=(-1)^k a_{1-k}$, the highpass filter pair are $b_0=\frac{1}{2\sqrt{2}},\ b_1=\frac{-1}{\sqrt{2}},\ b_2=\frac{1}{2\sqrt{2}}$ & $\tilde{b}_{-1}=\frac{1}{4\sqrt{2}},\ \tilde{b}_0=\frac{1}{2\sqrt{2}},\ \tilde{b}_1=\frac{-3}{2\sqrt{2}},\ \tilde{b}_2=\frac{1}{2\sqrt{2}},\ \tilde{b}_3=\frac{1}{4\sqrt{2}}$

Foundations of lifting scheme:

Consider to numbers *a*, *b* as two neighbouring samples of a sequence and then these have some correlation which we would like to take advantage. The simple linear transform which replaces *a* and *b* by average ***s*** and difference *d* i.e.

$$s=\frac{a+b}{2} \quad \& \quad d=\frac{a-b}{2}$$

The idea is that if *a* and *b* are highly correlated, the expected absolute value of their difference d will be small and can be represented with fever bits. In case that *a* = *b*, the difference is simply zero. We have not lost any information because we can always recover *a* and *b* from the gives *s* and *d* as:

$$a=s-\frac{d}{2} \quad \& \quad b=s+\frac{d}{2}$$

Finally, a wavelet transform built through lifting consists of three steps: split. Predict and update as given in the Figure 1.

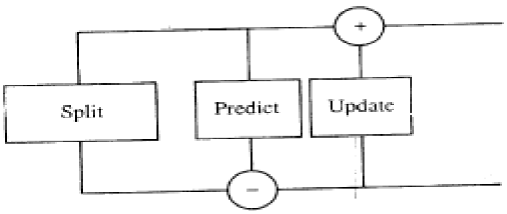


Fig. 1. Steps in lifting scheme

Split: Splitting the signal into two disjoint sets of samples.

Predict: If the signal contains some structure, then we can expect correlation between a sample and its nearest neighbors. i.e. $d=\text{odd}-\text{P(even)}$

Update: Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry to reflect our knowledge of the signal. i.e. $s=\text{even}+\text{U}(d)$

The detailed algorithm using different wavelets is given in the next section. The general lifting stages for decomposition and reconstruction of a signal are given in Figure 2.

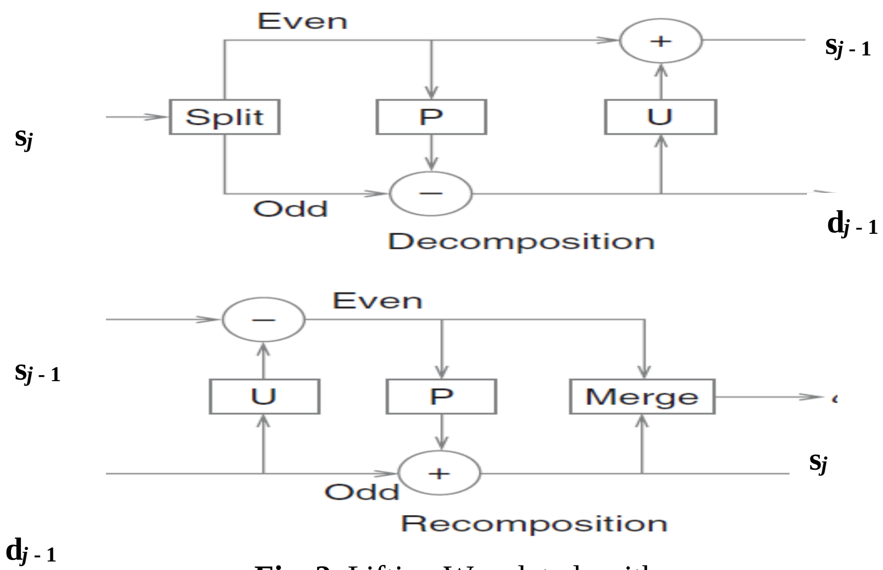


Fig. 2. Lifting Wavelet algorithm.

3. Method of solution

Consider one-dimensional parabolic PDEs (1) or (2), after discretizing this equation through the finite difference method (FDM), we get system of algebraic equations. Through this system we can write the system as

$$Au = b \tag{3.1}$$

where A is $N \times N$ coefficient matrix, b is $N \times N$ matrix and u is $N \times N$ matrix to be determined. where $N = 2^J$, N is the number of grid points and J is the level of resolution.

Solve Eq. (3.1) through the iterative method, we get approximate solution. Approximate solution containing some error, therefore required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are HWLS, DWLS BWLS etc. Now we are using the advanced technique based on different wavelets called as lifting scheme. Recently, lifting schemes are useful in the signal analysis and image processing in the area of science and engineering. But currently it extends to approximations in the numerical analysis [10]. Here, we are discussing the algorithm [11] of the lifting schemes as follows.

3.1 Haar wavelet Lifting scheme (HWLS)

In [11], Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [12]. The representation of Haar wavelet via lifting form presented as;

Decomposition:

Consider approximate solution $S = \tilde{P}_j$ like as signal, then apply the HWLS decomposition (finer to coarser) procedure as,

$$\left.\begin{aligned} & d^{(1)} = S_{2j} - S_{2j-1}, \\ & s^{(1)} = S_{2j-1} + \frac{1}{2} d^{(1)}, \\ & S_1 = \sqrt{2}\, s^{(1)} \quad \text{and} \\ & D = \frac{1}{\sqrt{2}} d^{(1)} \end{aligned}\right\} \tag{3.2}$$

In this stage finally, we get new approximation as,

$$S = [S_1 \;\; D] \tag{3.3}$$

Reconstruction:

Consider Eq. (3.3) and then apply the HWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned} d^{(1)} &= \sqrt{2}\ D, \\ s^{(1)} &= \frac{1}{\sqrt{2}} S_1, \\ S_{2j-1} &= s^{(1)} - \frac{1}{2} d^{(1)} \quad \text{and} \\ S_{2j} &= d^{(1)} + S_{2j-1} \end{aligned}\right\} \tag{3.4}$$

which is the required solution of the given equation.

3.2 Daubechies wavelet Lifting scheme (DWLS)

As discussed in the previous section 3.1, we follow the same procedure but we used different wavelet i.e., Daubechies 4th order wavelet coefficient. The DWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned} s^{(1)} &= S_{2j-1} + \sqrt{3} S_{2j}, \\ d^{(1)} &= S_{2j} - \frac{\sqrt{3}}{4} s^{(1)} - \left(\frac{\sqrt{3}-2}{4}\right) s_1^{(j-1)}, \\ s^{(2)} &= s^{(1)} - d_1^{(j+1)}, \\ S_1 &= \frac{\sqrt{3}-1}{\sqrt{2}} s^{(2)} \quad \text{and} \\ D &= \frac{\sqrt{3}+1}{\sqrt{2}} d^{(1)} \end{aligned}\right\} \tag{3.5}$$

Here, we get new approximation as,

$$S = [S_1 \;\; D] \tag{3.6}$$

Reconstruction:

Consider Eq. (3.6), then apply the DWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned} d^{(1)} &= \frac{\sqrt{2}}{\sqrt{3}+1} D, \\ s^{(2)} &= \frac{\sqrt{2}}{\sqrt{3}-1} S_1, \\ s_1^{(j)} &= s^{(2)} + d_1^{(j+1)}, \\ S_{2j} &= d^{(1)} + \frac{\sqrt{3}}{4} s_1^{(j)} + \frac{\sqrt{3}-2}{4} s_1^{(j-1)} \quad \text{and} \\ S_{2j-1} &= s^{(1)} - \sqrt{3} S_{2j} \end{aligned}\right\} \tag{3.7}$$

which is the required solution of the given equation.

3.3 Biorthogonal wavelet Lifting scheme (BWLS)

As discussed in the previous sections 3.1 and 3.2, we follow the same procedure here we used another wavelet i.e., biorthogonal wavelet (CDF(2,2)). The BWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned} d^{(1)} &= S_{2j} - \frac{1}{2}\left[S_{2j-1} + S_{2j+2}\right], \\ s^{(1)} &= S_{2j-1} + \frac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right], \\ D &= \frac{1}{\sqrt{2}} d^{(1)}, \\ S_1 &= \sqrt{2}\, s^{(1)} \end{aligned}\right\} \quad (3.8)$$

In this stage finally, we get new signal as,

$$S = [S_1 \;\; D] \quad (3.9)$$

Reconstruction:

Consider Eqn. (3.9), then apply the DWLS reconstruction (coarser to finer) procedure as

$$\left.\begin{aligned} s^{(1)} &= \frac{1}{\sqrt{2}} S_1, \\ d^{(1)} &= \sqrt{2}\, D, \\ S_{2j-1} &= s^{(1)} - \frac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right] \\ S_{2j} &= d^{(1)} + \frac{1}{2}\left[S_{2j-1} + S_{2j+2})\right], \end{aligned}\right\} \quad (3.10)$$

which is the required solution of the given equation.

The coefficients $s_1^{(j)}$ and $d_1^{(j)}$ are the average and detailed coefficients respectively of the approximate solution u_a . The new approaches are tested through some of the numerical problems and the results are shown in next section.

4. Numerical examples

In this section, we applied Lifting scheme for the numerical solution of linear and nonlinear parabolic partial differential equations and also demonstrate the competence and applicability of HWLS, DWLS and BWLS. The error is computed by $E_{\max} = \max|u_e - u_a|$, where u_e and u_a are exact and approximate solution respectively.

Test Problem 4.1 We consider the equation (1.1) with $f(u) = -2u$

$$\text{i.e.} \quad u_t = u_{xx} - 2u \quad (4.1.1)$$

subject to the I.C.:

$$u(x,0) = \sinh x \quad (4.1.2)$$

and

$$\text{B.C.s:} \quad u(0,t) = 0, \quad u(1,t) = \sinh(1)\times e^{-t} \quad (4.1.3)$$

Which has the exact solution $u(x,t) = \sinh(x)\times e^{-t}$ [4]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 1. The maximum absolute errors with CPU time of the methods are presented in table 1.

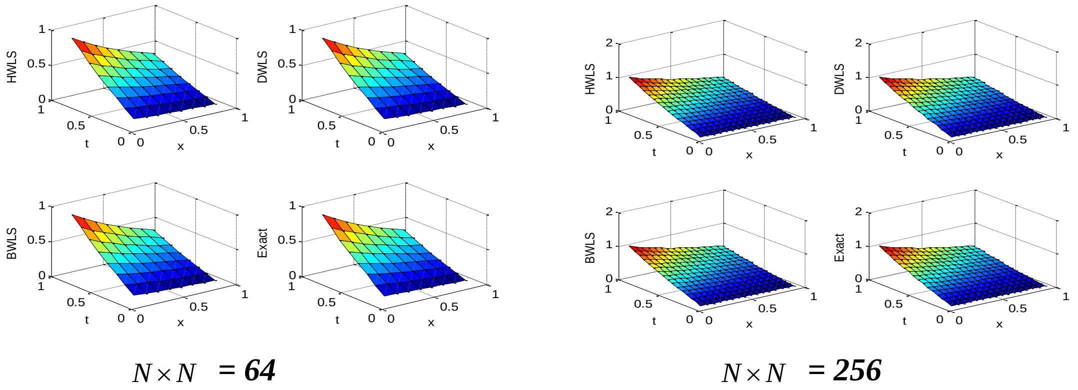


Fig. 1. Comparison of numerical solutions with exact solution of test problem 4.1 for $N \times N$ =64 & 256.

Table 1. Maximum error and CPU time (in seconds) of the methods of test problem 4.1

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
16	FDM	4.0195e-03	2.7089	0.0721	2.7840
	HWLS	4.0195e-03	0.0011	0.0020	0.0031
	DWLS	4.0195e-03	0.0009	0.0104	0.0113
	BWLS	4.0195e-03	0.0007	0.0044	0.0051
256	FDM	1.2802e-03	3.1700	01505	3.3205
	HWLS	1.2802e-03	0.0011	0.0021	0.0032
	DWLS	1.2802e-03	0.0009	0.0100	0.0109
	BWLS	1.2802e-03	0.0008	0.0044	0.0052
4096	FDM	3.4759e-04	50.1420	15.9000	66.0420
	HWLS	3.4759e-04	0.0013	0.0026	0.0039
	DWLS	3.4759e-04	0.0010	0.0117	0.0127
	BWLS	3.4759e-04	0.0008	0.0052	0.0060

Test Problem 4.2 We consider the equation (1.2) with $g(x,t)=\sin x$,

$$\text{i.e.} \quad u_t = u_{xx} + \sin x \tag{4.2.1}$$

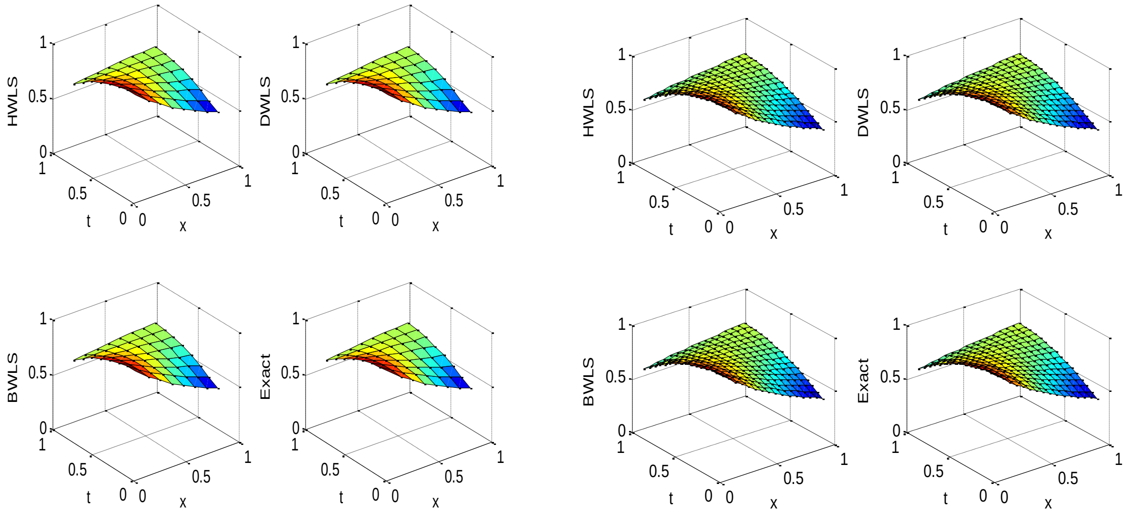
subject to the I.C.:

$$u(x,0) = \cos x \tag{4.2.2}$$

and

$$\text{B.C.s:} \quad u(0,t) = e^{-t}, \quad u(1,t) = \cos(1)e^{-t} + \sin(1)\left(1-e^{-t}\right) \tag{4.2.3}$$

which has the exact solution $u(x,t) = e^{-t}\cos x + \left(1-e^{-t}\right)\sin x$ [4]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 2. The maximum absolute errors with CPU time of the methods are presented in table 2.



$N \times N$ **= 64** $N \times N$ **= 256**

Fig. 2. Comparison of numerical solutions with exact solution of test problem 4.2 for $N \times N$ =64 & 256.

Table 2. Maximum error and CPU time (in seconds) of the methods of test problem 4.2

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
16	FDM	3.7828e-03	5.6612	0.0567	5.7179
	HWLS	3.7828e-03	0.0007	0.0013	0.0020
	DWLS	3.7828e-03	0.0006	0.0070	0.0076
	BWLS	3.7828e-03	0.0005	0.0029	0.0034
256	FDM	1.1534e-03	3.2005	0.1073	3.3078
	HWLS	1.1534e-03	0.0007	0.0013	0.0020
	DWLS	1.1534e-03	0.0006	0.0065	0.0071
	BWLS	1.1534e-03	0.0005	0.0038	0.0043
4096	FDM	3.1024e-04	49.4390	20.2830	69.7220
	HWLS	3.1024e-04	0.0011	0.0023	0.0034
	DWLS	3.1024e-04	0.0009	0.0111	0.0120
	BWLS	3.1024e-04	0.0008	0.0050	0.0058

Test Problem 4.3 We consider the equation (1) with $f(u) = 6u(1-u)$

i.e. $$u_t = u_{xx} + 6u(1-u) \tag{4.3.1}$$

subject to the I.C.: $$u(x,0) = 1\big/\left(1+e^{x}\right)^2 \tag{4.3.2}$$

and B.C.s: $$u(0,t) = 1\big/\left(1+e^{-5t}\right)^2, \quad u(1,t) = 1\big/\left(1+e^{1-5t}\right)^2 \tag{4.3.3}$$

Which has the exact solution $u(x,t) = 1\big/\left(1+e^{x-5t}\right)^2$ [13]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 3. The maximum absolute errors with CPU time of the methods are presented in table 3.

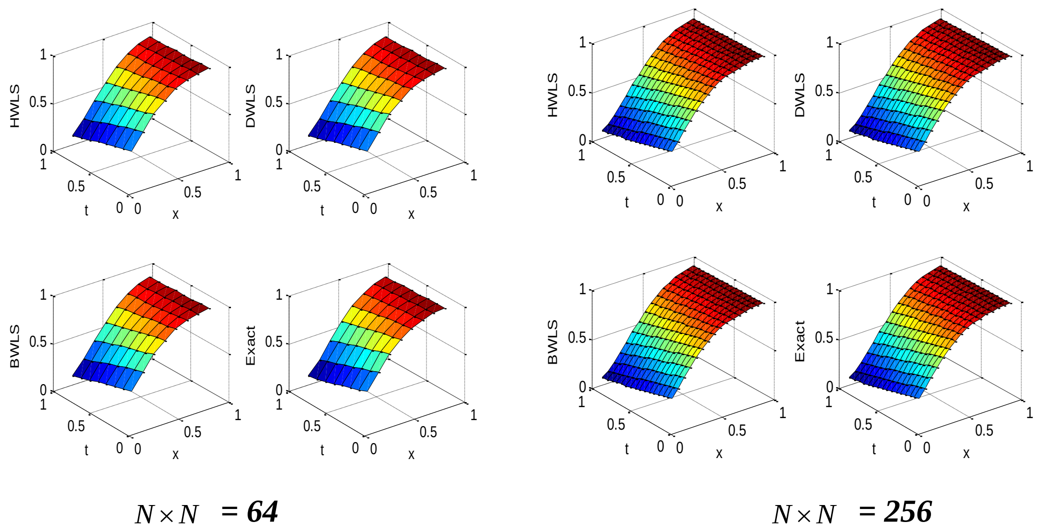


Fig. 3. Comparison of numerical solutions with exact solution of test problem 4.3 for $N \times N$ =64 & 256.

Table 3. Maximum error and CPU time (in seconds) of the methods of test problem 4.3

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
16	FDM	2.1332e-02	2.9144	0.0004	2.9148
	HWLS	2.1332e-02	0.0011	0.0017	0.0028
	DWLS	2.1332e-02	0.0010	0.0128	0.0138
	BWLS	2.1332e-02	0.0009	0.0044	0.0053
256	FDM	8.7386e-03	3.2582	0.0015	3.2597
	HWLS	8.7386e-03	0.0011	0.0017	0.0028
	DWLS	8.7386e-03	0.0010	0.0109	0.0119
	BWLS	8.7386e-03	0.0005	0.0027	0.0032
4096	FDM	2.6256e-03	10.1280	0.0034	10.1314
	HWLS	2.6256e-03	0.0007	0.0012	0.0019
	DWLS	2.6256e-03	0.0006	0.0083	0.0089
	BWLS	2.6256e-03	0.0005	0.0029	0.0034

Test problem 4.4. We consider the equation (1) with $f(u) = 2u^3$

$$\text{i.e.} \quad u_t = u_{xx} - 2u^3 \qquad (4.4.1)$$

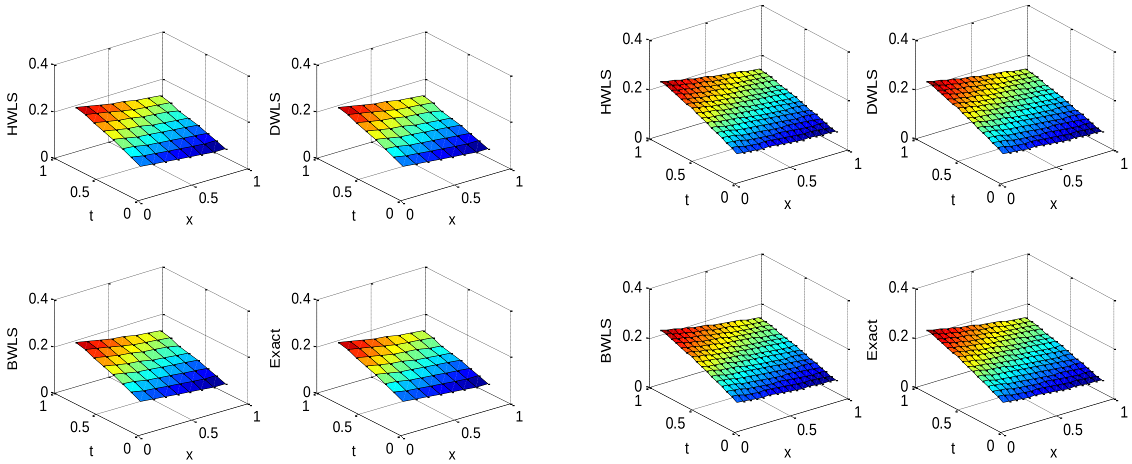
subject to the I.C.:

$$u(x,0) = {(2x+1)}\Big/{(x^2+x+10)} \qquad (4.4.2)$$

and B.C.s:

$$u(0,t) = 1\Big/(6t+10), \quad u(1,t) = 1\Big/(2t+4) \qquad (4.4.3)$$

Which has the exact solution $u(x,t) = {(2x+1)}\Big/{(x^2+x+6t+10)}$ [14]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 4. The maximum absolute errors with CPU time of the methods are presented in table 4.



$N \times N$ **= 64** $N \times N$ **= 256**

Fig. 4. Comparison of numerical solutions with exact solution of test problem 4.4 for $N \times N$ =64 & 256.

Table 4. Maximum error and CPU time (in seconds) of the methods of test problem 4.4

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
16	FDM	7.7266e-04	3.6717	0.0003	3.6720
	HWLS	7.7266e-04	0.0009	0.0016	0.0025
	DWLS	7.7266e-04	0.0008	0.0105	0.0113
	BWLS	7.7266e-04	0.0007	0.0041	0.0048
256	FDM	2.7103e-04	5.9599	0.0009	5.9608
	HWLS	2.7103e-04	0.0009	0.0011	0.0020
	DWLS	2.7103e-04	0.0006	0.0082	0.0088
	BWLS	2.7103e-04	0.0005	0.0026	0.0031
4096	FDM	7.4160e-05	6.4387	0.0023	6.4410
	HWLS	7.4160e-05	0.0007	0.0011	0.0018
	DWLS	7.4160e-05	0.0005	0.0086	0.0091
	BWLS	7.4160e-05	0.0005	0.0028	0.0033

5. Conclusions and future works

In this work, we applied the wavelet based lifting schemes for the numerical solution of linear and nonlinear parabolic partial differential equations. Figures show that the numerical solutions obtained by Lifting schemes are agrees with the exact solution. Also in the tables, convergence of the presented schemes is observed i.e. the error decreases when the level of resolution N increases and HWLS & BWLS shows significant advantages i.e. CPU time is lesser than FDM & DWLS. Hence, the presented Lifting schemes in particular HWLS & BWLS are very effective for solving linear and non-linear partial differential equations.

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