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Effect of Chemical reaction on Peristaltic blood flow of a Magneto-Jeffrey fluid with thermal radiation in a tapered asymmetric channel

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Abstract: In this paper, we investigate the effect of chemical reaction on peristaltic flow of a Jeffrey fluid in a tapered asymmetric channel with induced magnetic field and thermal radiation. The flow is analysed by long wavelength and low Reynolds number approximations. The reduced equations are solved by using the Adomian Decomposition Method and the expressions for velocity, stream function axial induced magnetic field and pressure gradient are obtained. The effect of pertinent parameters are illustrated graphically.

Keywords: Peristaltic transport, Magneto-Jeffrey fluid, chemical reaction, thermal radiation, tapered asymmetric channel.

1. Introduction

One of the major chemical mechanisms for fluid transport in many biological systems well known to physiologists is peristalsis. Peristalsis is a mechanism of series of wave-like muscle contractions and relaxations that moves bio-fluids in different processes. Some examples of peristaltic phenomenon include urine movement from kidney to gallbladder, bile transport in duct, chyme transport in small intestine, food processing in digestive tract, flow of blood in small vessels, locomotion of worms and many others. Peristalsis finds numerous applications in medical and industrial systems which include various devices like rollers, hose and tube pumps, dialysis, open-heart by pass and heart-lung machines etc. Latham [1] was the first who initially investigated the motion of fluid in peristaltic pump. Shapiro et al. [2] studied the peristaltic activity for flow of viscous fluid in a tube and channel employing the long wavelength and low Reynolds number approximations. Some recent contributions describing the peristaltic mechanism of Newtonian and non-Newtonian fluids may be mentioned in references [3-7].

Recently, peristaltic flow with magnetic particles has grabbed the attention of several researchers due to its emerging applications in engineering and medical processes. MHD (Magnetohydrodynamics) is employed in magnetic drug targeting, pumping of blood, reduction of bleeding during surgery, continue casting process, hyperthermia, magnetic resonance imaging (MRI) and magnetotherapy etc. Few of the industrial applications include heat exchangers, pump meters, radar systems, magnetic devices for cell separation, magnetic drug targeting and magnetic tracers. Kothandapani and Srinivas [8] investigated the peristaltic transport of a Jeffrey fluid under the effect of magnetic field. Later Hayat and Ali [9] studied the influence of magnetic field on Jeffrey fluid in a tube, Vajravelu et.al [10] analysed the peristaltic transport of a conducting Jeffrey fluid in an inclined asymmetric channel. Mahmouda et.al [11] explained the MHD peristaltic transport of a Jeffrey fluid in a porous medium.

The topic of blood flow (or Hemodynamics) problems have received a considerable attention due to its major importance in physiopathology. For a long time, blood is treated as a vital fluid. Blood circulation performs various types of function in the human body such as transport of nutrients, transport of oxygen, removal of metabolic products and removal of carbon dioxide. Mekheimer [12] examined the effects of magnetic field on peristaltic blood flow of couple stress fluid in a non-uniform channel. Akbar [13] analyzed the blood flow of Prandtl fluid model in tapered stenosed arteries.

Matter interacts to form new products through a process called chemical reaction. Everyday a lot of chemical reactions takes place in the human body. Furthermore various industrial processes include chemical reactions such as Haber's process (chemical binding of Nitrogen from air to make ammonia), disinfectant (chemical treatment to kill bacteria and viruses) and pyro processing (chemically combine materials like cement).Barika and Dash [14] studied the chemical reaction effect of a magneto-micropolar fluid in a porous medium. Hayat et.al [15] analysed the chemical reaction in peristaltic transport of a MHD couple stress fluid with Soret and Dufour effects. Hayat et.al [16] have analysed the Jeffrey fluid model for convective boundary conditions.

In thermodynamics, thermal radiation also known as heat is the emission of electromagnetic waves from all matter that has a temperature greater than absolute zero. Heat transfer takes place in the human body by conduction, convection, evaporation and radiation. Transport of heat by the circulatory system makes heat transfer in the body. Ajaz et.al [17] analysed the radiation effects on micropolar fluid. Hayat et.al [18] studied the entropy generation rate for a peristaltic pump of a Jeffrey fluid. Selvi et.al. [19] discussed the effect of heat transfer on peristaltic flow of Jeffrey fluid in an inclined porous stratum. Asha and Deepa [20] analysed the entropy generation for peristaltic blood flow of a magneto-micropolar fluid with thermal radiation in a tapered channel.

The word taper means diminish or reduce in thickness towards one end. The tapered asymmetric channel is normally created due to the intra-uterine fluid flow induced by myometrical contractions and it was stimulated by asymmetric peristaltic fluid flow in a two-dimensional channel. Ajaz [21] studied the peristaltic flow of nanofluids in a tapered asymmetric porous channel. Asha and Deepa [22] discussed the micro polar fluid flow in a tapered asymmetric channel. Asha and Deepa [23] analysed the impacts of hall and heat transfer with peristalsis. Asha and Deepa [24] also reported the thermo- diffusion and diffusion-thermo effects with peristaltic flow.

In view of the above discussion the aim of this study is to analyse the chemical reaction effect on peristaltic blood flow of a magneto-Jeffrey fluid with thermal radiation in a tapered asymmetric channel. The Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives, it represents a rheology different from the Newtonian. The problem is first modelled and then analysed by the long wavelength and low Reynolds number approximations. The reduced governing equations are solved by using the Adomian Decomposition Method (ADM) and the effect of various parameters are discussed and illustrated graphically.

2. Mathematical modelling

Consider the peristaltic flow of an incompressible, viscous and electrically conducting magneto-Jeffrey fluid through a tapered asymmetric two dimensional channel with thermal radiation effects. The flow is generated by sinusoidal wave trains propagating with constant speed ‘c’ along the channel walls.

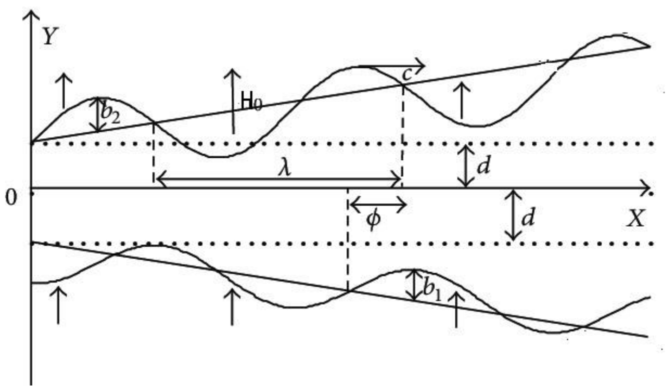

Figure 1 A physical sketch of the problem

We consider a rectangular coordinate system for the channel in which the X-axis is taken along the centreline of the channel and the Y-axis is transverse to it. An external transverse uniform constant magnetic field H_0, induced magnetic field $H'(h_x{}', h_y{}', 0)$ and therefore the total magnetic field $H'^{+}(h_x{}', H_0 + h_y{}', 0)$, where $h_x{}'$ and $h_y{}'$ are the components of induced magnetic field along the co-ordinated axes. The geometry of the wall surfaces is given by,

$$h'_1(X', t') = d + m'X' + a_1 sin\left[\frac{2\pi}{\lambda}(X' - ct')\right], \text{ upper wall} \quad (1)$$

$$h'_2(X', t') = -d - m'X' - a_2 sin\left[\frac{2\pi}{\lambda}(X' - ct') + \phi\right], \text{ lower wall} \quad (2)$$

Where a_1 and a_2 are the amplitudes of waves, λ is the wavelength, $\phi(0 \le \phi \le \pi)$ is the phase difference, m' is the coefficient of tapered parameter and d is the half-width of the channel.
The governing equations for magneto-Jeffrey fluid, are represented by the following set of equations

$$\nabla . \vec{v'} = 0, \tag{3}$$

$$\rho\left(\frac{\partial \vec{v'}}{\partial t'} + \left(\vec{v'} . \nabla\right)\vec{v'}\right) = divT - \nabla\left(p' + \frac{1}{2}\mu_e\left(H'^{+}\right)^2\right) - \mu_e\left(H'^{+} . \nabla\right)H'^{+}, \tag{4}$$

Where $T = -pI + S$in which the extra stress tensor for Jeffrey fluid is defined as

$$S = \frac{\mu}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right)$$

The Maxwell's equations,

$$\nabla \times H'^{+} = \vec{J'}, \nabla \times \vec{E'} = -\mu_e\left(\frac{\partial H'^{+}}{\partial t'}\right) \tag{5}$$

along with the Ohm's law

$$\vec{J'} = \sigma\left(\vec{E'} + \mu_e\left(\vec{v'} \times H'\right)\right), \tag{6}$$

In addition, it should be noted that

$$\nabla . H' = 0 \text{ and } \nabla . E' = 0, \tag{7}$$

Now, combining equations (5)-(7) we get the induction equation,

$$\frac{\partial H'^{+}}{\partial t'} = \nabla \times \left(\vec{v'} \times H'^{+}\right) + \frac{1}{\mu_e \sigma}\nabla H', \tag{8}$$

Where $\frac{1}{\mu_e\sigma}(=\eta)$ is the magnetic diffusivity and $\vec{v'}$ is the velocity vector, $p'$is the fluid pressure, ρ the fluid density, σ the electrical conductivity, μ_e is the magnetic permeability, E' is an induced electric field, J is the current density, I is the Cauchy stress tensor, S is the extra stress tensor, μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation time, λ_2 is the retardation time, and $\dot{\gamma}$ is the shear rate.
The equations that govern the fluid motion for unsteady flow of an incompressible magneto-Jeffrey polar fluid in the cartesian co-ordinate system may be represented as,

$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0, \tag{9}$$

$$\frac{\partial U'}{\partial t'} + U'\frac{\partial U'}{\partial X'} + V'\frac{\partial U'}{\partial Y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial X'} + \frac{\partial\left(S'_{X'X'}\right)}{\partial X'} + \frac{\partial\left(S'_{X'Y'}\right)}{\partial Y'} - \frac{\mu_e}{\rho}\left(h'_x\frac{\partial h_x'}{\partial X'} + h'_y\frac{\partial h_x'}{\partial Y'}\right) - \frac{\mu_e}{\rho}H_0\frac{\partial h_x'}{\partial Y'} \tag{10}$$

$$\frac{\partial U'}{\partial t'} + U'\frac{\partial U'}{\partial X'} + V'\frac{\partial U'}{\partial Y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial X'} + \frac{\partial\left(S'_{X'X'}\right)}{\partial X'} + \frac{\partial\left(S'_{Y'Y'}\right)}{\partial Y'} - \frac{\mu_e}{\rho}\left(h'_x\frac{\partial h_x'}{\partial X'} + h'_y\frac{\partial h_x'}{\partial Y'}\right) - \frac{\mu_e}{\rho}H_0\frac{\partial h_x'}{\partial Y'} \tag{11}$$

$$\rho\, C_p'\left(\frac{\partial T'}{\partial t'} + U'\frac{\partial T'}{\partial X'} + V'\frac{\partial T'}{\partial Y'}\right) = k\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right) - \frac{\partial q_r}{\partial y'} - Q'(T' - T_0) \tag{12}$$

$$\left(\frac{\partial C'}{\partial t'} + U'\frac{\partial C'}{\partial X'} + V'\frac{\partial C'}{\partial Y'}\right) = D_m\left(\frac{\partial^2 C'}{\partial X'^2} + \frac{\partial^2 C'}{\partial Y'^2}\right) + \frac{D_m K_T}{T_m}\left(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2}\right) - k_2(C' - C_0) \tag{13}$$

With the following dimensional boundary conditions,

$$\psi' = \frac{q'}{2}, \frac{\partial \psi'}{\partial y'} = 0, \frac{\partial \xi'}{\partial y'} = 0, T' - T_0 = 0 \text{ and } C' - C_0 = 0 \text{ at } y = h_1'$$

$$\psi' = -\frac{q'}{2}, \frac{\partial \psi'}{\partial y'} = 0, \frac{\partial \xi'}{\partial y'} = 0, T' - T_0 = 1 \text{and } C' - C_0 = 1 \text{ at } y = h_2' \tag{14}$$

Further the flow field in laboratory frame (X', Y') and wave frame (x', y') are treated as the unsteady and steady motion respectively. Considering the wave frame (x', y') moving with a velocity c away from a fixed frame (X', Y') that follows from the following transformations

$$x' = X' - ct',\ y' = Y',\ u'(x', y') = U' - c,\ v'(x', y') = V'$$

In which (u', v') and (U', V') are the respective velocity components in the laboratory and wave frames.
Introducing the following non-dimensional variables

$$x = \frac{x'}{\lambda},\ y = \frac{y'}{d},\ u = \frac{u'}{c},\ v = \frac{\lambda v'}{dc},\ w = \frac{dw'}{c},\ p = \frac{d^2 p'}{\lambda \mu c},\ t = \frac{ct'}{\lambda},\ \psi = \frac{\psi'}{cd},\ F = \frac{q'}{cd},$$

$$\xi = \frac{\xi'}{H_0 d},\ h_x = \frac{h_x'}{H_0},\ h_y = \frac{h_y'}{H_0},\ Re = \frac{\rho dc}{\mu},\ \delta = \frac{d}{\lambda},\ S_1 = \frac{H_0}{c}\sqrt{\frac{\mu_e}{\rho}},\ P_m = p + \frac{1}{2}Re\delta\left(\frac{\delta(H^+)^2}{\rho c^2}\right),$$

$$E = -\frac{E'}{\mu_e c H_0},\ R_m = \sigma\mu_e dc,\ h_1 = \frac{h_1'}{d},\ h_2 = \frac{h_2'}{d},\ a = \frac{a_1'}{d},\ b = \frac{a_2'}{d},\ m = \frac{\lambda m'}{\mu},\ \theta = \frac{T' - T_0}{T_1 - T_0} \tag{15}$$

$$\phi = \frac{C' - C_0}{C_1 - C_0},\ \gamma = \frac{\mu}{\rho}, P_r = \frac{\mu C_p}{K},\ S_r = \frac{\rho D_m K_T (T_1 - T_0)}{T_m \mu (C_1 - C_0)},\ S_c = \frac{\mu}{\rho D_m},\ N_r = \frac{16\sigma T_0^3}{3k'K}, S = \frac{k_2 d}{c}, Q_0 = \frac{Q' d^2}{C_p \mu},$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\delta\frac{\partial \psi}{\partial x},\ h_x = \frac{\partial \xi}{\partial y}, h_y = -\delta\frac{\partial \xi}{\partial x}$$

Where P_r is the Prandtl number, S_c is the Schmidt number, S_r is the Soret number, and N_r is the thermal radiation parameter, S is the coefficient of chemical reaction, Q_0 is the coefficient of heat addition or absorption, $S_1 = \frac{H_0}{c}\sqrt{\frac{\mu_e}{\rho}}$ the Strommer's number also known as magnetic force number, $R_m = \sigma\mu_e dc$ the magnetic Reynolds number, ξ is the magnetic force function.

The total pressure in the fluid, which is equal to the sum of the ordinary and magnetic pressure given by $P_m = p + \frac{1}{2}Re\delta\left(\frac{\delta\left(H'^{+}\right)^2}{\rho c^2}\right)$ and $E = -\left(\frac{E'}{\mu_e c H_o}\right)$ is defined as the electrical field strength in non-dimensional form.

Taking after the Rosseland diffusion approximation, the radiative flux q_r for radiation is sculpted as

$$q_r = \frac{-4\sigma'}{3k'}\frac{\partial T'^4}{\partial y'}, \tag{16}$$

Where σ' denotes Stefan-Boltzman constant and k′ is defined as mean absorption coefficient, K is the thermal conductivity of the fluid. Supposing that the temperature variances within the flow are adequately small such that T'^4 can be represented as the linear combination of temperature. This is practiced by expanding T'^4 by using Taylor series about T_0 as follows

$$T'^4 = T_0^4 + 4T_0^3(T' - T_0) + 6T_0^2(T' - T_0)^2 + \ldots \tag{17}$$

By neglecting the higher order terms (second order onwards) in $(T' - T_0)$ we arrive at

$$3T'^4 \approx -3T_0^4 + 4T_0^3 T', \tag{18}$$

Differentiating Eq. (16) with respect to y′ and using Eq. (18) to get

$$\frac{\partial q_r}{\partial y'} = -\frac{16\sigma' T_0^3}{3k'}\frac{\partial^2 T'}{\partial y'^2}, \tag{19}$$

Using the above non-dimensional quantities Equations. (10) – (13) reduce to

$$Re\delta\left[\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\psi_y\right] = -\frac{\partial P_m}{\partial x} + \delta\frac{\partial(S_{xx})}{\partial x} + \frac{\partial(S_{xy})}{\partial y} + ReS_1^2\xi_{yy} + ReS_1^2\delta\left[\left(\xi_y\frac{\partial}{\partial x} - \xi_x\frac{\partial}{\partial y}\right)\xi_y\right] \tag{20}$$

$$Re\delta^3\left[\left(\psi_x\frac{\partial}{\partial y} - \psi_y\frac{\partial}{\partial x}\right)\psi_x\right] = -\frac{\partial P_m}{\partial y} + \delta^2\frac{\partial(S_{yx})}{\partial x} + \delta\frac{\partial(S_{yy})}{\partial y} - ReS_1^2\delta^2\xi_{xy} - ReS_1^2\delta^3\left[\left(\xi_y\frac{\partial}{\partial x} - \xi_x\frac{\partial}{\partial y}\right)\xi_x\right] \tag{21}$$

$$\psi_y - \delta(\psi_x\xi_y - \psi_y\xi_x) + \frac{1}{R_m}(\xi_{yy} + \delta^2\xi_{xx}) = E \tag{22}$$

$$Re\delta\left[\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\theta\right] = \frac{1}{P_r}\left[\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta\right] + \frac{N_r}{P_r}\frac{\partial^2}{\partial x^2}[(1 - Q_0)\theta] \tag{23}$$

$$Re\delta\left[\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\phi\right] = \frac{1}{S_c}\left[\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi\right] + S_r\left[\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta\right] - S\phi \tag{24}$$

where $S_{xx} = \frac{2\delta}{1+\lambda_1}\left(1 + \frac{\lambda_2 c\delta}{d_1}\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\right)\psi_{xy}$

$$S_{xy} = \frac{1}{1+\lambda_1}\left(1 + \frac{\lambda_2 c\delta}{d_1}\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\right)(\psi_{yy} - \delta^2\psi_{xx})$$

$$S_{yy} = -\frac{2\delta}{1+\lambda_1}\left(1 + \frac{\lambda_2 c\delta}{d_1}\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\right)\psi_{xy}$$

Applying the assumptions of long wavelength $\delta \leq\!< 1$and low but finite Reynolds number neglecting the terms of order δ and higher the above equations (20) – (24) take the form,

$$-\frac{\partial p}{\partial x} + \frac{1}{(1+\lambda_1)}\frac{\partial^3\psi}{\partial y^3} + ReS_1^2\xi_{yy} = 0, \tag{25}$$

$$\frac{\partial p}{\partial y} = 0, \tag{26}$$

$$\frac{\partial\psi}{\partial y} + \frac{1}{R_m}\frac{\partial^2\xi}{\partial y^2} = E, \tag{27}$$

$$(1 + N_r)\frac{\partial^2\theta}{\partial y^2} - P_r Q_0\theta = 0, \tag{28}$$

$$\frac{\partial^2\phi}{\partial y^2} + S_c S_r\frac{\partial^2\theta}{\partial y^2} - S\phi = 0, \tag{29}$$

Now eliminating the total pressure from equations (25) and (26) we get,

$$\frac{1}{(1+\lambda_1)}\frac{\partial^4\psi}{\partial y^4} + ReS_1^2\xi_{yyy} = 0, \tag{30}$$

$$\frac{\partial^2\xi}{\partial y^2} = R_m\left(E - \frac{\partial\psi}{\partial y}\right), \tag{31}$$

$$(1 + N_r)\frac{\partial^2\theta}{\partial y^2} - P_r Q_0\theta = 0, \tag{32}$$

$$\frac{\partial^2\phi}{\partial y^2}+S_cS_r\frac{\partial^2\theta}{\partial y^2}-S\phi=0 \tag{33}$$

Using Eq. (31) in Eq. (30) we get,

$$\frac{\partial^3\psi}{\partial y^3}+H^2(1+\lambda_1)\left(E-\frac{\partial\psi}{\partial y}\right)=A_3 \tag{34}$$

With non-dimensional boundary conditions given by,

$$\psi=\frac{F}{2}\ ,\ \frac{\partial\psi}{\partial y}=0,\frac{\partial\xi}{\partial y}=0,\ \theta=0 \text{ and } \phi=0 \text{ at } y=h_1$$

$$\psi=-\frac{F}{2},\ \frac{\partial\psi}{\partial y}=0,\frac{\partial\xi}{\partial y}=0,\ \theta=1 \text{ and } \phi=1 \text{ at } y=h_2 \tag{35}$$

The instantaneous volumetric flow rate in the fixed frame is given by,

$$Q=\int_{h'_2}^{h'_1}U'(X',Y',t')dy', \tag{36}$$

Where h'_1 and h'_2 are functions of X' and t'

The rate of volume in the wave frame is found to be given by,

$$q=\int_{h'_2}^{h'_1}u'(x',y')dy', \tag{37}$$

Using the transformations into the equations (36) and (37), the relation between Q and q can be obtained as

$$Q=q+c\left(h'_1-h'_2\right), \tag{38}$$

The time mean flow over a period T at a fixed position X' is defined as

$$Q'=\frac{1}{T}\int_0^T Qdt, \tag{39}$$

Using (38) in (39) the flow rate Q' has the form

$$Q'=\frac{1}{T}\int_0^T q\,dt+c\left(h'_1-h'_2\right)=q+cd_1+cd_2, \tag{40}$$

The dimensionless mean flow rate related to dimensionless time mean flow rate is given by

$$\Theta=F+1+d, \tag{41}$$

Where $\Theta=\frac{Q'}{cd}$ and $F=\frac{q}{cd}$ such that

$$F=\int_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy=\psi(h_1)-\psi(h_2), \tag{42}$$

3. Method of Solution

Integrating twice equation (32) we get,

$$\theta=c_1e^{s_1y}+c_2e^{-s_1y}, \tag{43}$$

Substituting the value of θ in Eq. (33) we get

$$\phi=\frac{-(S_c\,S_r)\left(e^{-s_1h_1}e^{s_1y}+e^{s_1h_1}e^{-s_1y}\right)}{\left(e^{s_1h_1}e^{s_1h_2}-e^{s_1h_1}e^{-s_1h_2}\right)}-c_3y-c_4, \tag{44}$$

Where $s_1=\sqrt{\frac{Q_oP_r}{(1+N_r)}}$

Using the appropriate boundary conditions of equation (35) we get,

$$c_1=\frac{e^{s_1h_2}}{\left(e^{-s_1(h_1-h_2)}-e^{s_1(h_1-h_2)}\right)e^{s_1(h_1+h_2)}},\ c_2=\frac{e^{s_1h_1}}{\left(e^{-s_1(h_1-h_2)}-e^{s_1(h_1-h_2)}\right)},$$

$$c_3=-\frac{1}{(h_2-h_1)}-\frac{(S_c\,S_r)}{\left(e^{s_1h_1}e^{s_1h_2}-e^{s_1h_1}e^{-s_1h_2}\right)(h_2-h_1)}\left[\frac{e^{-s_1h_1}e^{s_1h_2}+e^{s_1h_1}e^{-s_1h_2}}{{h_2}^2}-\frac{e^{-s_1h_1}e^{s_1h_1}+e^{s_1h_1}e^{-s_1h_1}}{{h_1}^2}\right]$$

$$c_4=\frac{(S_c\,S_r)\left(e^{-s_1h_1}e^{s_1h_1}+e^{s_1h_1}e^{-s_1h_1}\right)}{\left(e^{s_1h_1}e^{s_1h_2}-e^{s_1h_1}e^{-s_1h_2}\right)}+\frac{h_1}{(h_2-h_1)}+\frac{(S_c\,S_r)}{\left(e^{s_1h_1}e^{s_1h_2}-e^{s_1h_1}e^{-s_1h_2}\right)(h_2-h_1)}\left[\frac{e^{-s_1h_1}e^{s_1h_2}+e^{s_1h_1}e^{-s_1h_2}}{{h_2}^2}-\frac{e^{-s_1h_1}e^{s_1h_1}+e^{s_1h_1}e^{-s_1h_1}}{{h_1}^2}\right]$$

Using the standard adomian decomposition method equation (34) in the operator form can be written as

$$L_{yyy}[\psi_{yyy}]=A_3-N^2E+N^2\psi_y, \tag{45}$$

where $N^2=H^2(1+\lambda_1)$

Applying the inverse operator ${L^{-1}}_{yyy}(.)=\int_0^y\int_0^y\int_0^y(.)dy\,dy\,dy$ to the above equation we get,

$$\psi=A+By+C\frac{y^2}{2!}+(A_3-N^2E)\frac{y^3}{3!}+N^2{L^{-1}}_{yyy}[\psi_y], \tag{46}$$

where A, B, C and A_3 are constants to be determined

Now we decompose ψ as $\psi=\sum_{n=0}^{\infty}\psi_n$, (47)

Substituting ψ in equation (46) we obtain,

$$\psi_0 = A + By + C\frac{y^2}{2!} + (A_3 - N^2E)\frac{y^3}{3!}, \tag{48}$$

and

$\psi_{n+1} = N^2 \iiint (\psi_n)_y \, dydydy$, where $n \geq 0$

Therefore,

$\psi_1 = N^2\left(B\frac{y^3}{3!} + C\frac{y^4}{4!} + (A_3 - N^2E)\frac{y^5}{5!}\right),$

$\psi_2 = N^2\left(B\frac{y^5}{5!} + C\frac{y^6}{6!} + (A_3 - N^2E)\frac{y^7}{7!}\right),$

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$\psi_n = N^{2n}\left(B\frac{y^{2n+1}}{(2n+1)!} + C\frac{y^{2n+2}}{(2n+2)!} + (A_3 - N^2E)\frac{y^{2n+3}}{(2n+3)!}\right),$

According to (47) the closed form of ψ can be written as,

$$\psi = A + sinh(Ny)\left(\frac{B}{N} + \frac{1}{N^3}(A_3 - N^2E)\right) - \frac{1}{N^2}(A_3 - N^2E)y + \frac{C}{N^2}(cosh(Ny) - 1)$$

which can be put in the simplest form as,

$$\psi = A_0 + A_1\,cosh(Ny) + A_2\,sinh(Ny) + Ey - \frac{A_3}{N^2}y \tag{49}$$

Where

$$A_0 = \frac{FN(h_2 + h_1) + tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2(h_2 + h_1) + \frac{FN^2(h_2 + h_1)}{1 + \lambda_1}\right)}{2N(h_2 - h_1) + 2\,tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)}$$

$$A_1 = \frac{F + (h_1 - h_2)\,sinh\left[N\left(\frac{h_1 + h_2}{2}\right)\right]sech\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{(h_1 - h_2)N - 2\,tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)}$$

$$A_2 = \frac{F + (h_1 - h_2)\,cosh\left[N\left(\frac{h_1 + h_2}{2}\right)\right]sech\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{(h_1 - h_2)N - tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)}$$

$$A_3 = \frac{\left(2N^2(E + 1) + \frac{N^4(F - h_1E + Eh_2)}{1 + \lambda_1}\right)tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{(h_1 - h_2)N - tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)}$$

Thus the expression for the axial velocity u is obtained as,

$$u = \left(N(F + h_1 - h_2)\right)\left(-1 + sech\left[N\left(\frac{h_1 - h_2}{2}\right)\right]cosh\left[N\left(\frac{h_1 + h_2}{2} - y\right)\right]\right) + \frac{\left(\frac{N^2(Eh_2 + (h_2 - h_1 - q))}{1 + \lambda_1}\right)tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{(h_1 - h_2)N - tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]\left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)} \tag{50}$$

Substituting the value of ψ in equation (31) and solving for the magnetic force function ξ we get,

$$\xi = \frac{A_3R_m}{N^2}\frac{y^2}{2} - R_m\left(\frac{A_1}{N}sinh(Ny) + \frac{A_2}{N}cosh(Ny)\right) + c_5y + c_6 \tag{51}$$

where

$$c_5 = \frac{R_m}{2}(h_1 + h_2)\left(\frac{N(Eh_2 - Eh_1 + F) + (2E + 1) + \frac{N^2(Eh_2 - Eh_1 + F)}{1 + \lambda_1}tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{N(h_1 - h_2) - \left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}\right)$$

$$c_6 = \frac{R_m}{N}(h_1 + h_2)\left(\frac{2(h_1 - h_2 + F) + N^2h_1h_2(Eh_2 - Eh_1 + F) + Nh_1h_2\left((2E + 1) + \frac{N^2(Eh_2 - Eh_1 + F)}{1 + \lambda_1}\right)tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}{2N(h_2 - h_1) + \left(2 + \frac{N^2(h_2 - h_1)}{1 + \lambda_1}\right)tanh\left[N\left(\frac{h_1 - h_2}{2}\right)\right]}\right)$$

The expression for axial- induced magnetic field across the channel is given by

$$h_x = \frac{\partial\xi}{\partial y} \tag{52}$$

The electric field strength E can be determined by integrating (31) and using the boundary conditions on ξ and ψ across the wall surface as,

$$E = \frac{F}{h_1(x) - h_2(x)}, \tag{53}$$

The pressure gradient is obtained from the dimensionless momentum equation for the axial velocity

$$\frac{dp}{dx} = \frac{1}{1+\lambda_1}\psi_{yyy} + H^2(E - \psi_y) \tag{54}$$

Now using the equations (49) and (50), the axial pressure gradient is given by,

$$\frac{dp}{dx} = \frac{N^3(h_2E-h_1E+F)+tanh\left[N\left(\frac{h_1-h_2}{2}\right)\right]\left(2N^2(E+1)+\frac{N^4(h_2E-h_1E+F)}{1+\lambda_1}\right)}{(1+\lambda_1)\,tanh\left[N\left(\frac{h_1-h_2}{2}\right)\right]\left(N(h_2-h_1)+\left(2+\frac{N^2(h_2-h_1)}{(1+\lambda_1)}\right)\right)} \tag{55}$$

The pressure rise per wavelength Δp in the non-dimensional form is given by,

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx, \tag{56}$$

4. Results and discussion

In this section, the effects of various parameters on the pumping characteristics of a magneto-Jeffrey fluid are investigated and illustrated graphically.

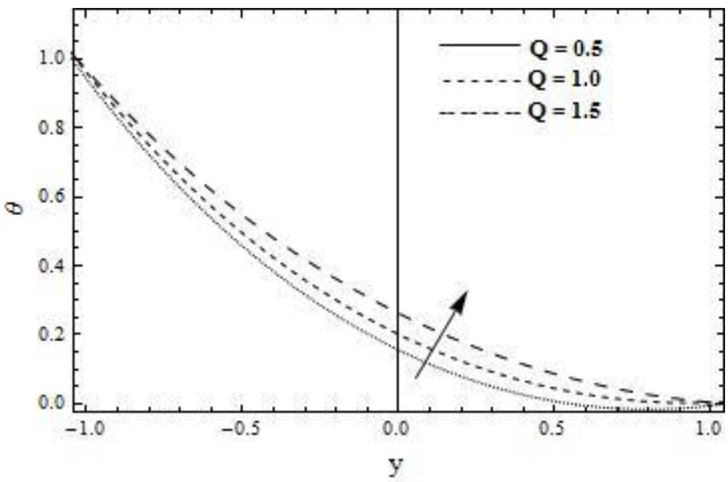

Figure.2: Temperature profile for different values of Q with a = 0.1, b = 0.2, d = 1, $\phi = 0$

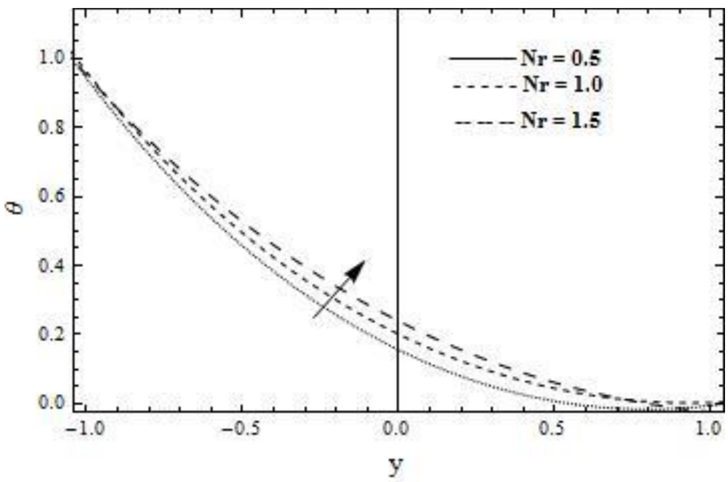

Figure.3: Temperature profile for different values of N_r with a = 0.1, b = 0.2, d = 1, $\phi = 0$

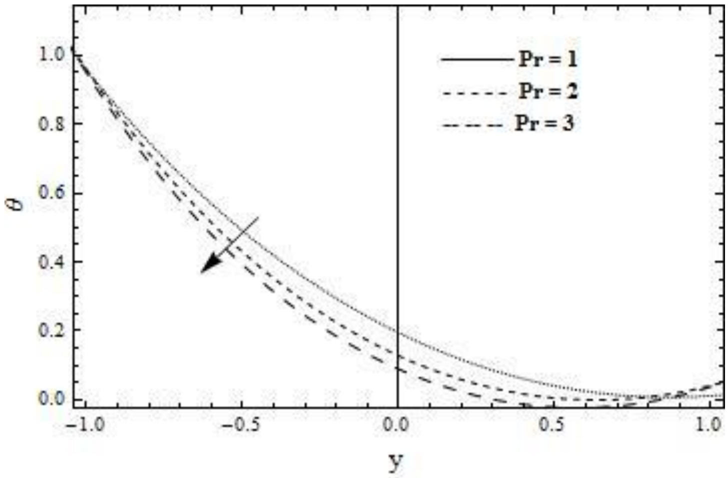

Figure.4: Temperature profile for different values of P_r with a = 0.1, b = 0.2, d = 1, $\phi = 0$

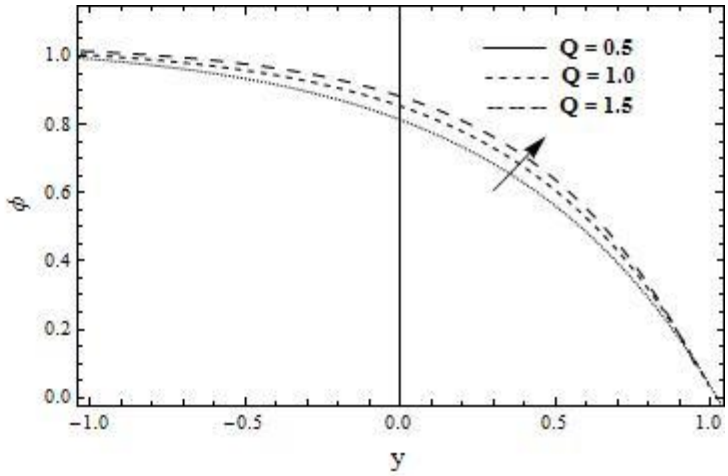

Figure.5: Concentration profile for different values of Q with a = 0.1, b = 0.2, d = 1, $\phi = 0$

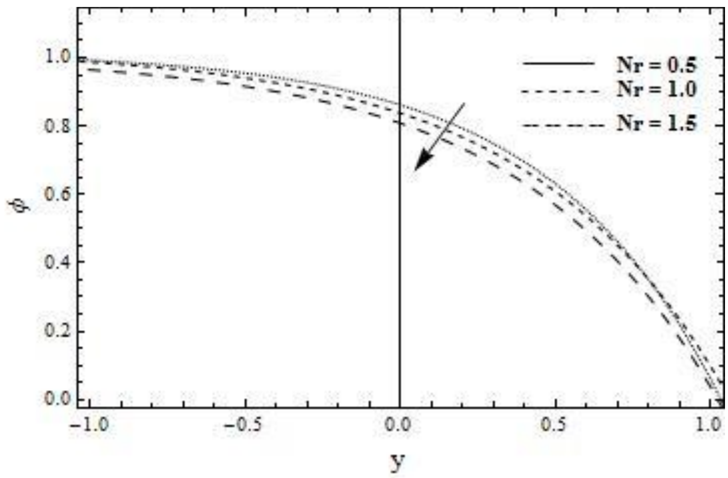

Figure.6: Concentration profile for different values of N_r with a = 0.1, b = 0.2, d = 1, $\phi = 0$

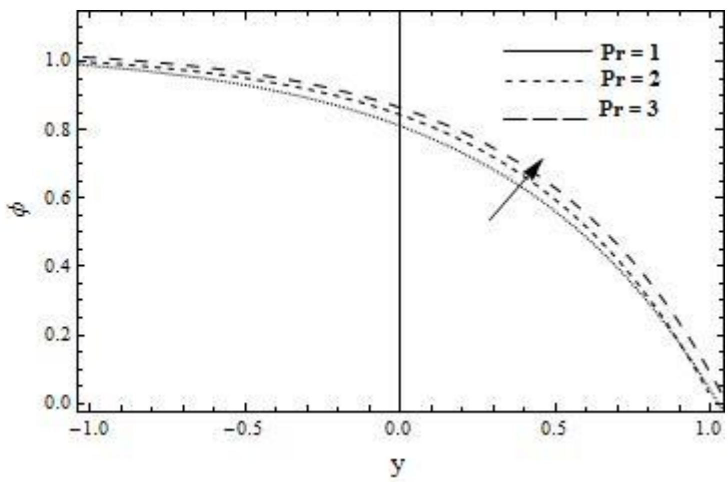

Figure.7: Concentration profile for different values of P_r with a = 0.1, b = 0.2, d = 1, $\phi = 0$

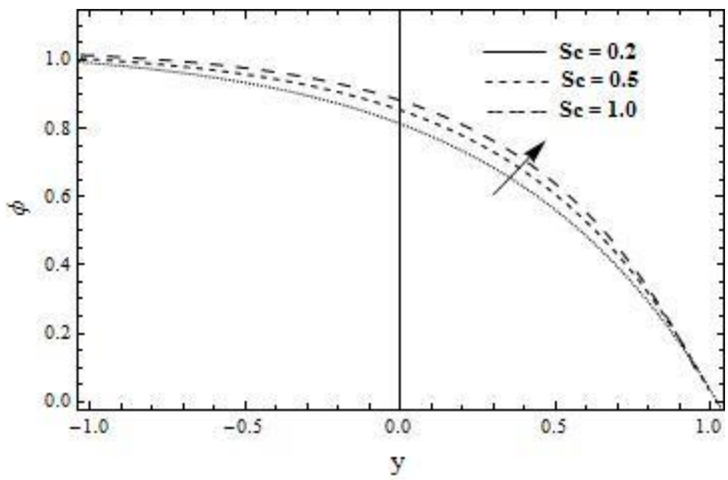

Figure.8: Concentration profile for different values of S_c with a = 0.1, b = 0.2, d = 1, $\phi = 0$

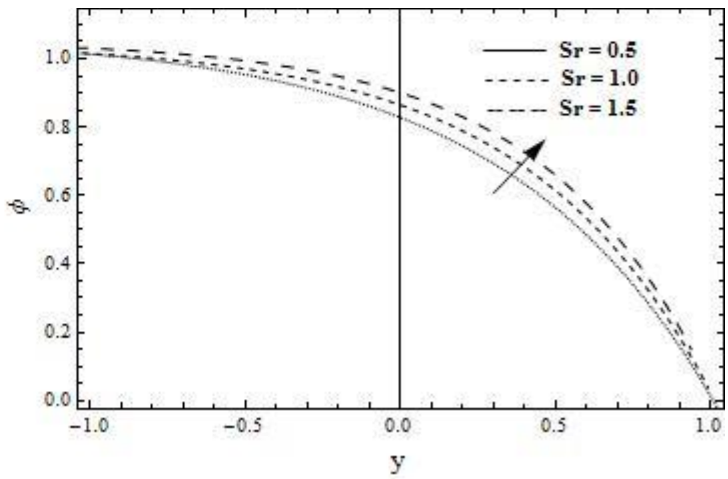

Figure.9: Concentration profile for different values of S_r with a = 0.1, b = 0.2, d = 1, $\phi = 0$

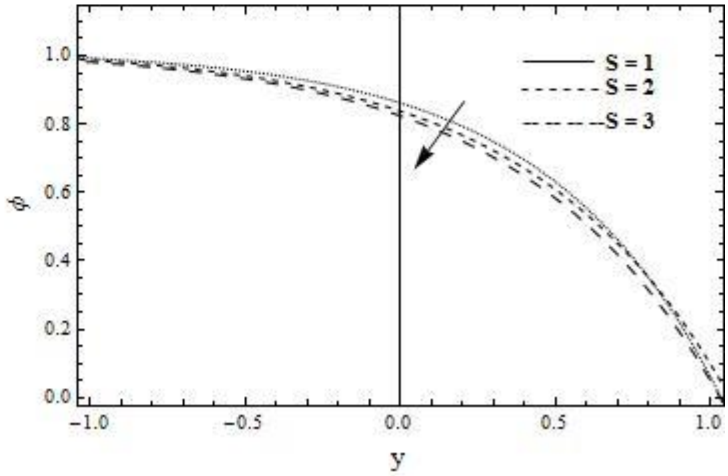

Figure.10: Concentration profile for different values of S with a = 0.1, b = 0.2, d = 1, $\phi = 0$

The variation of temperature and concentration distribution for different values of physical parameters such as heat source/sink parameter (Q), thermal radiation parameter (N_r), Prandtl number (P_r) and chemical reaction parameter S has been illustrated in Figures. (2) – (10). Figure.2 illustrates the temperature distribution for different values of Q, it is observed that the temperature distribution increases by increasing the heat source/sink parameter (Q) because both are directly proportional. From figures.3 and 4, it is observed that the temperature distribution increases by increasing the thermal radiation parameter N_r while an opposite trend is observed by Prandtl number P_r. Figure.5 shows the concentration distribution for different values of Q. It is observed that the heat source/sink parameter (Q) increases the concentration distribution. Figure. 6 shows that the concentration profile decreases by increasing the thermal radiation parameter N_r while an exactly opposite behaviour is observed in Figure. 7 with an increase in Prandtl number P_r. Figure. 8 shows that the concentration increases with an increase in the Schmidt's number S_c and Figure. 9 shows that the concentration increases with an increase in the Sorets number S_r

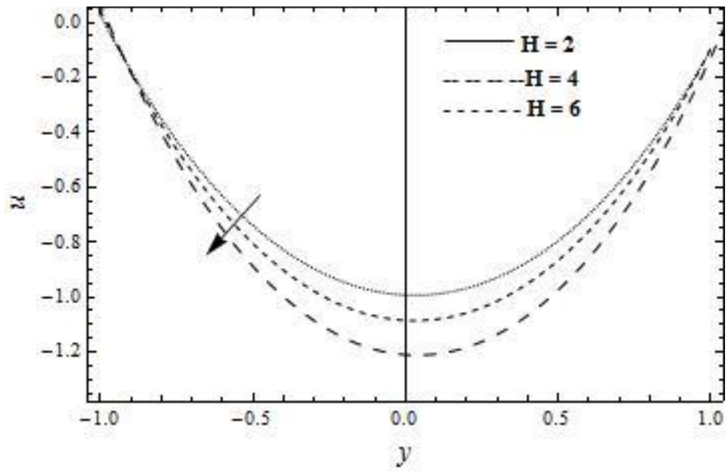

Figure.11: Velocity profile for different values of H with a = 0.1, b = 0.2, d = 1, λ_1= 0.5, $\phi = 0$

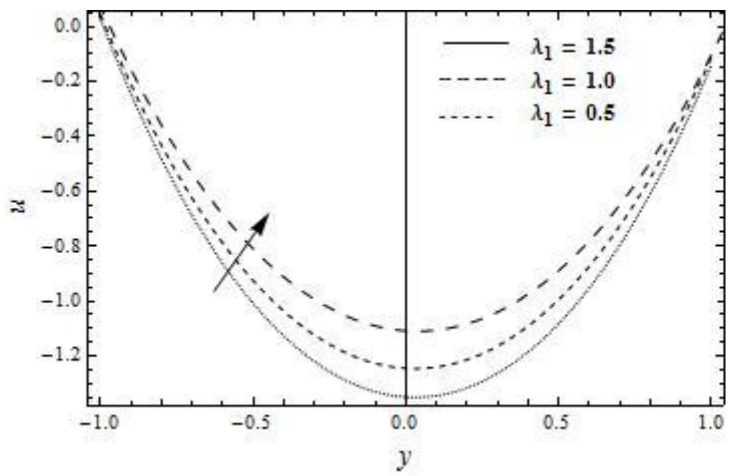

Figure.12: Velocity profile for different values of λ_1 with a = 0.1, b = 0.2, d = 1, H = 2.0, $\phi = 0$

Figures 11 and 12 represent the variation of axial velocity u for different values of the Hartmann number H and the Jeffrey fluid parameter λ_1. Figure 8 shows that the axial velocity decreases with increasing Hartmann number H. However the trend is reversed in the case of Jeffrey fluid parameter λ_1 as shown in Figure 12.

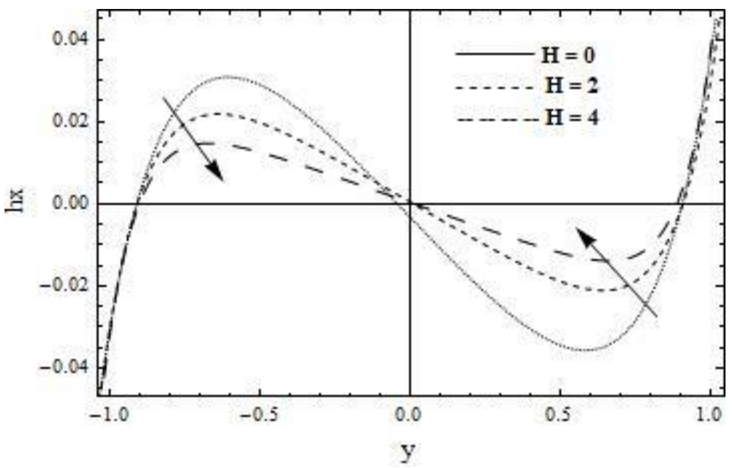

Figure.13: Axial induced magnetic field profile for different values of H with $\phi = 0$

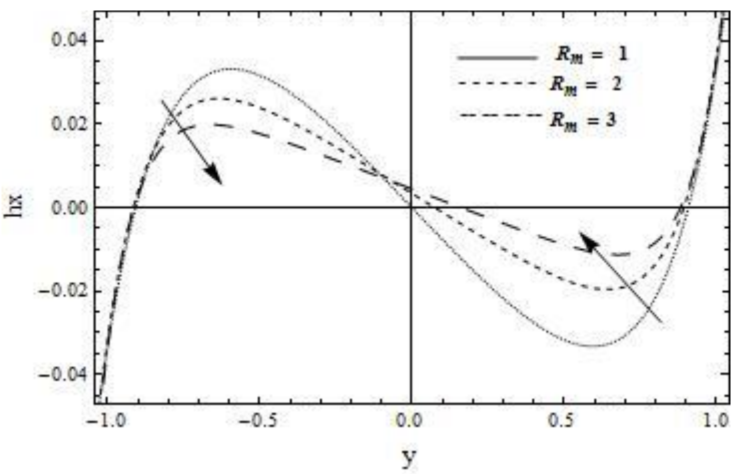

Figure.14: Axial induced magnetic field profile for different values of R_m with $\phi = 0$

The effect of induced magnetic field for the variations in Hartmann number H is illustrated in Figures. 13 and 14. It is observed that the magnitude of the axial induced magnetic field h_x decreases at the lower wall whereas induced magnetic field increases at the upper wall with the increasing values of Hartmann number H. The same trend is observed in Figure.14, with an increase in the magnetic Reynolds number R_m. Figures.15 and 16 show the variation of axial pressure gradient Δp along the length of the channel in one wavelength. From Figure.16 it is observed that both pressure rise and volume flow rate are inversely proportional to each other.

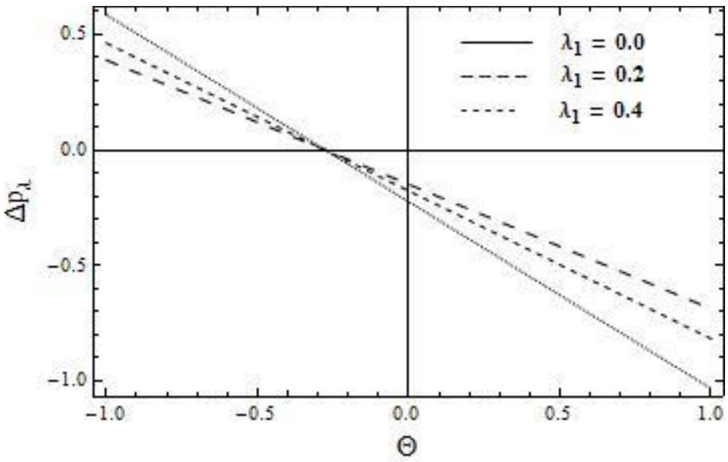

Figure.15: Pressure gradient profile for different values of λ_1 with H = 0.2, $\phi = 0$

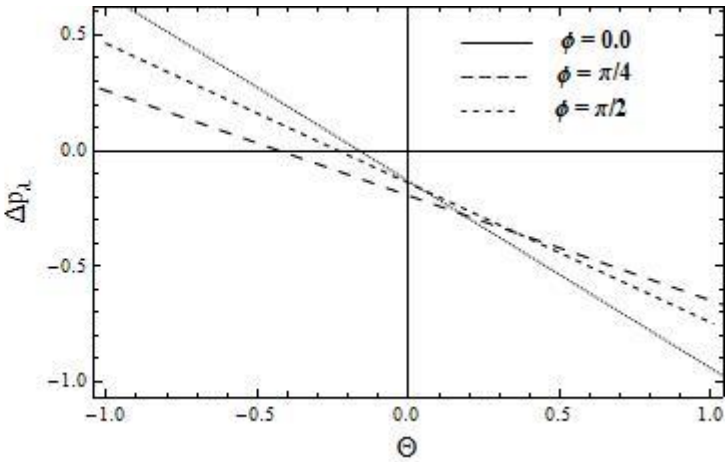

Figure.16: Pressure gradient profile for different values of ϕ with H = 2.0

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