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Improved Method of the Four-Pole Parameters for Calculating Transmission Loss on Acoustics Silence

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Abstract. The boundary element method (BEM) is sufficient for calculating Transmission Loss in the acoustics silencer systems. BEM will be implemented by either the conventional four-pole transfer matrix method or the recently developed three-point method, but there are some short for each method. The three-point method is easier to use, and faster than four-pole methods. It does not produce the 4-pole parameters, the section being evaluated cannot be inter-linked with other sections. In order to perform such an evaluation, in this study, an improved method based on the four-pole parameters for use in the BEM is presented. The major advantage of the improved method is that it not only provides a very fast method for computing the TL, but it also produces the four-pole parameters.

Key Words: Silencer, Transmission Loss, Boundary Element Method, Numerical Analysis

1. Introduction

Despite the benefits of thermal comfort provided by air-conditioning systems, there is an increasing concern about the noise produced by air duct systems. There are basically two types of noise control methods used in air duct systems, passive control and active control methods. The passive noise control method usually takes two forms. One is using viscous dissipative materials in air duct system and the other one is using wave reflection by discontinuity of impedance, i.e. expansion muffler. In order to simplify the study of the performance of expansion muffler, most of past studies assumed the use of a rigid duct wall [1, 2]. In this way, it is known that BEM is sufficient for predicting the performance, in terms of Transmission Loss, of an expansion muffler with rigid walls numerically [3~5]. BEM will be implemented by either the conventional four-pole method or the recently developed three-point method, but there are some short for each method. The three-point method is easier to use, and faster than four-pole methods. It does not produce the four-pole parameters and, as such, the section being evaluated cannot be inter-linked with other sections [6]. In order to perform such an evaluation, in this paper, an improved method based on the four-pole parameters for use in the BEM is presented.

2. BEM Model for Acoustics System

The linear three dimensional wave equation for propagation of sound in acoustics cavity is given by [8]

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2}\frac{\partial^2 p}{\partial^2 t}(\vec{x},t) = \rho_0 \frac{\partial \psi}{\partial t} \tag{1}$$

where p is the acoustic pressure, c is the speed of wave, ρ_0 is the density of the medium, and ψ is the acoustic source power-flux per unit volume.

Take the Laplace transform for equation (1), assuming zero initial conditions, obtain the spatial Helmholtz equation with source flux per unit volume as

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$$\nabla^2 \overline{p}(\vec{x},s) + k^2 \overline{p}(\vec{x},s) = i\omega\rho_0 \overline{\psi}(\vec{x},s) \tag{2}$$

where ∇^2 is the Laplace Operator, $\overline{p}$ is the Laplace transformed acoustic pressure, $k = \dfrac{\omega}{c}$ is wave number, $i = \sqrt{-1}$.

Using Green integral theorem we can obtain the boundary integral equation for an acoustic cavity

$$C\overline{p}(\vec{y}_0,s) + \iint_s \overline{p}(\vec{x},s)\frac{\partial \overline{G}}{\partial n}(\vec{x},\vec{y}_0,s)ds(\vec{x}) = \iint_s \overline{G}(\vec{x},\vec{y}_0,s)\frac{\partial \overline{p}}{\partial n}(\vec{x},s)ds(\vec{x}) - s\rho_0 \sum_{m=1}^{M} Q_m \overline{G}(\vec{x}_m,\vec{y}_0,s) \tag{3}$$

where $\overline{G}(\vec{x},\vec{y},s) = \dfrac{1}{4\pi} e^{\frac{sr}{c}}, r = \sqrt{x^2+y^2}$.

In order to solve the system equation, equation (3), for acoustics pressure by using BEM the boundary should be discretized for numerical integration. If the boundary discretized into N elements, the discretized boundary integral equation is given as

$$[H](p) - [G](q) = (b) \tag{4}$$

where

$$\left[\hat{H}\right]^{(ie)} = \left[\int_0^1\int_0^{1-\xi_2} \frac{\partial \overline{G}}{\partial n}[\phi^{(1)},\phi^{(2)},\phi^{(3)}]Jd\xi_1 d\xi_2\right]^{(ie)},$$

$$[G]^{(ie)} = \left[\int_0^1\int_0^{1-\xi_2} \overline{G}[\phi^{(1)},\phi^{(2)},\phi^{(3)}]Jd\xi_1 d\xi_2\right]^{(ie)}, \; b_i = -s\rho_0 \sum_{m=1}^{M} Q_m \overline{G},$$

$$[H]^{(ie)} = \left[\hat{H}\right]^{(ie)} + C[\xi]^{(ie)}, \; q = \frac{\partial p}{\partial n}.$$

3. Improved Method of the Four- Pole Parameters

When a muffler is modeled by the boundary element method, the transmission loss can be evaluated by either the conventional four-pole method or the recently developed three-point method [8]. The three-point method produces only the transmission loss and nothing else. On the other hand, the four-pole method has the advantage of retaining the transfer matrix of the muffler, which contains important parameters when the muffler is connected to another muffler or other components in the exhaust system. However, the major drawback of the conventional four-pole method is that it requires two separate boundary element runs due to the two different boundary conditions imposed on the outlet boundary. Therefore, it can take twice as long to get the TL when compared to the more efficient three-point method. In this paper, an improved method to derive the four-pole parameters for use in the BEM is introduced.

A muffler with an inlet and an outlet can be represented by a linear acoustic four-pole network

$$\begin{bmatrix} p_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ u_2 \end{bmatrix} \tag{5}$$

where A, B, C and D are the four-pole parameters. Rearrange equation (5) to get

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6}$$

where $A^* = p_1\big|_{u_1=1,u_2=0}$, $B^* = p_1\big|_{u_1=0,u_2=1}$, $C^* = p_1\big|_{u_1=1,u_2=0}$, $D^* = p_1\big|_{u_1=0,u_2=1}$.

Two BEM runs are still needed to get the above four parameters. The first BEM run produces A^* and C^*, while the second BEM run produces B^* and D^*. Nevertheless, only one BEM matrix needs to be solved at each frequency, because two linear systems of equations of their BEM runs share the same coefficient matrix. The second BEM run uses only a different velocity condition, and therefore, requires only a trivial Backsubstitution stage. Actually, the two BEM runs can be done simultaneously because the two right side vectors for their linear system of equations corresponding to the two different velocity boundary conditions may be formed at the same time. Compared to the three-point method, this improved method is even faster because it does not require any field-point solution.

The original four-pole parameters in equation (5) can be obtained by solving equation (6) for p_1 and v_1 in terms of p_2 and v_2. Doing so yields

$$A = A^*/C^*, \quad B = B^* - A^*D^*/C^*$$
$$C = 1/C^*, \quad D = -D^*/C^* \tag{7}$$

With the four-pole parameters available, the TL can then be calculated by equation as [9]

$$TL = 20\log_{10}\left\{\frac{1}{2}\left|A + \frac{B}{z_0} + Cz_0 + D\right|\right\} \tag{8}$$

ALGORITHM

Step 1 Input the initial data of the duct, boundary condition, mesh element split, and different controls parameter;

Step 2 To discretize the numerical integration (1) and boundary condition by using a numerical integration method such as the Guassian quadrature formula;

Step 3 Calculate the sound pressure *p* and normal derivative of the sound pressure *v* via a Gauss elimination;

Step 4 Obtain four-pole parameters by solving equations (5~7) for $p_1$and v_1 in terms of p_2 and $v_{2,}$ calculating the transmission loss using equations (8);

Step 5 Output the plot of the frequency spectrum transmission loss, the four-pole parameters, the sound pressure *p*, etc.; End.

4. Numerical Experiment

The TL behavior of an expansion chamber silencer with a large diameter is investigated, as shown for Fig. 1.

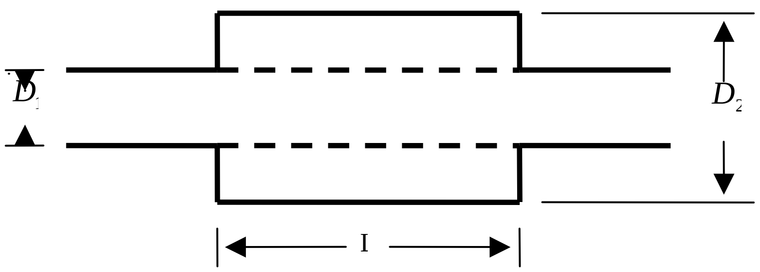

Fig. 1 A Simple Expansion Chamber Silencer with chamber length $L = 15.24cm$ and expansion ratio $m = S_2/S_1 = 22.22$, S_1 and S_2 are respectively cross-section of inlet(outlet) tube and the main chamber body.

All numerical experiments in this paper, the computer we used is Pentium®4 CPU 1.80GHz, and the operating system is Windows XP Home Edition. The tools of numerical computations are the C++ Programming Language and Matlab. We consider the range of frequencies about 0 ~ 1800 Hz. The theoretical TL curve can be calculated by [7]

$$TL = 10\log_{10}[1 + 0.25(m - \frac{1}{m})^2 \sin^2(kL)]$$
$$= 10\log_{10}[1 + 0.25(m - \frac{1}{m})^2 \sin^2(\frac{2\pi f}{c}L)] \tag{9}$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ is wave number, c is speed of sound.

The calculating TL traditional laboratory method base on plan wave theory, hereafter the traditional method referred to as plan wave. Assuming the shell of the silencer is rigid, a mesh of 196 BEM triangular elements is utilized. The predicted TL results by the combined BEM and improved four-pole method are shown in Fig. 2. The calculating TL values are compared with plan wave. We use Four-point Guassian quadrature formula for the integral of BEM.

At the low frequencies, $f \leq 250$ Hz, the TL predicated by the BEM and improved four-pole method agrees well with the plan wave theory interior of the expansion chamber.

However at the higher frequencies, $f > 250$ Hz, the wave transmission in the expansion chamber is not a plane wave but become more complex. In fact, we know that there is a primacy portrait acoustics modular stimulant at 1125Hz and primacy radial acoustics modular stimulant at 1360Hz [8]. Therefore the plane wave theory is not valid for frequencies above some cutoff frequency. Therefore the plane wave theory is not valid for frequencies above some cutoff frequency.

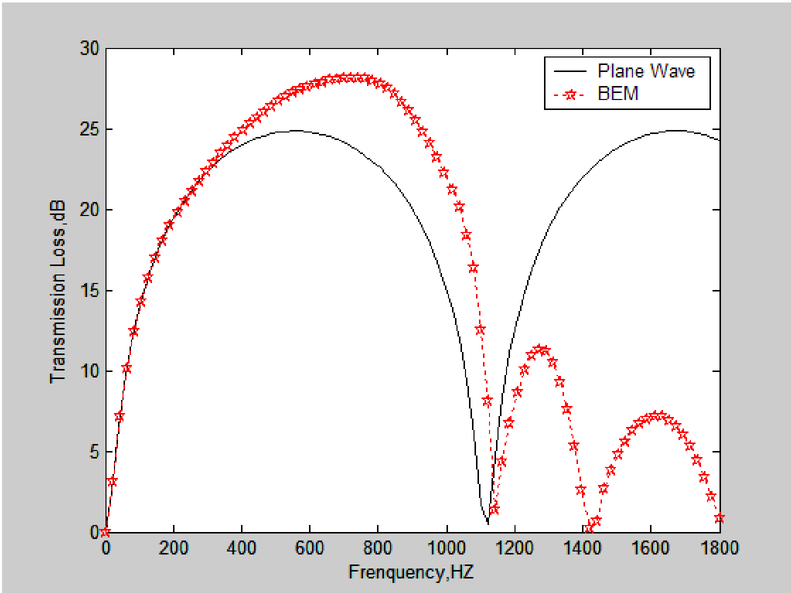

Fig. 2 The TL values for a muffler with rigid walls

On the other hand, to demonstrate the efficiency of the improved method for deriving the four-pole parameters, the CPU time comparison of the three different TL methods at three individual frequencies is shown. The second muffler model is used as the test case. Since the muffler has a plane of symmetry, one only models one half of the geometry. At 200 Hz, 600 Hz and 1200 Hz, the number of elements is 92, 133 and 196, respectively. The CPU time in Seconds comparison is shown in Table 1.

Table 1 Comparison of computation time in seconds on three Methods of Calculating TL

Number of elements	Four-pole Method	Three-point Method	Improved Four-pole
92	36	25	25
133	89	62	62
196	327	231	229

From the Table 1, we can see that the improved four-pole method is even a little faster than the three-point method for the 196 elements model. Since the entire integral equation is not reformulated for the second BEM run even in the conventional four-pole method the CPU time of the conventional four-pole method is not exactly twice of the three-point method or the improved four-pole method. However, when the frequency goes up and the size of the matrix becomes bigger and bigger, solving the matrix will dominate the entire process. Then, the CPU time of either the three-point method or the improved four-pole method will eventually reach 50% of the conventional four-pole method.

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