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Dynamics Analysis of a 4D Neuron model under Periodic Disturbance

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Abstract: Based on Hindmarsh-Rose neuron model, a 4D neuron model is addressed. Dynamical behaviors of the proposed neuron model are investigated under external periodic disturbance. The diversity of dynamics of the neuron is revealed via altering the parameters in the external periodic disturbance. The research results may be beneficial for further exploring the dynamic behaviors relative to the mode transition of neuron network.

Keywords: Hindmarsh-Rose neuron model; periodic disturbance; bursting

1. Introduction

To study the dynamical behaviors of neurons, a variety of simplified neuron models have been put forward for theoretical analysis and numerical research. Based on the electrophysiological experiments, Hodgkin and Huxley established the well-known Hodgkin-Huxley (HH) model considering the discharge of neuron [1], which revealed the electrophysiological mechanism of neurophysiological activity and can make people better understand the characteristics of neurons' electrical activity. On the basis of HH model, some other neuron models have been established to describe the rich discharge modes of neurons, such as FitzHugh-Nagumo (FHN) model [2], Hindmarsh-Rose (HR) model [3], Morris-Lecar (ML) model [4], Chay model [5], etc.

Some dynamical behaviors of above neurons have been discussed and presented [6-14]. For example, the bifurcation of two coupled FHN neurons was studied [6] and the dynamical properties of FHN neuron system was reproduced by adjusting the resistive-capacitive-inductance Josephson junction (RCLSJ) model [7]. The bifurcation diagram of HR neuron model in a two-dimensional parameter space was reported and the complex bifurcation structure in the diagram was pointed out [8]. Period-adding bifurcation (with or without chaos) and intermittent chaos phenomenon (periodic and intermittent chaotic) in a modified HR neuron model was observed [9]. Various responses of ML neuron under multiple stimuli were discussed [10] and the dynamics of ML neuron driven by channel noise was analyzed along with the mechanism about ion channel noise generating spontaneous action potentials being given [11]. The parameter regions for different firing patterns in Chay neural model were pointed out and the bifurcation of electric activities were analyzed [12]. The transitions between the firing activity modes in Chay neuron system were explored by depolarizing current [13] and different types of bursting in it were surveyed [14].

Among the presented neuron models, HR model is more suitable for bifurcation analysis and its output can better simulate the behaviors of some mollusk neurons. Therefore, HR neuron model has been discussed and many results about the dynamics of it have been obtained [15-18]. By discussing the pattern formation of neurons, a result is obtained that the dynamics modes could be adjusted by altering the external forcing current [19, 20].

In the existing results, the addressed neuron model, the term to describe the electro-magnetic radiation is always nonlinear, but in some real cases, the electro-magnetic radiation may be linear, which is simple and almost not being reported. Therefore, to further investigate the electric activities of neuron and reveal the dynamical behaviors of more neurons, a simplified neuron model considering magnetic flux is addressed and the dynamical behaviors of it are to be explored under periodic disturbance. Other parts of this paper will be given as follows. In Section 2, based on HR neuron model, a simplified neuron model considering magnetic flux is put forward. Section 3 depicts the simulations to illustrate the dynamical behaviors of the proposed neuron model. Conclusions are drawn in Section 4.

2. Model description

The famous HR model [3] can be described as

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z + I_{ext} \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x+1.6) - z] \end{cases}, \quad (1)$$

where x is the membrane potential, y is the slow current for recovery variable, and z is the adaption current. I_{ext} is the external forcing current. When system parameters of (1) are chosen as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.006$, $s = 4$ and initial values are taken as x =-1.5, y =0.7, z =0.9, the dynamical behaviors of variable x in system (1) are pictured in Fig.1 for $I_{ext} = 0.3$, 1.3, 1.4, 2, 2.2, 2.7, 3, 3.5, respectively. From Fig.1, it is easy to know that dynamics of membrane potential variable x in neuron system (1) shows diversity with I_{ext} increasing, such as quiescent state, spiking, regular bursting and chaotic bursting.

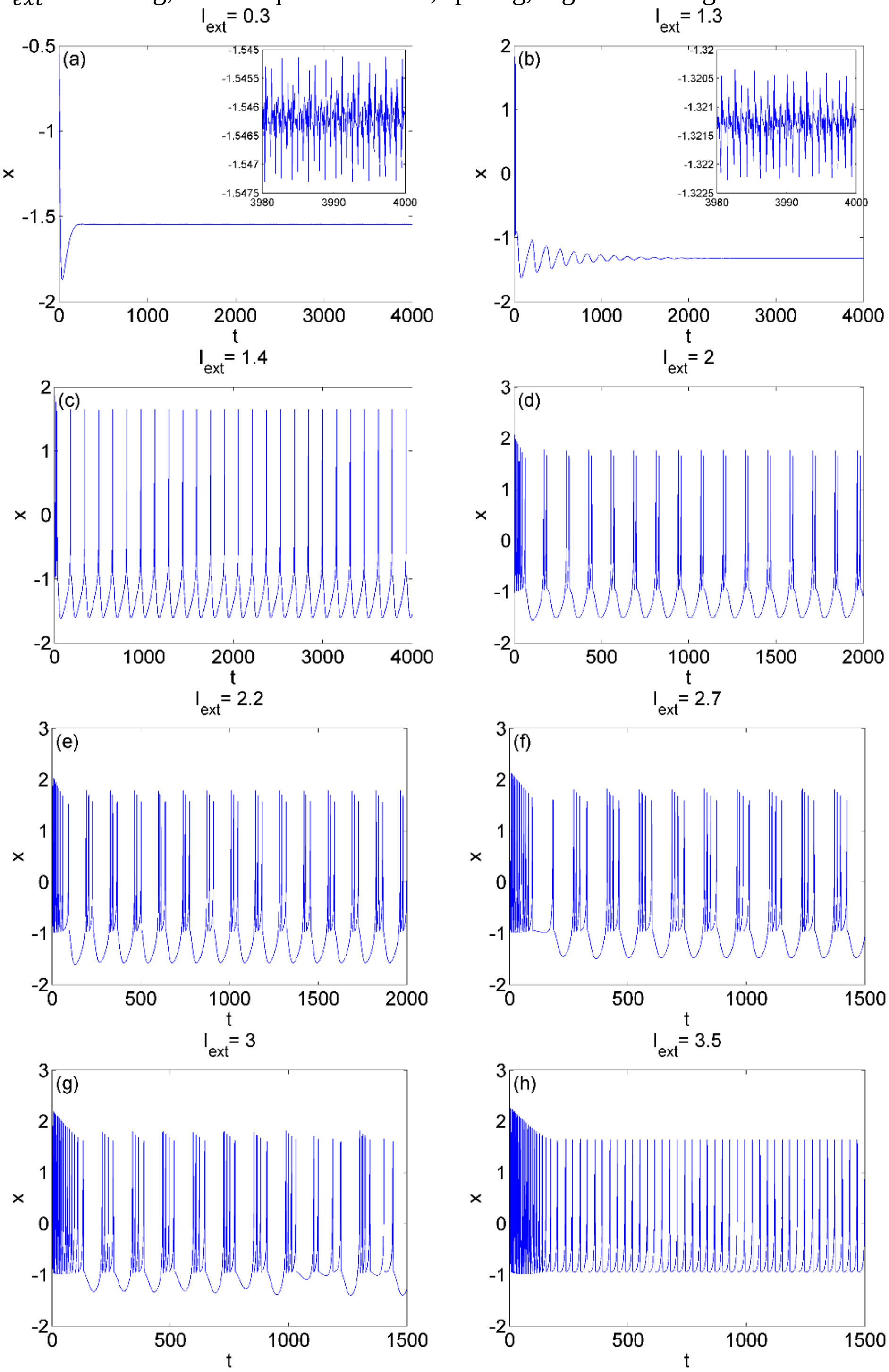


Fig.1 Dynamics of variable x in system (1) for different values of I_{ext},
(a) $I_{\text{ext}} = 0.3$, (b) $I_{\text{ext}} = 1.3$, (c) $I_{\text{ext}} = 1.4$, (d) $I_{\text{ext}} = 2$,

(e) $I_{\text{ext}} = 2.2$, (f) $I_{\text{ext}} = 2.7$, (g) $I_{\text{ext}} = 3$, (h) $I_{\text{ext}} = 3.5$.

To investigate the effect of electromagnetic induction on the discharge mode of neuron, a four-variable HR neuron model is proposed by introducing magnetic flux as a new variable [21], which is described as

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z + I_{ext} - k_1\rho(w)x \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \\ \dot{w} = x - k_2 w \end{cases}, \tag{2}$$

where $\rho(w) = (\alpha + 3\beta w^2)$ with α, β being fixed parameters [22]. k_1 and k_2 describe the interaction between membrane potential and magnetic flux. $k_1\rho(w)x$ means the induction current according to the previous works [21]. Results in reference [22] suggest that the dynamical behaviors of membrane potential in system (2) can be adjusted by altering the magnetic flux. It means that multiple modes of dynamical behaviors can be observed when changing the external forcing current, which makes parameters have wider region for generating complex dynamics.

Inspired by the work in [23], considering a linear coupling between membrane potential x and magnetic flux w, a simplified neuron model considering magnetic flux can be simplified as

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z + I_{ext} - \alpha x - \beta w \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \\ \dot{w} = x - k_1 w \end{cases}, \tag{3}$$

where α, β, k_1 are fixed parameters describing the interaction between membrane potential x and the new variable w.

3. Numerical simulations

In this section, numerical simulations are carried out to illustrate the dynamical behaviors of system (3). Take $\alpha = 0.004$, $\beta = 0.012$, $k_1 = 6.2$, and other parameters are the same as those in system (1), initial values are chosen as x =-1.5, y =0.7, z =0.9, w=0.2, the dynamics of membrane potential in neuron system (3) are depicted in Fig.2 for I_{ext} = 0.3, 1.3, 1.4, 2, 2.2, 2.8, 3, 3.5, respectively. From Fig.2, it is obvious to see that membrane potential variable x in neuron system (3) can also show multiple modes with I_{ext} changing and the dynamics characteristics is similar to that of HR neuron model (1). To further investigate this phenomenon, Fig.3 depicts the bifurcation of x in neuron system (3) versus I_{ext} at y=-2.5, from which it is known that the dynamical behaviors of x in neuron system (3) experiences adding periodic bifurcations to chaos, which confirms the result in Fig.2. Therefore, it can be said that the proposed neuron model (3) may represent a kind of neuron and the investigation on it can help people understand the rhythm of nervous system.

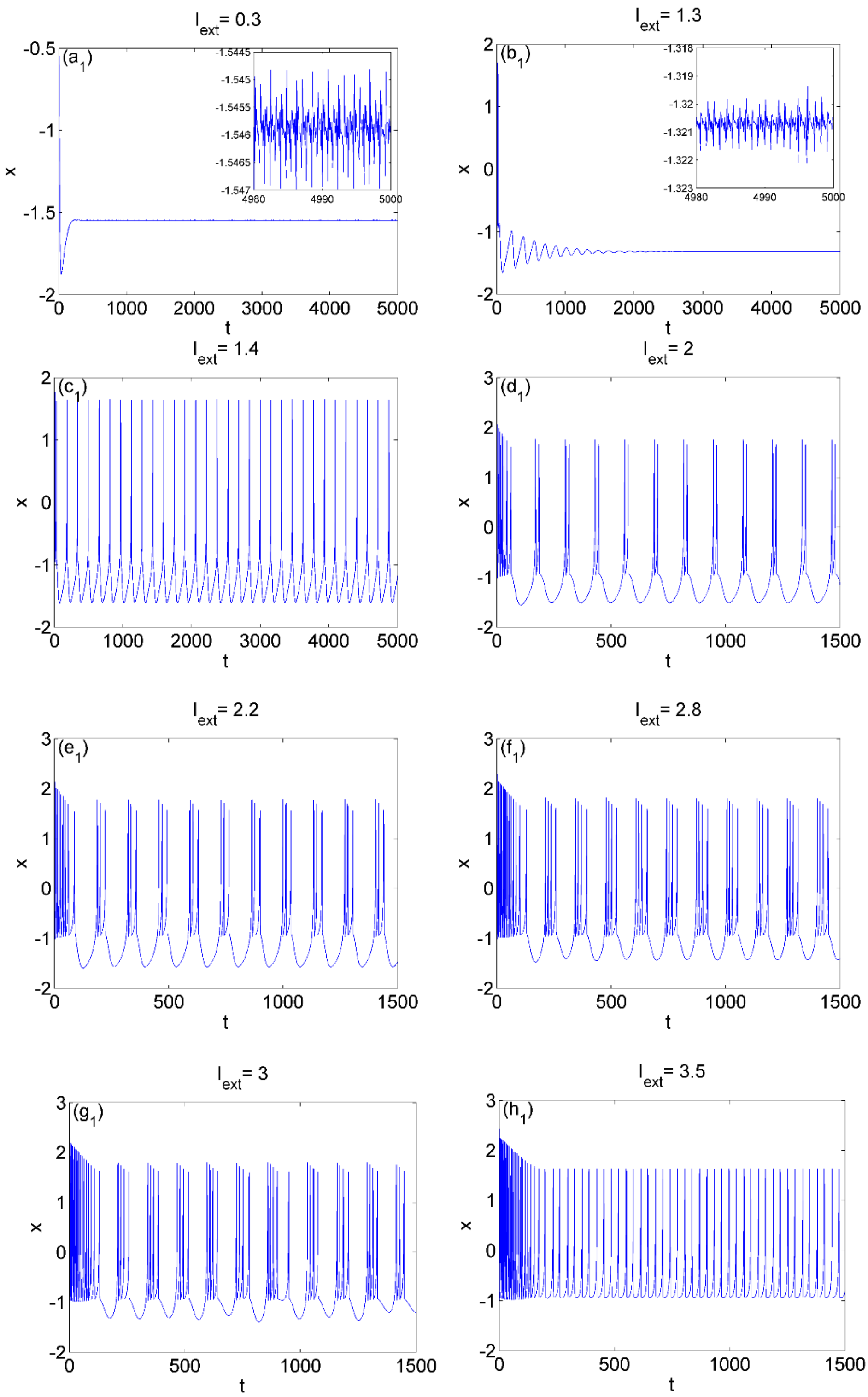


Fig.2 Dynamics of membrane potential in neuron system (3) for different external forcing current, (a1) $I_{\text{ext}} = 0.3$, (b1) $I_{\text{ext}} = 1.3$, (c1) $I_{\text{ext}} = 1.4$, (d1) $I_{\text{ext}} = 2$, (e1) $I_{\text{ext}} = 2.2$, (f1) $I_{\text{ext}} = 2.8$, (g1) $I_{\text{ext}} = 3$, (h1) $I_{\text{ext}} = 3.5$.

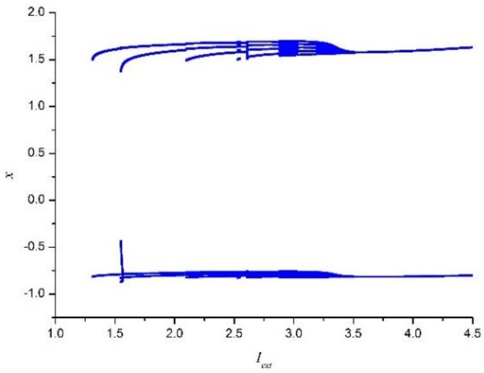


Fig.3 Bifurcation diagram of x (the Poincare section at y=-2.5) in neuron system (3) versus I_{ext}.

Additionally, due to the wide existence of biological electricity, almost all neurons are disturbed by external disturbance. It is necessary to explore the effect of external disturbance on the dynamics of the neuron. Here, it is supposed that neuron system (3) is stimulated by periodic external forcing current

$$I_{ext} = I + Asin(\omega t + \phi). \quad (4)$$

Then the proposed neuron system (3) can be rewritten as

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z - \alpha x - \beta w + I + Asin(\omega t + \phi) \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \\ \dot{w} = x - k_1 w \end{cases}, \quad (5)$$

where I is the constant part of the external forcing current, A, ω and ϕ are the amplitude, the angular frequency, initial phase in the periodic part, respectively.

Simulations are demonstrated to discuss the discharge mode of neuron (5) for different external forcing currents imposed on it. To realize the simulations, fourth Runge-Kutta algorithm is used to resolve model (5) with selected parameters. Time step is taken as Δh=0.01. To study the effect of external forcing current on the dynamics of neuron (5), simulations of system (5) are carried out for two cases.

Case 1 Initial phase $\phi = 0$

Firstly, we consider the dynamics of system (5) by changing the value of constant forcing current I and Fig.4 gives the dynamical behaviors of membrane potential x for I=0.3, 1.3, 1.4, 2, 2.2, 2.6, 3, 3.5, respectively, from which we know that the dynamical behaviors of x show diversity with the change of I. In particular, for I=0.3 and 1.3, the dynamics of membrane potential x reach quiescent state revolving around the equilibrium point with small amplitude (The largest amplitude is about 0.002, see Fig.4 (a)-(b)). While for I=1.4, 2, 2.2, 2.6, neuron system (5) shows period-adding bursting (Fig.4 (c)-(f)) and for I=3.0, chaotic bursting is observed. With I increasing, period-1 bursting can be found again for I=3.5 but with smaller period than for I=1.4. Fig.4 suggests that altering the constant part in periodic forcing current can adjust the discharge modes of neuron system (5). Therefore, suitable electric modes can be selected by applying appropriate constant signals.

In fact, the change of amplitude in the forcing current can also make membrane potential x show multiple electric activities. By adjusting amplitude A, the discharge mode of x in neuron system (5) is given in Fig.5, from which it is easy to see that membrane potential x in neuron system (5) has also complicated dynamics, such as periodic bursting (Fig.5 (a)), hybrid bursting (spiking and bursting state occur alternatively (Fig.5 (b))), spiking accompanied by bursting state(Fig.5 (c)-(c1)).

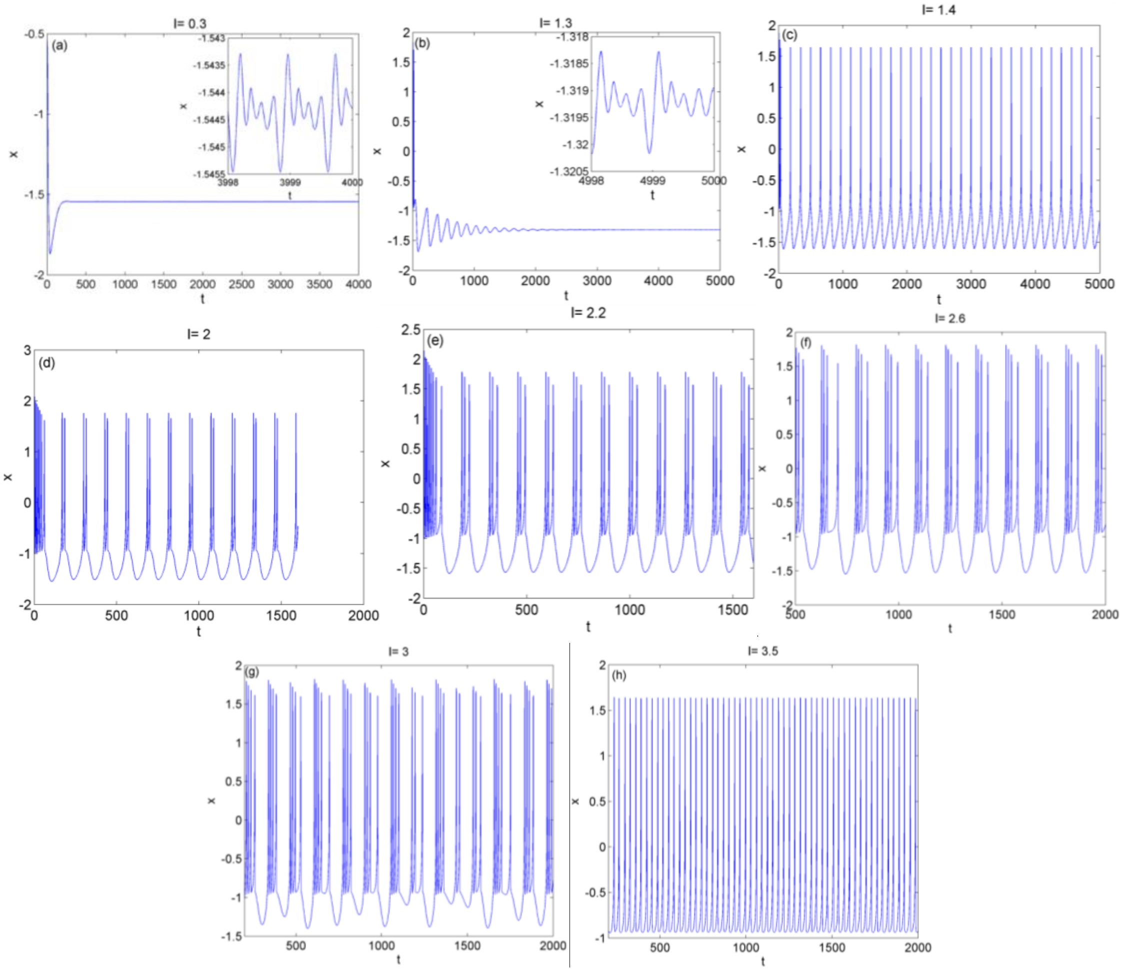


Fig.4 Dynamics of membrane potential x in neuron system (5) at $\alpha = 0.004$, $\beta = 0.012$, $\omega = 0.001$, $A = 0.5$ for (a) $I = 0.3$ (b) $I = 1.3$, (c) $I = 1.4$, (d) $I = 2$, (e) $I = 2.2$, (f) $I = 2.6$, (g) $I = 3$, (h) $I = 3.5$.

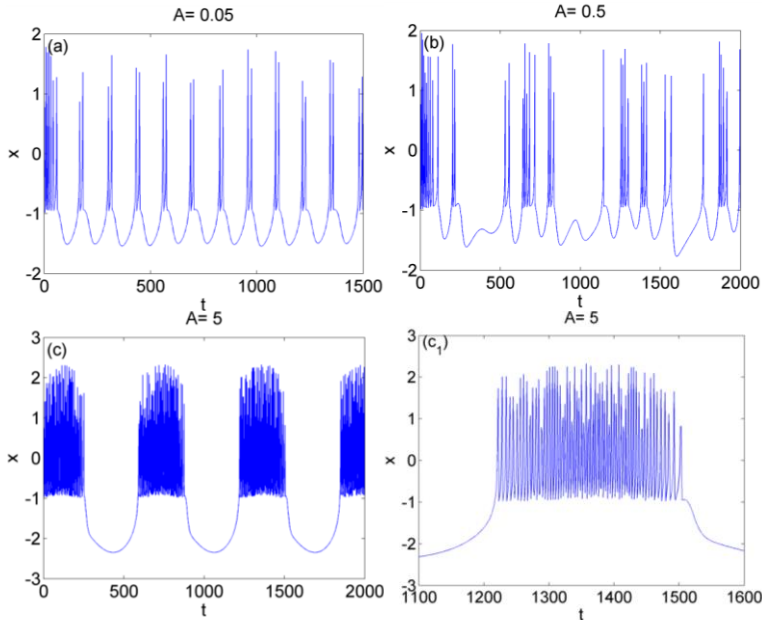
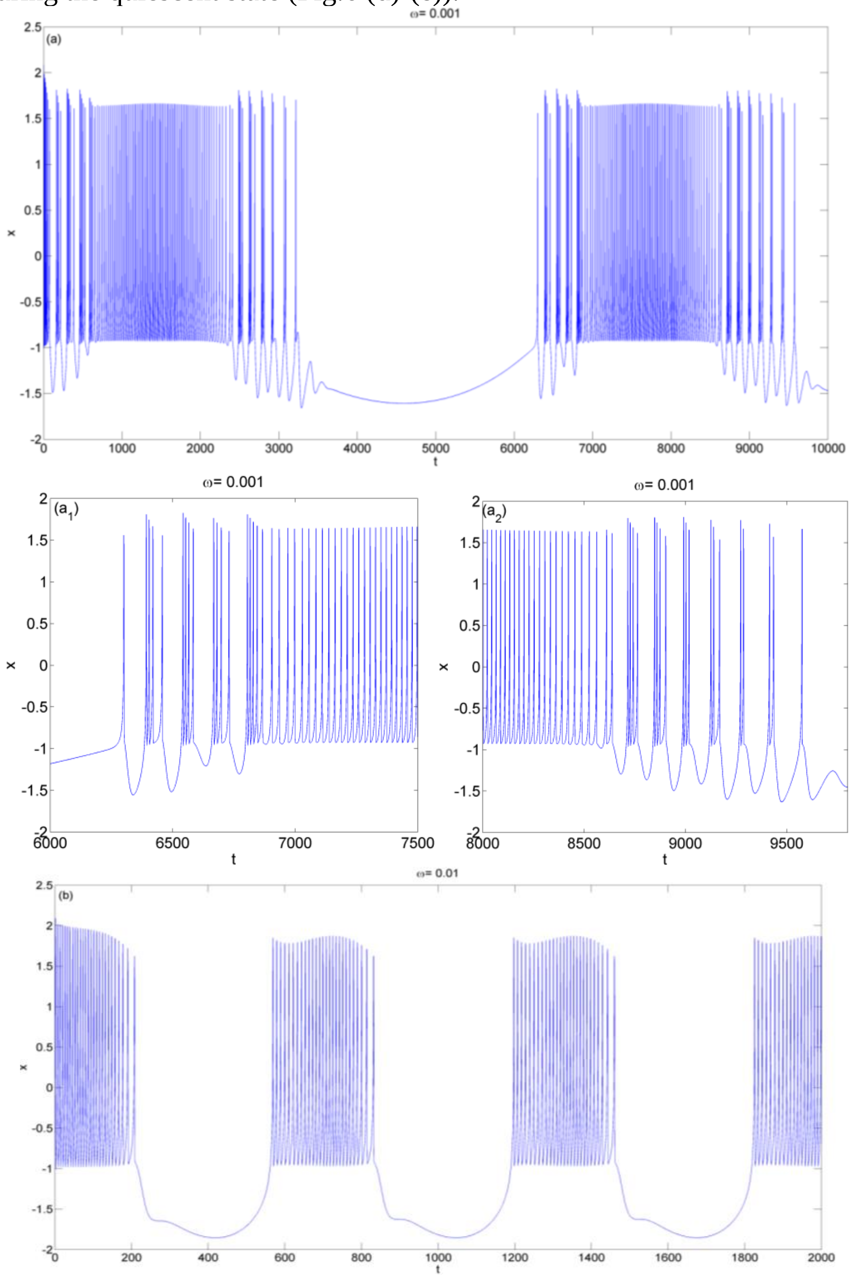


Fig.5 Dynamics of x in neuron system (5) at $\alpha = 0.004$, $\beta = 0.012$, $\omega = 0.01$, $I = 2.0$ for (a) $A = 0.05$, (b) $A = 0.5$, (c)$A = 5$, (c1) Partial enlargement of (c).

Results in Fig.5 confirmed that dynamics of neuron system (5) can alternate between different modes with the change of amplitude in the external forcing current. Potential mechanism for this phenomenon may be that the external forcing current is involved in constant and periodic forcing signal, which are imposed on the system simultaneously via multi-channel and makes the dynamical behaviors be adjusted synchronously by various signals. This phenomenon is very interesting.

Then, the effect of angular frequency on the electric activities of neuron system (5) is described in Fg.6, from which we know that, different angular frequency in the external forcing current can also result in the diversity of discharge mode. Fig.6 (a) indicates that the electric activity of neuron system (5) changes between quiescent, bursting and spiking for $\omega = 0.001$. For $\omega=0.01$ and $\omega = 0.1$, neuron (5) shows regular bursting (Fig.6(b)-(c)). When $\omega = 1$ and $\omega=60$, a phenomenon can be noticed that there is oscillation with small amplitude during the quiescent state (Fig.6 (d)-(e)).



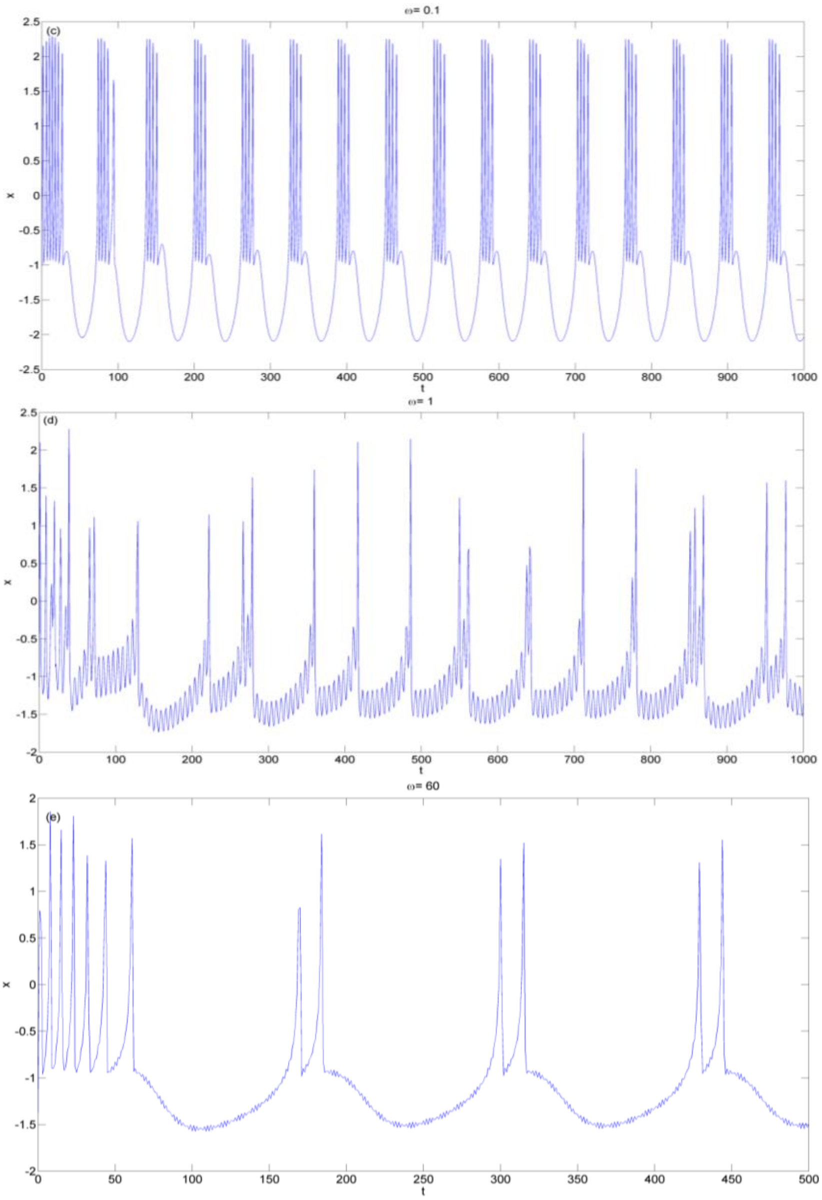


Fig.6 Dynamics of neuron system (5) at $\alpha = 0.004$, $\beta = 0.012$, $A = 2.0$, $I = 2.0$, and for (a) $\omega = 0.001$ and (a1), (a2) are the partial enlargements of (a), (b) $\omega = 0.01$, (c) $\omega = 0.1$, (d) $\omega = 1$, (e) $\omega = 60$.

Case 2 Initial phase $\phi \neq 0 (0 \leq \phi < 2\pi)$.

Because the initial phase ϕ only exists in the periodic part of the external forcing current, we only consider the effects of amplitude A and angular frequency ω on the dynamics of membrane potential for different initial phase ϕ.

Without loss of generality, initial phase is taken as $\pi/2$, the dynamical behaviors of variable x in neuron system (5) are given in Fig.7. From Fig.7, it can be seen that the membrane potential also performs multiple discharge modes, such as regular bursting, hybrid-period bursting, etc. This phenomenon is similar to that for $\phi = 0$.

To study the effect of angular frequency ω on the dynamics of membrane potential x in system (5), the dynamics of x with ω changing is calculated and given in Fig. 8, which indicates that the various values of angular frequency can lead to multiple discharge modes of the neuron system (5) as well.

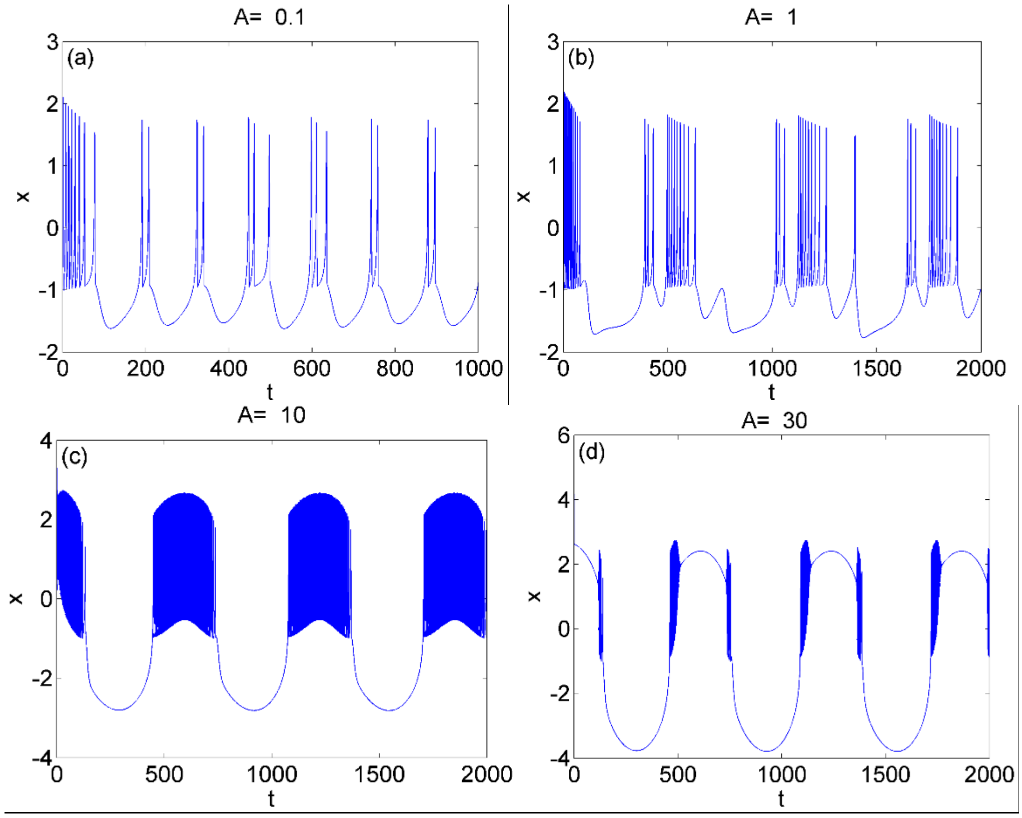


Fig.7 Dynamics of variable x in system (5) at $\alpha = 0.004$, $\beta = 0.012$, $I = 2.0$, $\phi = \pi/2$, $\omega = 0.01$, for different values of A, (a) $A = 0.1$, (b) $A = 1$, (c) $A = 10$, (d) $A = 30$.

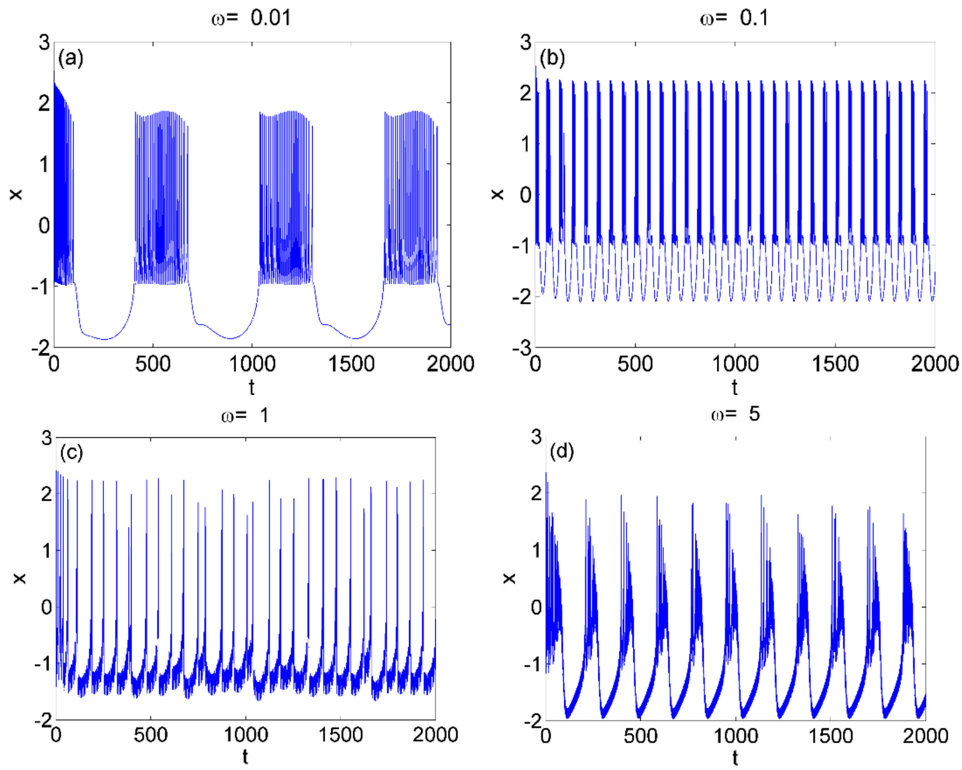


Fig.8 Dynamics of variable x in system (5) at $\alpha = 0.004$, $\beta = 0.012$, $I = 2.0$, $\phi = \frac{\pi}{2}$, $A = 2$, for different values of ω. (a) $\omega = 0.01$, (b) $\omega = 0.1$, (c) $\omega = 1$, (d) $\omega = 5$.

4. Conclusions

Based on HR neuron model, a simplified neuron model considering magnetic flux is proposed and the electric activities of it are discussed in presence of periodic disturbance. Results suggest that the dynamical behavior of membrane potential variable shows diversity when constant and periodic signals are imposed on the neuron simultaneously. It means that mode transition of electric activities not only depends on the constant current, but also relative to the properties of periodic signal, such as amplitude and angular frequency. Therefore, by choosing periodic or constant signals, suitable electrical activity modes could be obtained.

Acknowledgments

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