

PERSONNEL APPOINTMENTS: A PYTHAGOREAN FUZZY SETS APPROACH USING SIMILARITY MEASURE

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Abstract: This paper explores the advantage of Pythagorean fuzzy sets in personnel appointments by employing normalized Euclidean similarity to find the similarity between applicants to each positions. The choice of Euclidean similarity for Pythagorean fuzzy sets by incorporating the three traditional parameters, is because it gives a reliable similarity with respect to other similarity measures for Pythagorean fuzzy sets that incorporate the three traditional parameters as studied in literature. By finding the similarity of the applicants and positions (both in Pythagorean fuzzy pairs/values), in the light of the qualifications require by the organisation, we determine the suitable applicants for the available positions. Also, we propose the notions of level sets of Pythagorean fuzzy sets and Pythagorean fuzzy pairs.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Personnel appointments, Pythagorean fuzzy pairs, Pythagorean fuzzy set, Similarity measure

1. Introduction

The adventure into fuzzy sets by Zadeh [35] dawn a new beginning in non-classical sets. Out of the several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov [1, 2] called intuitionistic fuzzy sets (IFSs) is quite interesting and useful. IFS incorporates both membership function, μ and non-membership function, ν with hesitation margin, π (that is, neither membership nor non-membership functions) such that $\mu + \nu \leq 1$. Fuzzy sets are IFSs but the converse is not necessarily true [2]. In fact, there are situations where IFS theory is more appropriate to deal with. Sequel to the introduction of IFSs, a lot of attentions have been paid on developing similarity measures for IFSs, as a way to apply them to solving many decision-making problems. As a result, some similarity measures were proposed, see [6, 12, 17, 24, 26, 32]. Some applications of IFSs have been carried out using measures, as can be found in [7, 11-13, 25, 26].

Notwithstanding, there are cases when $\mu + \nu \geq 1$. This situation can only be captured by a construct, called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed in [28-30] is a new tool to deal with vagueness considering the membership grade, μ and non-membership grade, ν satisfying the condition $\mu + \nu \geq 1$ or $\mu + \nu \leq 1$ such that, $0 \leq \mu^2 + \nu^2 \leq 1$. That is, PFS generalizes IFS. Honestly speaking, the origin of Pythagorean fuzzy sets emanated from intuitionistic fuzzy sets of second type (IFSST) introduced in [3]. As a generalized set, PFS has close relationship with IFS. PFSs can be used to characterize the uncertain information more sufficiently and accurately than IFSs. Since inception, the theory of PFSs has been extensively researched [22, 23, 31]. Pythagorean fuzzy set has attracted great attentions of many scholars, and the concept has been applied to several application areas [8, 9, 14, 15, 18, 19, 28, 30, 34].

Similarity and dissimilarity measures for PFSs have been studied from different perspectives. Some authors have researched on measures for PFSs by considering four or more parameters in [16, 21, 33], which are not the traditional parameters of PFSs as noted in [28-31]. In [27], some similarity measures between PFSs based on the cosine function were proposed by considering the degree of membership, degree of non-membership and degree of hesitation, and applied to pattern recognition and medical diagnosis. A similarity measure for PFSs based on the combination of cosine similarity measure and Euclidean distance measure featuring only membership and non-membership degrees were introduced in [20]. Of recent, some dissimilarity and similarity measures for PFSs which satisfied the metric distance conditions were introduced in [10] by incorporating the three conventional parameters of PFSs.

Personnel appointment is one of the most uncertain exercise in the domain of decision-making. A failure in personnel appointment will lead to the liquidation of an organization. Thus, this justifies the reason why we attempt to solve recruitment exercise using the notion of PFSs, which have been proven to be resourceful in tackling uncertainty more effectively than IFSs.

This paper studies PFSs, and presents an exploration into an application of PFSs to personnel appointments using normalised Euclidean similarity for applicants and available positions to determine which applicant is suitable for a particular position. The notion of Pythagorean fuzzy pairs is introduced (as extension of intuitionistic fuzzy pairs in [4, 5]). We reiterate the concept of similarity measure for PFS. When the reliability test of the measures were conducted, it follows that normalized Euclidean similarity for PFS yields the best similarity; this informs its choice in the study. The rest of the paper is thus presented; Section 2 provides some preliminaries on fuzzy sets, IFS and PFS, while Section 3 covers some similarity measures for PFSs with their numerical verifications. In Section 4, we present an application of PFS to personnel appointment using Euclidean similarity measure. Finally, Section 5 concludes the paper and provides direction for future studies.

2. Basic notions of Pythagorean fuzzy sets

We recall some basic notions of fuzzy sets, IFSs and PFSs.

Definition 2.1 [35]. Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A: X \rightarrow [0,1]$.

That is,

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \text{ is totally in } X \\ 0, & \text{if } x \text{ is not in } X \\ (0,1), & \text{if } x \text{ is partly in } X \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{ \langle \mu_A(x) \rangle \mid x \in X \}$ or $A = \{ \langle \frac{\mu_A(x)}{x} \rangle \mid x \in X \}$

where the function

$$\mu_A(x): X \rightarrow [0,1]$$

defines the degree of membership of the element, $x \in X$.

Definition 2.2 [2]. Let a nonempty set X be fixed. An IFS A of X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

or

$$A = \{ \langle \frac{\mu_A(x), \nu_A(x)}{x} \rangle \mid x \in X \},$$

where the functions

$$\mu_A(x): X \rightarrow [0,1] \text{ and } \nu_A(x): X \rightarrow [0,1]$$

define the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For each A in X ,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is the intuitionistic fuzzy set index or hesitation margin of x in X . The hesitation margin $\pi_A(x)$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus,

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1.$$

Definition 2.3 [28]. Let X be a universal set. Then, a Pythagorean fuzzy set A which is a set of ordered pairs over X , is defined by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

or

$$A = \{ \langle \frac{\mu_A(x), \nu_A(x)}{x} \rangle \mid x \in X \},$$

where the functions

$$\mu_A(x): X \rightarrow [0,1] \text{ and } \nu_A(x): X \rightarrow [0,1]$$

define the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$,

$$0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1.$$

Supposing $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by

$$\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]} \text{ and } \pi_A(x) \in [0,1]. \text{ In what follows, } (\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1.$$

Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$.

We denote the set of all PFSs over X by $PFS(X)$.

Example 2.4. Let $A \in PFS(X)$. Suppose $\mu_A(x) = 0.7$ and $\nu_A(x) = 0.5$ for $X = x$. Clearly, $0.7 + 0.5 \not\leq 1$, but $0.7^2 + 0.5^2 \leq 1$. Thus $\pi_A(x) = 0.5099$, and hence $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$.

Definition 2.5 [28, 29]. Let $A, B \in PFS(X)$. Then we have the following:

- (i) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}.$
- (ii) $A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle \mid x \in X \}.$
- (iii) $A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle \mid x \in X \}.$
- (iv) $A \oplus B = \{ \langle x, \sqrt{(\mu_A(x))^2 + (\mu_B(x))^2 - (\mu_A(x))^2(\mu_B(x))^2}, \nu_A(x)\nu_B(x) \rangle \mid x \in X \}.$
- (v) $A \otimes B = \{ \langle x, \mu_A(x)\mu_B(x), \sqrt{(\nu_A(x))^2 + (\nu_B(x))^2 - (\nu_A(x))^2(\nu_B(x))^2} \rangle \mid x \in X \}.$

Remark 2.6 [10]. Let $A, B, C \in PFS(X)$. By Definition 2.5, the following properties hold:

$$(A^c)^c = A$$

$$A \cap A = A$$

$$A \cup A = A$$

$$A \oplus A \neq A$$

$$A \otimes A \neq A$$

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

$$A \oplus B = B \oplus A$$

$$A \otimes B = B \otimes A$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

$$A \otimes (B \otimes C) = (A \otimes B) \otimes C$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

$$A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$$

$$\begin{aligned}
A \otimes (B \cup C) &= (A \otimes B) \cup (A \otimes C) \\
A \otimes (B \cap C) &= (A \otimes B) \cap (A \otimes C) \\
(A \cap B)^C &= A^C \cup B^C \\
(A \cup B)^C &= A^C \cap B^C \\
(A \oplus B)^C &= A^C \otimes B^C \\
(A \otimes B)^C &= A^C \oplus B^C.
\end{aligned}$$

Definition 2.7 [28]. Let A and B be PFSs of X . Then

$$A = B \Leftrightarrow \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x) \forall x \in X,$$

and

$$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \text{ or } \nu_A(x) \leq \nu_B(x) \forall x \in X.$$

We say $A \subset B \Leftrightarrow A \subseteq B$ and $A \neq B$.

Definition 2.8 [10]. Let $A, B \in PFS(X)$. Then A and B are comparable to each other if $A \subseteq B$ and $B \subseteq A$.

Now, we define level or ground set of a PFS and the notion of Pythagorean fuzzy pairs.

Definition 2.9 [13]. Let $A \in PFS(X)$. Then the level/ground set or support of A is defined by

$$A_* = \{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 1 \forall x\},$$

and the set A^* is defined by

$$A^* = \{x \in X \mid \mu_A(x) \geq 0, \nu_A(x) \leq 1 \forall x\}.$$

Certainly, A_* and A^* are subsets of X .

Definition 2.10. Pythagorean fuzzy pairs (PFPs) or Pythagorean fuzzy values (PFVs) is an object in the form $\langle a, b \rangle$, where $a, b \in [0, 1]$, and $a^2 + b^2 \leq 1$. PFPs are used for the evaluation of objects or processes and which components (a and b) are interpreted as degrees of membership and non-membership or degrees of validity and non-validity or degrees of correctness and non-correctness.

Example 2.11. Let $A \in PFS(X)$ for $X = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose

$$A = \left\{ \left\langle \frac{0.6, 0.4}{x_1} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\}.$$

We can rewrite A as

$$A = \left\{ \left\langle \frac{0.6, 0.4}{x_1} \right\rangle, \left\langle \frac{0.0, 1.0}{x_2} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.0, 1.0}{x_4} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\}.$$

Then, $A_* = \{x_1, x_3, x_5\}$ and $A^* = \{x_1, x_2, x_3, x_4, x_5\} = X$.

PFPs are

$$x_1 = \langle 0.6, 0.4 \rangle, x_2 = \langle 0.0, 0.1 \rangle, x_3 = \langle 0.8, 0.4 \rangle, x_4 = \langle 0.0, 0.1 \rangle, x_5 = \langle 0.7, 0.2 \rangle.$$

3. Similarity measures for Pythagorean fuzzy sets

Similarity measure (SM) for PFSs is a dual concept of distance measure for PFSs, which has been studied hitherto. Firstly, we recall the axiomatic definition of similarity measure of Pythagorean fuzzy sets proposed in [10].

Definition 3.1 [10]. Let X be nonempty set and $A, B, C \in PFS(X)$. The similarity measure s between A and B is a function $s: PFS \times PFS \rightarrow [0, 1]$ satisfies

- (i) $0 \leq s(A, B) \leq 1$ (boundedness)
- (ii) $s(A, B) = 1$ iff $A = B$ (separability)
- (iii) $s(A, B) = s(B, A)$ (symmetric)
- (iv) $s(A, C) + s(B, C) \geq s(A, B)$ (triangle inequality).

By incorporating the three parameters of PFSs, the following similarity measures for PFSs were proposed in [10]. Let $A, B \in PFS(X)$ such that $X = \{x_1, \dots, x_n\}$, then

$$\begin{aligned}
 s_1(A, B) &= 1 - \frac{1}{2n} \sum_{i=1}^n [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| \\
 &\quad + |\pi_A(x_i) - \pi_B(x_i)|], \\
 s_2(A, B) &= 1 - \left(\frac{1}{2n} \sum_{i=1}^n [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 \right. \\
 &\quad \left. + (\pi_A(x_i) - \pi_B(x_i))^2] \right)^{\frac{1}{2}}, \\
 s_3(A, B) &= 1 - \frac{1}{2n} \sum_{i=1}^n [(\mu_A(x_i))^2 - (\mu_B(x_i))^2 + (\nu_A(x_i))^2 - (\nu_B(x_i))^2 \\
 &\quad + |(\pi_A(x_i))^2 - (\pi_B(x_i))^2|]
 \end{aligned}$$

3.1 Numerical examples

We now verify whether these similarity measures satisfy the conditions in Definition 3.1.

Example 3.2. Let $A, B, C \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$. Suppose

$$\begin{aligned}
 A &= \left\{ \left\langle \frac{0.6, 0.2}{x_1}, \left\langle \frac{0.4, 0.6}{x_2}, \left\langle \frac{0.5, 0.3}{x_3} \right\rangle \right\rangle \right\}, \\
 B &= \left\{ \left\langle \frac{0.8, 0.1}{x_1}, \left\langle \frac{0.7, 0.3}{x_2}, \left\langle \frac{0.6, 0.1}{x_3} \right\rangle \right\rangle \right\} \text{ and} \\
 C &= \left\{ \left\langle \frac{0.9, 0.2}{x_1}, \left\langle \frac{0.8, 0.2}{x_2}, \left\langle \frac{0.7, 0.3}{x_3} \right\rangle \right\rangle \right\}.
 \end{aligned}$$

Calculating the similarity using the itemized similarity measures above, we have

$$\begin{aligned}
 s_1(A, B) &= 1 - \frac{1}{6} \sum_{i=1}^3 [0.6 - 0.8 + 0.2 - 0.1 + |0.7746 - 0.5916| \\
 &\quad + |0.4 - 0.7| + |0.6 - 0.3| + |0.6928 - 0.6481| \\
 &\quad + |0.5 - 0.6| + |0.3 - 0.1| + |0.8124 - 0.7937|] \\
 &= 0.7589, \\
 s_2(A, B) &= 1 - \left(\frac{1}{6} \sum_{i=1}^3 [(0.6 - 0.8)^2 + (0.2 - 0.1)^2 + (0.7746 - 0.5916)^2 \right. \\
 &\quad \left. + (0.4 - 0.7)^2 + (0.6 - 0.3)^2 + (0.6928 - 0.6481)^2 \right. \\
 &\quad \left. + (0.5 - 0.6)^2 + (0.3 - 0.1)^2 + (0.8124 - 0.7937)^2] \right)^{\frac{1}{2}} \\
 &= 0.7706, \\
 s_3(A, B) &= 1 - \frac{1}{6} \sum_{i=1}^3 [0.6^2 - 0.8^2 + 0.2^2 - 0.1^2 + |0.7746^2 - 0.5916^2| \\
 &\quad + |0.4^2 - 0.7^2| + |0.6^2 - 0.3^2| + |0.6928^2 - 0.6481^2| \\
 &\quad + |0.5^2 - 0.6^2| + |0.3^2 - 0.1^2| + |0.8124^2 - 0.7937^2|] \\
 &= 0.7600.
 \end{aligned}$$

That is,

$$s_1(A, B) = 0.7589, s_2(A, B) = 0.7706, s_3(A, B) = 0.7600.$$

Similarly, we have

$$s_1(A, C) = 0.6702, s_2(A, C) = 0.6726, s_3(A, C) = 0.6100 \text{ and} \\ s_1(B, C) = 0.8113, s_2(B, C) = 0.8368, s_3(B, C) = 0.8133$$

Tables 1 contains all the values of similarity measures using Example 3.2, and explains the properties of the discussed similarity measures.

Table 1: Example 3.2

SM	$s(A, B)$	$s(A, C)$	$s(B, C)$
s_1	0.7589	0.6702	0.8113
s_2	0.7706	0.6726	0.8368
s_3	0.7600	0.6100	0.8133

3.1.1 Discussion

The following observations are made from Tables 1:

1. s_2 is the most accurate of the similarity measures discussed, since it provides the greatest similarity between A and B , A and C , and B and C , respectively.
2. s_1 , s_2 and s_3 satisfy the conditions of Definition 3.1 and hence, they are appropriate similarity measures for PFSs.
3. Since s_2 is the most accurate of the discussed similarity measures, we adopt it for the applications to be considered.

4. Pythagorean fuzzy sets in personnel appointments

In this section, we present an application of PFS to personnel appointments. Suppose an organisation wants to either reshuffle or employ executives, the challenge is how to appoint suitable officers into different positions assuming there are more than enough candidates for the positions. PFSs approach provides the solution because of its competency in handling uncertainties in decision making.

In an aptitude test for an employment, assume Q is a set of qualifications, P is a set of positions and A is a set of applicants vying for the available positions. Recall that, in an organisation setting, all positions do not have the same range of qualifications.

Let

$$Q = \{q_1, \dots, q_n\}, P = \{p_1, \dots, p_n\} \text{ and } A = \{a_1, \dots, a_n\}$$

be finite sets of qualifications, positions and applicants, respectively.

Suppose A and P are PFS of Q , and s be a normalized Euclidean similarity of A and P . Then $s(A, P)$ is given by

$$s(A, P) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n [(\mu_A(q_i) - \mu_P(q_i))^2 + (v_A(q_i) - v_P(q_i))^2 + (\pi_A(q_i) - \pi_P(q_i))^2]}, \quad (1)$$

where $q_i \in Q$ and n is the number of qualifications.

An applicant, A is suitable for a position, P if $s(A, P)$ is the greatest. To see the application using Equation 1, let us make use of a hypothetical case.

4.1 Case study

Let $A = \{\text{John, Mike, Deby, Lil}\}$ be the set of applicants vying for positions, $P = \{P_1, P_2, P_3, P_4, P_5\}$ be the set of positions and $Q = \{\text{honesty, team spirit, hardworking, transparency, academic fitness}\}$ be the set of qualifications expected by the applicants.

Assume the applicants are interviewed by an impartial panel comprises of ten members, and the scores of the interview are captured in Pythagorean fuzzy pairs as contain in Table 2.

Table 2: Applicants vs Qualifications

	honesty	team spirit	hardworking	transparency	acad. fitness
John	$\langle 0.5, 0.1 \rangle$	$\langle 0.6, 0.1 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.5, 0.2 \rangle$
Mike	$\langle 0.8, 0.1 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.4, 0.5 \rangle$
Deby	$\langle 0.5, 0.2 \rangle$	$\langle 0.6, 0.1 \rangle$	$\langle 0.5, 0.1 \rangle$	$\langle 0.5, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$
Lil	$\langle 0.7, 0.1 \rangle$	$\langle 0.5, 0.3 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.7, 0.1 \rangle$

Suppose Table 3 is the stipulated standing qualifications by the interview panel, for each of the positions.

Table 3: Qualifications vs Positions

	P_1	P_2	P_3	P_4	P_5
honesty	$\langle 0.7, 0.2 \rangle$	$\langle 0.7, 0.3 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.6, 0.3 \rangle$
team spirit	$\langle 0.8, 0.1 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.8, 0.0 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$
hardworking	$\langle 0.8, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$
transparency	$\langle 0.7, 0.2 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.9, 0.0 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.7, 0.1 \rangle$
acad. fitness	$\langle 0.9, 0.1 \rangle$	$\langle 0.9, 0.0 \rangle$	$\langle 0.6, 0.3 \rangle$	$\langle 0.7, 0.2 \rangle$	$\langle 0.5, 0.3 \rangle$

In Tables 2 and 3, the first entry is the membership degree, μ representing the scores allocated to the questions answered and the second entry is the non-membership degree, ν representing the scores allocated to the questions failed. After computing the indeterminacy degree, π representing the scores loss due to hesitation, the similarity of A and P is calculated using Equation 1, and the results are shown in Table 4.

Table 4: Applicants vs Positions

$s(A, P)$	P_1	P_2	P_3	P_4	P_5
John	0.7635	0.7604	0.8523	0.8460	0.8558
Mike	0.7357	0.7299	0.7670	0.8324	0.8309
Deby	0.7915	0.8014	0.7294	0.7889	0.7719
Lil	0.8162	0.8395	0.7664	0.8706	0.8141

4.2 Decision on appointments

The decisions made based on the position an applicant is suitable for, is determined by the greatest value of $s(A, P)$. From Table 4, John is suitable for P_5 , Mike is suitable for P_4 , Deby is suitable for P_2 , and Lil is suitable for P_4 (more suitable for the position than Mike).

Making the decision from a vertical view, P_1 , P_2 and P_4 are suitable for Lil with similarity of 0.8162, 0.8395 and 0.8706, respectively, and P_3 and P_5 are suitable for John with similarity of 0.8523 and 0.8558, respectively.

Remark 4.1. From the ongoing, the following observations are made;

- (i) Lil is the most qualified applicant, and follows by John.
- (ii) Vertical decision is competitive compare to horizontal decision. It should be applicable in an instance, when the employer has no sufficient fund to pay staff (however, the employed staff can be wore out for overworking).
- (iii) Horizontal decision is applicable if the employer can pay all the applicants. It also encourages efficiency since the staff would not be overworked.

5. Conclusion

The concept of PFSs is very much applicable in real-life problems because of its ability to cope with imbedded imprecision more effective than IFSs. Some applications of PFSs have been explored in literature. The notion of Pythagorean fuzzy pairs has been proposed, and exemplified. We have looked at an

application of PFSs in a more pressing decision-making problem (because of the high rate of embedding uncertainty) using normalized Euclidean similarity of applicants and available positions. We assert that PFS theory is a viable tool in decision science because of its leverages for indeterminacy encounter in day to day decision-making problems.

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