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Basic Theory in the New Real Line-scale Rough Function Model

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Abstract. The basic concepts of Pawlak rough function model are improved. The concepts of double approximation operators that are scale upper (lower) approximation and real line upper (lower) approximation are defined and their properties and antithesis characteristics are analyzed. Scale bijection theorem as well as relative propositions and conclusions are proposed furthermore. Based on the indiscernibility relation, the new real line-scale rough function model is established by generalizing the double approximation operators into two-dimensional space. That deepens and generalizes rough function model based on rough set theory, and makes the scheme of rough function theory more distinct and completed. The transformation of real function analysis from real line to scale is achieved therefore, which provides necessary theoretical foundation and technical support for further discussion of properties and practical application of rough function model.

Keywords: Rough set, Scale, Indiscernibility relation, Rough number, Rough function

1. Introduction

The theory of rough sets [1], proposed by Polish mathematician Zdzislaw Pawlak in 1982, is a kind of data analysis theory. As an effective mathematical tool and technical means transacting indefinite and incomplete knowledge, rough set theory has been found to have quite successful applications in the fields of artificial intelligence such as machine learning, pattern recognition, decision analysis, process control and knowledge discovery, etc [2-4].

With more than twenty years development, deepness, perfection and generalization, rough set theory has become more and more mature and complete gradually. However, although many theoretical and practical problems relating to data analysis have been solved successfully in the scheme of rough set theory, due to the limitation of rough set theory being based on set theory, a large number of theoretical and applicable problems relating to function theory such as the synthesis and analysis of rough controllers, the generation and optimization of discrete dynamic system [5-6], etc., could not be described and solved only by the lower and upper approximate sets in rough set theory. Therefore, Pawlak generalize the concepts of rough sets to real numbers domain, and provide the rough function description in real numbers domain [7-10].

Taking the indiscernibility relation defined on real numbers set as the basic starting point, the concepts of lower rough (discrete) and upper rough (discrete) representations are defined in rough function model which is based on rough set theory. A series of discrete properties of the lower and upper representations corresponding to real functions are discussed to investigate the relationship between real and discrete functions, especially how does the discretization of real line influence basic properties of real functions, etc. There are some similarities between rough function model and numerical method as well as approximate method, e.g. approximation of one function by another one, but differences in nature between them exist, that is, rough function model is based on function theory in which functions are defined and valued in the set of integers, while numerical and approximate method are still based on real function theory, and are not related directly to discrete mathematics needed in fields such as computer simulation, etc. The applications of real functions are achieved by the establishment of rough function . Furthermore, on the basis of fault allowed theory and rough sets theory, if the rough functions of some real function is gained in advance, then the

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changing states of this real function can be depicted by analyzing the properties of rough functions[11].

Pawlak rough function model did not proposed the discrete forms in two metrics of real line and scale. The definition of rough function in it can not reflect the obvious characteristics of rough functions which are defining and valuing in integral sets. This kind of definition is not strict from mathematical point of view; while from applied point of view, rough functions in this kind of definition are not applicable for computer and rough control, etc. In this paper, the concepts of double approximation operators that are scale upper (lower) approximation and real line upper (lower) approximation are defined. Their properties and antithesis characteristics are analyzed. Scale bijection theorem as well as relative propositions and conclusions are proposed furthermore. Based on the indiscernibility relation, the new real line-scale rough function model is established by generalizing the double approximation operators into two-dimensional space. That deepens and generalizes rough function model based on rough set theory, and makes the scheme of rough function theory more distinct and completed. The transformation of real function analysis from real line to scale is achieved therefore, which provides necessary theoretical foundation and technical support for further discussion of properties and practical application of rough function model in such fields as the analysis of discrete dynamic system, the synthesis and analysis of rough controller, computer simulation, etc.

2. Real line-scale rough function model and its properties

2.1. Concepts and properties of rough sets on the real line

Physical phenomena are usually described by differential equations. Solutions of these equations are real valued functions, i.e., functions which are defined and valued on continuous points. However, due to limited accuracy of measurements and computations, we are unable to observe (measure) or compute (simulate) exactly the abstract solutions. Consequently, we deal with approximate rather than exact solutions, i.e., we are using discrete and not continuous variables and functions[7-10]. In order to investigate the relationship in nature between the above two approaches, document [10] proposed the concept of scale based on rough set theory.

Definition 1[10] Let $\left[n\right]=\left\{0,1,2,...,n\right\}$ be a finite set of integers, R R be the set of real numbers. If the strictly monotonic function $d:\left[n\right]\to R$ satisfies $:\forall i,j\in\left[n\right],i<j$, implies $d\left(i\right)<d\left(j\right)$, then d is referred to as a scale.

In practical applications, the finite integers set $\left[n\right]$ in definition 1 can be used as a set of measurement values with respect to a certain measurement unit such as kg, km, hr, etc. The scale d is a mapping of measurement values into the set of real numbers. Elements in the inverse image of the scale, i.e., measurement values can be understood as approximations of real numbers, in accessible due to our lack of infinite precision of measurement of computation. The concept of the scale is similar to that of the landmark[12-13] in qualitative reasoning methods, but both concepts are used differently.

Assume any scale $d:\left[n\right]\to R$ determine a finite sequence of reals $S=\left\{x_0,x_1,x_2,...x_n\right\},s.t.,x_0<x_1<...<x_n$, where $x_i=d\left(i\right),i\in\left[n\right]$. It is easy to prove that the scale satisfied the theorem as follows:

Theorem 1. (Scale bijection theorem) The scale d is a bijection of $\left[n\right]$ to R_n , where $d\left(i\right)\in R_n\in\left[x_0,x_n\right],i\in\left[n\right]$; The scale d is a bijection of $\left[n\right]$ to S , where $d\left(i\right)=x_i\in S,i\in\left[n\right]$.

There is a one-to-one correspondence between any scale d and a finite increasing sequence S according to theorem 1. d can be viewed as a discretization of the closed interval $R_n=\left[d\left(0\right),d\left(0\right)\right]=\left[x_0,x_n\right]$.

The following two approximate operators are defined corresponding to a given scale d. They map any real number in R into integers set and give two forms of mapping from reals to integers.

Definition 2. Given a scale $d:\left[n\right]\to R,\forall x\in R_n$, the d -scale lower approximation of $xd_*\left(x\right)$ and the d-scale upper approximation of $xd^*\left(x\right)$ are defined as follows:

$$d_*\left(x\right)=\max\left\{i\left|i\in\left[n\right],x_i\le x\right.\right\}$$

$$d^*(x) = \min\left\{ i \middle| i \in [n], x_i \ge x \right\}$$

$\delta_d(x) = d^*(x) - d_*(x)$ is referred to as d -scale approximate error of x .

Both of d^* and d_* are denoted by $*d$ for convenience.

The relationship between the scale d and the operator $*d$ can be obtained according to definition 1 and 2.

Proposition 1. $\forall i \in [n], d(i) \le x \le d(i+1) \Leftrightarrow i \le *d(x) \le i+1$

Proof: According to definition 1, $d(i) \le x \le d(i+1) \Leftrightarrow x_i \le x \le x_{i+1}$. According to definition 2, $x_i < x < x_{i+1} \Leftrightarrow d_*(x) = i, d^*(x) = i+1$; $x = x_i \Leftrightarrow *d(x) = i$; $x = x_{i+1} \Leftrightarrow *d(x) = i+1$. Thus, $d(i) \le x \le d(i+1) \Leftrightarrow i \le *d(x) \le i+1$. The proof of proposition 1 is completed.

Proposition1 shows that the real value and the scale value of x are correspondent. It is impossible for them to jump from one interval to another. Proposition 1 illustrate the one-to-one correspondence between $[n]$ and S from another aspect. See Fig.1.

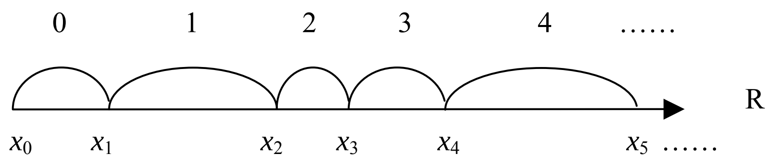

Fig. 1. The one-to-one correspondent relationship between $[n]$ and S

It is not difficult to obtain the following corollary in terms of proposition 1.

Corollary 1. $\delta_d(x)$ is equal to either 0 or 1, i.e.,

$$\delta_d(x) = \begin{cases} 0 & \text{When there exists } i \in [n] \text{ makes } x = x_i \\ 1 & \text{Otherwise} \end{cases}.$$

The indiscerbility relation on closed interval $R_n = [x_0, x_n]$ corresponding to d is defined as follows by $*d$.

Definition 3. [10] $\forall x \in R_n$, the indiscerbility relation I_d on closed R_n corresponding to d is defined as:

$$I_d = \{(x, y) | x, y \in R_n \wedge d_*(x) = d_*(y) \wedge d^*(x) = d^*(y)\} .$$

$\forall x \in R_n$, the indiscerbility relation I_d generates equivalence classes $[x]_d$ on R_n :

$$[x]_d = I_d(x) = \{y \in R_n | d_*(x) = d_*(y) \wedge d^*(x) = d^*(y)\}$$

It can be represented simplified as $[x]_d = I_d(x) = \left\{ y \in R_n \middle| *d(x) = *d(y) \right\}$.

$[x]_d$ constitutes a partition on R_n , which is denoted as $\pi_d(S)$. Thus all equivalence classes on R_n generated by I_d are:

$$\pi_d(S) = \{\{x_0\}, (x_0, x_1), \{x_1\}, (x_1, x_2), \{x_2\}, \Lambda , (x_{n-1}, x_n), \{x_n\}\}$$

The following proposition can be drawn from above:

Proposition 2. $\forall i \in [n]$, when $x_i < x < x_{i+1}, [x]_d = (x_i, x_{i+1})$; when $x = x_i, [x]_d = \{x_i\}$.

Definition 4. Given d and $S = \{x_0, x_1, x_2, ..., x_n\}, \forall x \in R_n$, let

$$I_{d*}(x) = \max\left\{ x_i \middle| x_i \in S, x_i \le x \right\}$$

$$I_d*(x) = \min\{x_i | x_i \in S, x_i \geq x\}$$

then $I_{d*}(x), I_d*(x)$ are referred to as -real line lower and upper approximation of x respectively. $\delta_{Id}(x) = I_d*(x) - I_{d*}(x)$ is referred to as d -real line approximate error of x.

Both of I_d* and I_{d*} are denoted by $*I_d$ for convenience.

Definition 5. $\forall x \in R_n$, if $I_{d*}(x) \neq I_d*(x)$, then x is referred to as a d -rough number. Otherwise, i.e., $I_{d*}(x) = I_d*(x)$, x is referred to as a d -exact number.

Denote $A^1 = (R, d), A^2 = (R, I_d), A^1$ and A^2 are referred to as scale and real line approximate space with respect to d. A^1 and A^2 are two kinds of different description of the same study object in different metrics and backgrounds, where both approximate operators $*d$ and $*I_d$ correspond to the scale d. According to the theorem of scale bijection (theorem 1), it is easy to observe that the two approximate operators have one-to-one correspondent antithetic characteristics, which is expressed as follows in detail.

Property 1. For $x, y \in R_n, xI_d y \Leftrightarrow *d(x) = *d(y) \Leftrightarrow *I_d(x) = *I_d(y)$.

Proof: The proof is not difficult to be obtained from definition 3 and proposition 2 , omitted. □

Property 2. $I_{d*}(x) = d(d_*(x)), I_d^*(x) = d(d^*(x))$.

Proof: By definition 2, we have $d_*(x) = \max\{i | i \in [n], x_i \leq x\}, d_*(x) \in [n]$. Taking operation d on $d_*(x)$, we can deduce: $d(d_*(x)) = \max\{x_i | x_i \in S, x_i \leq x\}$. Therefore, $I_{d*}(x) = d(d*(x))$ by definition 4.

$I_d^*(x) = d(d^*(x))$ can be verified in the same way.

Property 3. x is a d -rough number $\Leftrightarrow d_*(x) \neq d^*(x) \Leftrightarrow \delta_{Id}(x) \neq 0 \Leftrightarrow \delta_{Id}(x) = 1$.

Proof: It follows by property 2 that $d_*(x) \neq d^*(x) \Leftrightarrow d(d_*(x)) \neq d(d^*(x)) \Leftrightarrow I_{d*}(x) \neq I_d^*(x)$.By definition 4, we have $I_{d*}(x) \neq I_d^*(x) \Leftrightarrow \delta_{Id}(x) = I_d^*(x) - I_{d*}(x) \neq 0$ .By corollary 1, we can obtain $I_{d*}(x) \neq I_d^*(x) \Leftrightarrow x \neq x_i \forall i \in [n] \Leftrightarrow \delta_d(x) = 1$. Thus, the proof of property 3 can be completed.

Example 1. Let $x, y \in (x_3, x_4) \in \pi_d(S)$.It is clear that $x \ I_d \ y$, i.e., x and y is equivalent under the indiscerbility relation determined by d. While $d_*(x) = d_*(y) = 3, d^*(x) = d^*(y) = 4$. Meanwhile

$$I_{d*}(x) = d(d_*(x)) = x_3 = I_{d*}(y) = d(d_*(y)),$$
$$I_d^*(x) = d(d^*(x)) = x_4 = I_d^*(y) = d(d^*(y)),$$

i.e.,

$$I_{d*}(x) = I_{d*}(y) \neq I_d^*(x) = I_d^*(y),$$

or

$$d_*(x) = d_*(y) \neq d^*(x) = d^*(y),$$

or

$$\delta_{Id}(x) = \delta_{Id}(y) \neq 0,$$

or

$$\delta_d(x) = \delta_d(y) = 1.$$

Therefore, both real number x and y are rough numbers, and can be approximately represented by measurement value 3 (lower approximate) and 4 (upper approximation) of the scale d.

Any scale determines a partition on real line uniquely, determines an equivalence relation on reals set, i.e., indiscernibility relation. Elements in the same equivalence class are indiscernible with respect to the scale; they can be represented approximately as the measurement value of the scale. Hence, real parameters are substituted approximately by integral with respect to the given scale.

In term of the definitions and antithetic characteristics of approximate operators $*d$ and $*I_d$, their properties of theirs can be obtained as follows.

Proposition 3.

(1) $I_{d*}(x) \le x \le I_d^*(x)$;

(2) $x \le y \Rightarrow *I_d(x) \le *I_d(y)$;

(3) $*I_d(x \vee y) = *I_d(x) \vee *I_d(y)$;

(4) $*I_d(x \wedge y) = *I_d(x) \wedge *I_d(y)$;

(5) $*I_d(I_{d*}(x)) = I_{d*}(x)$;

(6) $*I_d(I_d^*(x)) = I_d^*(x)$;

Proof: (1) $\forall i \in [n]$, by proposition 2, when $x \in (x_i, x_{i+1}), I_{d*}(x) < x < I_d^*(x)$; when $x_i = x_{i+1}$, $*I_d(x) = \{x_i\}$. Therefore, $I_{d*}(x) \le x \le I_d^*(x)$ is hold.

(2) When *x*=*y*, it is obvious that the equality is hold.

When *x*≠*y*, various situations of *x*<*y* are discussed in details:

(a) When $x_i \le x < y < x_{i+1}$, $I_{d*}(x) = x_i = I_{d*}(y)$, $I_d^*(x) = x_{i+1} = I_d^*(y)$;

(b) When $x_i \le x < y = x_{i+1}$, $I_{d*}(x) = x_i < I_{d*}(y) = x_{i+1}$, $I_d^*(x) = x_{i+1} = I_d^*(y)$;

(c) When $x_i \le x < x_{i+1} \le y < x_{i+2}$, $I_{d*}(x) = x_i < I_{d*}(y) = x_{i+1}$, $I_d^*(x) = x_{i+1} < I_d^*(y) = x_{i+2}$;

(d)When $x_i \le x < x_{i+1}$, $x_j \le y < x_{j+1}$, $j > i+1$, $I_{d*}(x) = x_i < I_{d*}(y) = x_j$, $I_d^*(x) = x_{i+1} < I_d^*(y) = x_{j+1}$.

To sum up, when $x \le y$, we have $*I_d(x) \le *I_d(y)$.

(3) If *x*≤*y*, it is clear that $I_{d*}(x \vee y) = I_{d*}(y)$. Then by (2), we have $I_{d*}(x) \le I_{d*}(y)$, hence $I_{d*}(x) \vee I_{d*}(y) = I_{d*}(y)$. Thus we can see that $I_{d*}(x \vee y) = I_{d*}(x) \vee I_{d*}(y)$ .When *x*≥*y*, the conclusion can be verified similarly.

$I_d^*(x \vee y) = I_d^*(x) \vee I_d^*(y)$ can be proved in the same way.

(4) It is similar to the proof of (3).

(5) Let $I_{d*}(x) = x_i$ might as well, $i \in [n]$. By proposition 2, $*I_d(x_i) = x_i$, so $*I_d(I_{d*}(x)) = I_{d*}(x)$.

(6) It is similar to the proof of (5).

2.2. Real line-scale Rough function model

Given two scales $d:[n] \to R$ and $e:[m] \to R$. Let real function f be $f: R_n \to R_m$, where $R_n = [x_0, x_n]$, $R_m = [y_0, y_m]$. Generalizing approximate operators in definition 2 and definition 4 to two-dimensional space, the concepts of approximate operators of f in approximate space $B^1 = (R, e)$ and $B^2 = (R, I_e)$ are defined respectively.

Definition 6. The e-real line lower approximate $\overline{f}_*: R_n \to R_m$ of function f is defined as: $\overline{f}_*(x) = I_{e*}(f(x))$; The e-real line upper approximate $\overline{f}^*: R_n \to R_m$ of function f is defined as: $\overline{f}^*(x) = I_e^*(f(x))$. $\delta_{Ie}^f(x) = \overline{f}^*(x) - \overline{f}_*(x)$ is referred to as e-real line approximate error of f.

Both $\overline{f}_*$ and $\overline{f}^*$ are denoted by $*\overline{f}$ for convenience.

Definition 7. The e -scale lower approximate $f_*: [n] \to [m]$ of function f is defined as: $f_*(i) = e_*(f(d(i)))$; The e-scale lower approximate $f^*: [n] \to [m]$ of function f is defined as: f^* (i)=$e^*(f(d(i)))$. f_* and f^* are also called lower and upper rough representations of f. $\delta_e^f(i) = f^*(i) - f_*(i)$ is referred to as e-scale approximate error of f.

Both f_* and f^* are denoted by $*f$ for convenience.

For any real function, two discrete functions defined and valued in common reals set in B^1 are corresponded, while two discrete functions defined and valued in integers set in B^2 are corresponded. In the following, two forms of definitions of rough functions in different spaces are given, and establish the new real line-scale rough function model.

Definition 8. For $f: R_n \to R_m$, when $\overline{f}_*(x) \neq \overline{f}^*(x)$, f is referred to as a real line-rough function at point x, and is referred to as a rough function at point x in simplification. Otherwise, i.e., when $\overline{f}_*(x) = \overline{f}^*(x)$, f is referred to as an exact function at point x. In other words, if and only if $\delta_{Ie}^f(x) \neq 0$, f is a rough function at point x; Otherwise, f is an exact function at point x.

Definition 8′. For $f: R_n \to R_m$, when $f_*(i) \neq f^*(i)$, f is referred to as a scale-rough function at point i, and is referred to as a rough function at point i in simplification. Otherwise, i.e., when $f_*(i) = f^*(i)$, f is referred to as an exact function at point i. In other words, if and only if $\delta_e^f(i) = 1$, f is a rough function at point i; Otherwise, f is an exact function at point i.

Both approximate operators $*f$ and $*\overline{f}$ in approximate space $B^1 = (R, e)$ and $B^2 = (R, I_e)$ correspond to the scale $e: [m] \to R$. Assume $e: [m] \to R$ determines a finite increasing sequence of reals $S' = \{y_0, y_1, y_2, ..., y_n\}$, where $y_j = d(j)$, $j \in [m]$, then the scale e is a bijection from $[m]$ to S'. That is to say the scale *satisfies* the theorem of scale bijection (theorem 1). Therefore, the double approximate operators in approximate space B^1 and B^2 also nave one-to-one correspondent antithetic characteristics similar to property 1 to 3. Thus, definition 8 and 8′are equivalent and we have the following proposition:

Proposition 4. $\overline{f}_*(x) = e(e_*(f(x)))$, $\overline{f}^*(x) = e(e^*(f(x)))$.

Proof: By the definition of the scale lower approximation of reals, we have $e_*(f(x)) = \max\{j \mid j \in [m], y_j \le f(x)\}$, $e_*(f(x)) \in [m]$. Taking the operation e on $e_*(f(x))$, we can induce that $e(e_*(f(x))) = \max\{y_j \mid y_j \in S', y_j \le f(x)\}$. By the definition of the scale lower approximation of reals, we obtain $e(e_*(f(x))) = I_{e*}(f(x))$. Therefore, $\overline{f}_*(x) = I_{e*}(f(x)) = e(e_*(f(x)))$ is hold by definition 6.

$\overline{f}^*(x) = e(e_*(f(x)))$ can be verified in the same way.

The real line-scale rough functions also have a series of properties similar to proposition 3, which can be achieved only by transforming the variable there into function value, at the same time changing the scale on axis of ordinates into the scale e. They will not be spreading out here.

Definition 8 is the transition of definition 8′.It is the intermediate form of real function and the discrete function under scale metric. Both of them connect each other, and represent the discrete state and form of the real function under different metrics meanwhile. The real line-scale rough function model composed thus reflects this character. In fact, the approximate representations of real function f is placed into the set of lattice points (i.e. points having integer coordinates) in two-dimensional space determined by the scale d and e. Any real function f can be represented by two discrete functions defined and valued only on lattice points.

3. Conclusion

The new real line-scale rough function model is established by generalizing the Pawlak rough function model. The double approximate operations of real line-scale rough function model that are scale upper (lower) approximation and real line upper (lower) approximation have one-to-one correspondent antithetic characteristics. Any scale determines uniquely an equivalence relation on the set of real numbers, i.e., the indiscernibility relation. The elements in the same equivalence class can be represented approximately by the measurement value of the scale. Therefore, real parameters (or functions) can be represented approximately by integral parameters (or functions) under the given scale. The transformation of real function analysis from real line to the scale is thus achieved. The theory and practice of rough functions is a completely new research field. It has highly important study significance for practical applications in such fields as the analysis of discrete dynamic system, the synthesis and analysis of rough controller, computer simulation, etc.

Setting up the real line-scale rough function model, a series of concepts and properties such as rough continuous, rough derivatives, rough integrals, etc., which are similar to real functions can be given. The relationship between real functions and discrete functions which are viewed as measurement results can be studied in order to provide necessary theoretical foundation and technical support for further discussion of properties and practical application of rough function model. Due to the limitation of the length of the paper, further properties discussion and their applications of the real line-scale rough function model are arranged to be studied in the latter articles.

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