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Stability Analysis of Investor Emotional Propagation Model Based on SIR Model on Homogeneous Network

Wente Liu[1]
[1] faculty of science, Jiangsu University, Zhenjiang,212013, China
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Abstract. In the stock market, the investor's investment sentiment has a serious impact on other individual investors. In the past, people used the infectious disease transmission model to conduct in-depth research on the spread of various diseases. In this paper, we will use the infectious disease model. Based on and improved, the time delay caused by information delay and the saturation rate due to people's limited social ability are added to the homogenous network, which makes the established model more realistic. First, we establish the goal of emotional communication. The positive sentiment-based SIR propagation model is then proved by a series of mathematical methods to the local stability and global stability of the model. Finally, we use the numerical simulation study to verify the results we obtained, and the results show that: When the number of people stays in equilibrium and the information lagging factors are kept under control, the market will remain stable.

Keywords: investor sentiment, saturation rate, global stability, local stability

1. Introduction

With the rapid development of the world economy, more and more people now use investment as a means of income, and this group of investors is even larger. Under the theory of finance, investors' expectations for the future will be systematically biased. This biased expectation is called investor sentiment. It is an important indicator reflecting investor psychology. Nowadays, it is getting more and more Many scholars have studied this topic, making investors' behavior more rational. In today's stock market, it is obvious that there are complex systems with different types of agents, and individual investors are an important agent in the market, and their investment behavior may greatly affect the fluctuation of market prices. With the development of the network, the world has gradually developed into a whole, and different investors can interact easily, so investors' emotions will affect their decision-making, which may be largely through the network. Spread quickly.[1] In 1926, Kermack and McKendrick constructed the famous SIR warehouse model in order to study the prevalence of the 1665-1666 Black Death in London and the prevalence of the 1906 plague in Mumbai. Since then, infectious disease models have been widely used in various fields, such as emotional communication, information dissemination and so on. Liu used the infectious disease model SEIR in a heterogeneous network to discuss the propagation threshold of rumors in Weibo.[2] Wang et al. discuss the spread of emotions in complex networks in the article, and use models and simulations to more deeply verify their own ideas.[3] Xia et al. studied the spread of rumors in complex social networks, adopted the SEIR model with hesitation mechanism considering the attractiveness and uncertainty, and conducted steady-state analysis and investigated the propagation threshold and the size of the final rumor, and calculated the spread. The scale follows the opposite relationship.[4] Many people have studied the application of mathematical models in various fields, and have proposed methods that can solve many problems.[5-10] Dai discusses a computer virus propagation delay model with saturation. [11]Infectious disease models can also play an important role in controlling the spread of rumors [12-15] .

This paper aims to study the fluctuation of investor sentiment in the stock market, use the infectious disease model and add reasonable parameters to clarify the mechanism and ways of investor sentiment communication, better grasp and grasp the investment sentiment, and use mathematical methods to Modeling. Because there are still some differences between the conventional infectious disease model and emotional transmission, this paper introduces the time delay and the incidence of nonlinear saturation. Through the analysis of the system model, the stability of each equilibrium point and the existence conditions of the equilibrium point are discussed. Improve [16] and use the method of differential dynamic equation to study the effect of time delay and saturation rate on the dynamics of the whole system.

The rest of paper is organized as follows: In the second part, we built a model of investor sentiment in a homogeneous network and explained the meaning of each parameter. In the third part. We discussed the sentiment-free equilibrium and its existence. In the fourth part, we discuss the sentiment equilibrium and its existence. Finally, the full text is summarized in the fifth part.

2. The SIR model

In the stock market, we regard all investors as a homogeneous network, so investors will hold two attitudes in the operation of the stock market: excessive optimism or excessive pessimism. In terms of income, if the stock market is in a bull market, then the stock market volatility is mostly affected by the pessimism of investors, and the good news will generate more fluctuations than the bad news; if it is in the bear market, the investors Optimism will increase the volatility of earnings, pessimism will reduce the volatility of earnings, and bad news is more shocking than good news. Here we only consider the situation in which investors are overly optimistic in the market. So we divide investors into three groups, susceptible(S),infected(I) and removed(R).Among them, susceptible represents an investor who has idle funds but does not open an account at a brokerage firm; infected indicates a person who uses his own funds to invest in the stock market and actively spreads investment sentiment; removed means no longer participate in stocks Investors are investors who are delisting. When susceptible is in contact with neighboring node infected, because susceptible's own perception of the stock market and the gains in the stock market will affect the infected investment sentiment, infected will understand the stock market with probability β and start investment transactions. When infected in the stock market for a period of investment, they will exit the stock market with probability λ and become rational investors because of their own economic situation, stock returns or some other factors. When susceptible is turned into infected, it may be subject to external interference, which delays the time to enter the stock market, so we introduce time delay. And because everyone's social skills are limited, we introduce the concept of saturation incidence rate $\frac{rS(t)I(t)}{1+\alpha I(t)}$.

In order to establish the feasibility of the network, we have the following assumptions:

Assume that the ratio of entrants to exitors in the stock market is equal and both are $\underline{\mu}$.

Assume that the stock market is a homogeneous network with a network average of $\bar{k}$.

Therefore, from the above, the equation of the system can be given as:

$$\frac{dS(t)}{dt} = u - \beta\bar{k}S(t)\int_0^{\infty} f(s)I(t-s)ds - \frac{rS(t)I(t)}{1+\alpha I(t)} - uS(t) - hS(t)$$
$$\frac{dI(t)}{d(t)} = \beta\bar{k}S(t)\int_0^{\infty} f(s)I(t-s)ds + \frac{rS(t)I(t)}{1+\alpha I(t)} - uI(t) - \lambda I(t) \quad (2.1)$$
$$\frac{dR(t)}{d(t)} = \lambda I(t) - uR(t) + hS(t)$$

with the normalization condition $S(t)+I(t)+R(t)=1$.In the model, we change the latency of emotional transmission from discrete time lag to continuous time lag, and express the incidence of emotional transmission through the following formula:$\beta\bar{k}S(t)\int_0^{\infty} f(s)I(t-s)ds$.Here $f(s)$ represents the proportion of the emotion with the latency of s in the whole emotional infection, so that $f(s)$ is non-negative, is integrable at $[0,\infty)$,and satisfies the condition:$\int_0^{\infty} f(s)ds = 1, \int_0^{\infty} sf(s)ds < \infty$.And h is the vaccination efficient of the susceptible.

3. The sentiment-free equilibrium stability analysis

In this part, the local stability and global stability of the equilibrium without emotions will be solved and proved.

3.1. Local stability analysis of no emotional balance point

Can simply know, there is always a sentiment-free balance $E_0 = \left(\frac{u}{u+h}, 0, \frac{h}{u+h}\right)$, in the system (2.1). Let's verify the conditions for the existence of no emotional balance.

The no emotional balance point always satisfies the following equation:

$$\begin{cases} u-\beta\bar{k}S(t)\int_0^{\infty} f(s)I(t-s)ds-\frac{rS(t)I(t)}{1+\alpha I(t)}-uS(t)-hS(t)=0 \\ \beta\bar{k}S(t)\int_0^{\infty} f(s)I(t-s)ds+\frac{rS(t)I(t)}{1+\alpha I(t)}-uI(t)-\lambda I(t)=0 \\ \lambda I(t)-uR(t)+hS(t)=0 \end{cases} \tag{3.1}$$

In System (2.1), the basic number of regenerations is:

$$R_0=\frac{u\left(\beta\bar{k}+r\right)}{(u+\lambda)(u+h)}$$

Theorem 3.1 When $R_0<1$, the system is partially progressively stable at sentiment-free equilibrium point $E_0=\left(\frac{u}{u+h},0,\frac{h}{u+h}\right)$; when $R_0>1$, the system is unstable at sentiment-free equilibrium point $E_0=\left(\frac{u}{u+h},0,\frac{h}{u+h}\right)$.

Proof First for system (2.1), we can get its Jacobian matrix at equilibrium point E_0 as:

$$J(E_0)=\begin{pmatrix} -(u+h) & \frac{\beta u\bar{k}}{u+h}-\frac{ru}{u+h} & 0 \\ 0 & \frac{\beta u\bar{k}}{u+h}-\frac{ru}{u+h}-(u+\lambda) & 0 \\ h & \lambda & -u \end{pmatrix}$$

Thus obtaining the determinant of $|J(E_0)-\omega I|$ is:

$$f(\omega)=|J(E_0)-\omega I|=\begin{vmatrix} -(u+h)-\omega & -\frac{\beta u\bar{k}}{u+h}-\frac{ru}{u+h} & 0 \\ 0 & \frac{\beta u\bar{k}}{u+h}+\frac{ru}{u+h}-(u+\lambda)-\omega & 0 \\ h & \lambda & -u-\omega \end{vmatrix}$$

Obviously, the two eigenvalues of determinant $f(\omega)$: $\omega_1=-(u+h)<0$ $\omega_2=-u<0$ .Therefore, just consider another eigenvalue of $f(\omega)$, the corresponding characteristic polynomial is:

$$\frac{\beta u\bar{k}}{u+h}+\frac{ru}{u+h}-(u+\lambda)-\omega_3=0$$

which is:

$$\frac{u\left(\beta\bar{k}+r\right)-(u+\lambda)(u+h)}{u+h}-\omega_3=0$$

So, when $R_0<1$, $\omega_3<0$; when $R_0>1$, $\omega_3>0$.According to Routh-Hureitz stability judgement, system (2.1) is locally progressively stable at no emotional equilibrium point E_0 if and only if $R_0<1$. When $R_0>1$, the balance point E_0 is unstable.

3.2. Global stability analysis of no emotional balance point

Next, we discuss the global stability of the non-emotional equilibrium point $E_0=\left(\frac{u}{u+h},0,\frac{h}{u+h}\right)$ of System (2.1).

Theorem 3.2 If $R_0<1$, the system (2.1) is globally stable at equilibrium point E_0.

Proof Defining the Lyapunov function $L(t)=a_1S(t)+a_2I(t)$, then we can get:

$$\begin{aligned}
\frac{dL(t)}{dt} &= a_1\frac{dS(t)}{dt}+a_2\frac{dI(t)}{dt}\\
&= a_1\left(u-\beta S(t)\bar{k}\int_0^\infty f(s)I(t-s)ds-\frac{rS(t)I(t)}{1+\alpha I(t)}-uS(t)-hS(t)\right)+a_2\left(\beta S(t)\bar{k}\int_0^\infty f(s)I(t-s)ds+\frac{rS(t)I(t)}{1+\alpha I(t)}-uI(t)-\lambda I(t)\right)\\
&= a_1u-a_1\beta S(t)\bar{k}\int_0^\infty f(s)I(t-s)ds-a_1r\frac{S(t)I(t)}{1+\alpha I(t)}-a_1S(t)(u+h)+a_2\beta S(t)\bar{k}\int_0^\infty f(s)I(t-s)ds+a_2r\frac{S(t)I(t)}{1+\alpha I(t)}-a_2I(t)(u+\lambda)\\
&\le a_1u-a_1\beta S(t)\bar{k}-a_1r\frac{S(t)}{1+\alpha I(t)}-a_1(u+h)+a_2\beta S(t)\bar{k}+a_2r\frac{S(t)}{1+\alpha I(t)}-a_2(u+\lambda) \qquad (3.2)\\
&\le a_1u-a_1\beta\bar{k}+a_1r-a_1(u+h)+a_2\beta\bar{k}+a_2r-a_2(u+\lambda)\\
&= a_2(u+\lambda)\left(\frac{a_1}{a_2}\frac{u-\left(\beta\bar{k}-r\right)}{u+\lambda}-1\right)+a_1(u+h)\left(\frac{a_2}{a_1}\frac{\beta\bar{k}+r}{u+h}-1\right)
\end{aligned}$$

Next order $\frac{a_1}{a_2}=\frac{\beta\bar{k}+r}{u+h}$,and substituted into equation (3.2) above, can get:

$$\begin{aligned}
\frac{dL(t)}{dt} &\le a_2(u+\lambda)\left(\frac{\left(\beta\bar{k}+r\right)\left[u-\left(\beta\bar{k}+r\right)\right]}{(u+h)(u+\lambda)}-1\right)\\
&\le a_2(u+\lambda)\left(\frac{u\left(\beta\bar{k}+r\right)}{(u+h)(u+\lambda)}-1\right)\\
&= a_2(u+\lambda)(R_0-1)
\end{aligned}$$

Let $L(t)=\eta(t)$, consider the equation: $\frac{d\eta(t)}{dt}=a_2(u+\lambda)(R_0-1)$, from 0 to t integral, can get: $\eta(t)=\eta(0)e^{a_2(u+\lambda)(R_0-1)}$. When $R_0<1$,we have $t\to\infty,\eta(t)\to 0$. So from the comparison theorem of functional differential equations:$0\le L(t)\le\eta(t)$, $t>0$. Therefore,$t\to\infty$, $L(t)\to 0$, so the system (2.1) is attracting the whole place. In summary, the system (2.1) is globally stable at the point of sentiment-free balance.

4. The positive equilibrium point stability analysis

In this section, local and global stability analysis will be performed on the positive equilibrium point. We suppose that $E^*=(S^*,I^*,R^*)$ is the solution of system (2.1). Then E^* always satisfies Equation (3.1). By calculation, you can get:

$$S^*=\frac{u-(u+\lambda)I^*}{u+h},\quad R^*=\frac{\lambda(u+h)I^*-(u+\lambda)hI^*+uh}{u(u+h)}$$

And I^* is the solution of the following equation:

$$AI^{*2}+BI^*+C=0$$

Among them,

$$A=-\alpha\beta\bar{k}(u+\lambda)$$

$$B=\alpha\beta u\bar{k}-\beta\bar{k}(u+\lambda)-(u+\lambda)(u+h)\alpha-r(u+\lambda)$$

$$C=\beta u\bar{k}+ru-(u+\lambda)(u+h)$$

Thus, we have the following conclusion:

Theorem 4.1

(1) When $R_0>1$,the system has a local balance point E_1^*.

(2) When $R_0=1$ and $B<0$, then the system has only one local balance point $E_2^*=(S_2^*,I_2^*,R_2^*)$, in which $I_2^*=-\frac{B}{A}$.

(3) When $R_0 < 1$ and $B \geq 0$, the system has no balance point.

(4) When $R_0 < 1$ and $B < 0$, $B^2 - 4AC \geq 0$, the system has two balance points.

Thus we can get the Jacobian matrix of system (2.1) at equilibrium point E^* as:

$$J\left(E^*\right)=\begin{pmatrix} -\beta\bar{k}I^* - \frac{rI^*}{1+\alpha I^*} - (u+h) & -\beta S^*\bar{k} - \frac{rS^*}{(1+\alpha I^*)^2} & 0 \\ \beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*} & \beta S^*\bar{k} + \left(\frac{rS^*}{(1+\alpha I^*)^2} - (u+\lambda)\right) & 0 \\ h & \lambda & -u \end{pmatrix}$$

In order to prove that the corresponding feature value of $J(E^*)$ is negative, we can consider that $J(E^*)$ is a negative fixed matrix. So we give the following lemma.

Lemma 4.1 The necessary and sufficient conditions for the matrix to be negative are: the odd-order main sub-form is negative, and the even-order main sub-form is positive.

For the sequential principal subforms of $J(E^*)$,

$$D_1 = \left|-\beta\bar{k}I^* - \frac{rI^*}{1+\alpha I^*} - (u+h)\right| = -\beta\bar{k}I^* - \frac{rI^*}{1+\alpha I^*} - (u+h) < 0$$

$$D_2 = \begin{pmatrix} -\beta\bar{k}I^* - \frac{rI^*}{1+\alpha I^*} - (u+h) & -\beta S^*\bar{k} - \frac{rS^*}{(1+\alpha I^*)^2} \\ \beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*} & \beta S^*\bar{k} + \left(\frac{rS^*}{(1+\alpha I^*)^2} - (u+\lambda)\right) \end{pmatrix}$$

$$= \left[-\beta\bar{k}I^* - \frac{rI^*}{1+\alpha I^*} - (u+h)\right]\cdot\left[\beta S^*\bar{k} + \left(\frac{rS^*}{(1+\alpha I^*)^2} - (u+\lambda)\right)\right] - \left[-\beta S^*\bar{k} - \frac{rS^*}{(1+\alpha I^*)^2}\right]\cdot\left[\beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*}\right]$$

$$> (u+\lambda)\left(\beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*}\right) - (u+h)\left(\beta\bar{k} + \frac{r}{1+\alpha I^*}\right) + (u+h)(u+\lambda)$$

$$> (u+\lambda)\left(\beta\bar{k} + r\right) - (u+h)\left(\beta\bar{k} + r\right) + (u+h)(u+\lambda)$$

$$= (\lambda - h)\left(\beta\bar{k} + r\right) + (u+h)(u+\lambda)$$

$$D_3 = -uD_2$$

When condition $R_0 > 1$ and $\lambda - h > 0$ are satisfied, judgment by negative definite matrix, Jacobian matrix $J(E^*)$ is a negative matrix. Thus by theorem 4.1, we can get the following theorem.

Theorem 4.2 The system (2.1) is locally progressively stable at equilibrium point $E^* = (S^*, I^*, R^*)$.

Further we will discuss the global stability of E^*, is equivalent to the stability of the zero point in the above system. Let $V_1(t) = \tau_1^2(t)$, and calculate the derivative along the system (2.1) as:

$$\frac{dV_1(t)}{dt} = -2\left(\beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*} + u + h\right)\tau_1^2 - 2\beta S^*\bar{k}\tau_1\int_0^\infty f(s)\tau_2 ds - 2\frac{r\tau_1\tau_2 S^*}{1+\alpha I^*}$$

$$\leq -2\left(\beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*} + u + h\right)\tau_1^2 - 2\beta S^*\bar{k}\tau_1\tau_2 - 2\frac{rS^*}{1+\alpha I^*}\tau_1\tau_2$$

$$\leq -2\left(\beta\bar{k}I^* + \frac{rI^*}{1+\alpha I^*} + u + h\right)\tau_1^2 + \left(\beta S^*\bar{k} + \frac{rS^*}{1+\alpha I^*}\right)\tau_1^2 + \left(\beta S^*\bar{k} + \frac{rS^*}{1+\alpha I^*}\right)\tau_2^2$$

Let $V_2(t) = \tau_2^2(t)$, and calculate the derivative along the system (2.1) as:

$$\frac{dV_2(t)}{dt} = 2\left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_1\tau_2 + 2\beta S^* \bar{k}\,\tau_2 \int_0^\infty f(s)\tau_2 ds + 2\left(\frac{rS^*}{1+\alpha I^*} - u - \lambda\right)\tau_2^2$$
$$\le 2\left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_1\tau_2 + 2\beta S^* \bar{k}\,\tau_2^2 + 2\left(\frac{rS^*}{1+\alpha I^*} - u - \lambda\right)\tau_2^2$$
$$\le \left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_1^2 + \left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_2^2 + 2\left(\beta S^* \bar{k} + \frac{rS^*}{1+\alpha I^*} - u - \lambda\right)\tau_2^2$$

Let $V_3(t) = \tau_3^2(t)$, and calculate the derivative along the system (2.1) as:

$$\frac{dV_3(t)}{dt} = 2\lambda\tau_2\tau_3 - 2u\tau_3^2 + 2h\tau_1\tau_3$$
$$\le \lambda\tau_2^2 + \lambda\tau_3^2 - 2u\tau_3^2 + h\tau_1^2 + h\tau_2^2$$
$$= h\tau_1^2 + (\lambda + h)\tau_2^2 + (\lambda - 2u)\tau_3^2$$

Definition of Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

Then it is clearly positive and the derivative along the system (2.1) is:

$$\frac{dV(t)}{dt} = -2\left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*} + u + h\right)\tau_1^2 - 2\beta S^* \bar{k}\,\tau_1 \int_0^\infty f(s)\tau_2 ds - 2\frac{r\tau_1\tau_2 S^*}{1+\alpha I^*} + 2\left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_1\tau_2 + 2\beta S^* \bar{k}\,\tau_2 \int_0^\infty f(s)\tau_2 ds$$
$$+ 2\left(\frac{rS^*}{1+\alpha I^*} - u - \lambda\right)\tau_2^2 + 2\lambda\tau_2\tau_3 - 2u\tau_3^2 + 2h\tau_1\tau_3$$
$$\le -2\left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*} + u + h\right)\tau_1^2 + \left(\beta S^* \bar{k} + \frac{rS^*}{1+\alpha I^*}\right)\tau_1^2 + \left(\beta S^* \bar{k} + \frac{rS^*}{1+\alpha I^*}\right)\tau_2^2 + \left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_1^2$$
$$+ \left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*}\right)\tau_2^2 + 2\left(\beta S^* \bar{k} + \frac{rS^*}{1+\alpha I^*} - u - \lambda\right)\tau_2^2 + h\tau_1^2 + (\lambda + h)\tau_2^2 + (\lambda - 2u)\tau_3^2$$
$$= \left(-\beta \bar{k} I^* - \frac{rI^*}{1+\alpha I^*} - 2u - h + \beta S^* \bar{k} + \frac{rS^*}{1+\alpha I^*}\right)\tau_1^2 + \left(\beta \bar{k} I^* + \frac{rI^*}{1+\alpha I^*} + 3\beta S^* \bar{k} + 3\frac{rS^*}{1+\alpha I^*} - 2u - \lambda + h\right)\tau_2^2 + (\lambda - 2u)\tau_3^2$$

Thus the following theorem can be obtained.

Theorem 4.3 If $R_0 > 1$, and at the same time meet the following conditions:

$$\begin{cases} -\beta \bar{k} I^* - \dfrac{rI^*}{1+\alpha I^*} - 2u - h + \beta S^* \bar{k} + \dfrac{rS^*}{1+\alpha I^*} < 0 \\ \beta \bar{k} I^* + \dfrac{rI^*}{1+\alpha I^*} + 3\beta S^* \bar{k} + 3\dfrac{rS^*}{1+\alpha I^*} - 2u - \lambda + h < 0 \\ \lambda - 2u < 0 \end{cases}$$

So $\frac{dV(t)}{dt}$ is negative. Therefore, the balance point $E^* = (S^*, I^*, R^*)$ is globally progressively stable.
We got the abo sentiment-free equilibrium stability and positive equilibrium point stability.

5. Conclusion

The transmission of emotions in the network is a very complicated process. We improved the traditional infectious disease model and established an SIR model of emotion transmission when investors invest.Through our proof, we can know that when the investment information can be transmitted in time and the number of investor groups reaches a saturated value, the market is in a stable state. That is to say, if we want to control the stability of the market, the relevant supervision Departments must strictly control the number of people entering the market to ensure the timeliness of information transmission, so that more investors can learn relevant information in a timely manner.

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