7659, England, UK
Journal of Information and Computing Science
Vol. 2, No. 1, 2007, *pp*. 71-76

The Unconventional Boundary Value Problems and its Superposition Solution

Feng Tian, Xia Li, JianShi Li

College of Computer Science and Engineering, Guizhou University, China, 550025

(*Received August 31, 2006, Accepted November 2, 2006*)

Abstract. This paper presents the unconventional boundary-value problems and it's superposition solution for the physical geometric nonlinear plane beam system .firstly, this paper takes AK type microswitchs as example and builds the model of the installment process and working process. Secondly, this paper builds a general mathematic model and points out the superposition solution for the unconventional boundary value problems. Lastly, it presents the computing example for the static process of the AK type microswitchs .

Keywords: unconventional problem, superposition solution, computing example

1. Introduction

The conventional boundary-value problems are saluted displacement by known load. Based on low-pressure electric switch, [1] firstly point out the concept "unconventional"; [2, 3] applied this concept into microswitch CAD and the analysis of dynamic-static switching process; [4, 5, 6] further applied it into the precise mechano-electronic components, electric-contact process analysis and built a dynamic model; [7] again use the three design thinking into the design of microswitch. Based on [1]−[7] studies, this paper take dispersed precise mechano-electronic components into geometry and physic nonlinear plane beam system.

The remainder of the paper is organized as follows, section 2 contains the finite element model of the physical and geometric nonlinear plane beam system; section 3 constructs the unconventional finite element model as the AK type microswitch's static process; section 4 presents the general unconventional model. Section 5 presents the superposition solution of unconventional problems; section 6 gives a typical example about AK type microswitch's static process; section 7 present the conclusion.

2. The conventional finite element model formulas of nonlinear plane beam system

In order to save space, we have left out the detailed derivation process and given only the finite element formulas of the element increment load of the physical and geometric collinear plane beam:

$$\left({}_{t}^{t}\mathrm{K}_{L}+{}_{t}^{t}\mathrm{K}_{NL}\right)U^{(i+1)}=P+{}^{t}r+{}^{t+\Delta t}r_{w}^{(i)} \tag{1}$$

Where

$${}_{t}^{t}K_{L}=\begin{pmatrix} C_1 & 0 & 0 & -C_1 & 0 & 0 \\ & C_2 & C_3 & 0 & -C_2 & C_4 \\ & & C_5 & 0 & -C_3 & C_6 \\ & & & C_1 & 0 & 0 \\ & & & & C_2 & -C_4 \\ & & & & & C_7 \end{pmatrix} \text{ Linear element stiffness matrix at t moment.} \tag{2}$$

$${}_{t}^{t}K_{NL}=\frac{{}^{t}p_{x}}{L}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & C_8 & C_9 & 0 & -C_8 & C_9 \\ & & C_{10} & 0 & -C_9 & C_{10}/4 \\ & & & 0 & 0 & 0 \\ & & & & C_8 & -C_9 \\ & & & & & C_{10} \end{pmatrix} \text{ Geometric nonlinear element stiffness matrix at t} \tag{3}$$

$C_1=\frac{E}{2}(\frac{h}{L})(b_1+b_2)$, $C_2=\frac{E}{2}(\frac{h}{L})^3(b_1+b_2)$, $C_3=\frac{E}{6}(\frac{h}{L})^3(2b_1+b_2)$, $C_4=\frac{E}{6}(\frac{h}{L})^3 L(b_1+2b_2)$,

$$C_5=\frac{E}{12}(\frac{h}{L})^3L^2(3b_1+b_2),\ C_6=\frac{E}{12}(\frac{h}{L})^3L^2(b_1+b_2),\ C_7=\frac{E}{12}(\frac{h}{L})^3L^2(b_1+3b_2),$$

$$C_8=\frac{6}{5},\ C_9=\frac{L}{10},\ C_{10}=\frac{2L^2}{15}.$$

h, b_1, b2 and L are the thickness, left-hand width, right-hand width and length of element pole respectively.

$$U=\begin{pmatrix}u_1 & \omega_1 & \theta_1 & u_2 & \omega_2 & \theta_2\end{pmatrix}^T \text{ Element node displacement increment} \quad (4)$$

$$P=\begin{pmatrix}p_{x1} & p_{y1} & M_1 & p_{x2} & p_{y2} & M_2\end{pmatrix} \text{ Element node force increment} \quad (5)$$

$${}^tr=\left({}^tp_{x1}+{}^tp_x,{}^tp_{y1}+\frac{{}^tm_2-{}^tm_1}{L},{}^tM_1-{}^tm_1,{}^tp_{x2}-{}^tp_x,{}^tp_{y2}-\frac{{}^tm_2-{}^tm_1}{L},{}^tM_2-{}^tm_2\right)^T \quad (6)$$

${}^tp_x$ denoting element axial force, ${}^tm_i$ i=1,2 denoting element pole-end internal bending moment $r_\omega^{(0)}=0$,

$$r_\omega^{(i)}==\left(0,\frac{m_{\omega2}^{(i)}-m_{\omega1}^{(i)}}{L},m_{\omega1}^{(i)},0,\frac{m_{\omega2}^{(i)}-m_{\omega1}^{(i)}}{L},m_{\omega2}^{(i)}\right)^T \text{ physical nonlinear correction load} \quad (7)$$

$m_\omega^{(x)}=m_E^{(x)}-m_{(x)}$ (Ref. 〔2〕) $m_E^{(x)}$ denoting the nominal elastic bending moment of the cross-section, $m_{(x)}$ denoting the plastic correction bending moment, $m_{\omega k}^{(i)}$ k=1,2 denoting element pole-end plastic correction bending moment, (i) being the i-time equilibrium increment within that increment load step at t+⊿t moment.

$$m_{\omega k}^{(i)}=m_e(1-\frac{E'}{E})\left[\frac{1}{Z_k^{(i)}}-\frac{3}{2}+\frac{Z_k^{2(i)}}{2}\right] \quad (8)$$

$m_e=\frac{bh^2}{6\sigma_s}$ elastic bending limit of the rectangular cross-section, being the material elastic limit.

$$Z_k^{(i)}=\begin{cases}\frac{m_e}{m_{kE}^{(i)}} & \text{If } {}^{t+\Delta t}m_{kE}^{(i)}>m_e, \text{ and } {}^{t+\Delta t}m_{kE}^{(i)}>{}^tm_k \text{ (plastic increment load)}\\ 1 & \end{cases} \quad (9)$$

$Z_k^{(i)}$ denoting the plastic degree at the element pole-end in the i-time equilibrium iteration within the increment load step at t+⊿t moment. ${}^{t+\Delta t}m_{KE}^{(i)}={}^tm_K+m_{KE}^{(i)}$ denoting the nominal elastic bending moment at the element pole-end in the i-time equilibrium iteration at t+⊿t moment. (10)

${}^{t+\Delta t}m_K^{(i)}={}^tm_K+m_{KE}^{(i)}-m_{K\omega}^{(i)}$ denoting the pole-end internal bending moment at the element in the i-time equilibrium iteration at t+⊿t moment. (11)

${}^{t+\Delta t}\sigma_{xE}^{(i)}=\frac{{}^{t+\Delta t}P_x}{A}+\frac{Y}{I}\left[1-\frac{x}{L},\frac{x}{L}\right]\begin{bmatrix}{}^{t+\Delta t}m_{1E}^{(i)}\\ {}^{t+\Delta t}m_{2E}^{(i)}\end{bmatrix}$ denoting the element nominal stress in the i-time equilibrium iteration at t+⊿t moment. (12)

$\sigma_{x\omega}^{(i)}=\frac{Y}{I}\left[1-\frac{x}{L},\frac{x}{L}\right]\begin{bmatrix}m_{\omega1}^{(i)}\\ m_{\omega2}^{(i)}\end{bmatrix}$ denoting the element plastic correction stress in the i-time iteration at t+⊿t moment. (13)

${}^{t+\Delta t}\sigma_x^{(i)}={}^{t+\Delta t}\sigma_{xE}^{(i)}-\sigma_{x\omega}^{(i)}$ denoting the element stress in the i-time iteration at t+⊿t moment. (14)

3. Example: Building the ‘unconventional’ finite element model of AK type microswitch’s static process

3.1 Computing chart

The switch, in reference to its structure and its increment load feature, can be reduced to a place beam system, of which node 15 is permitted limited horizontal and vertical displacements X and Y. Touch point 11 permitted a limited bouncing amplitude $h_1 - h_2$. Because Switch is made of different compositions of the variables X, Y, h1, h2, b (the ratio between the side-stripe and the mid-stripe in the leaf spring with to the width) and t (the thickness of the leaf spring), it will be unavoidable plastic yielding in some of them. To secure the elastic element feature of the leaf spring, we must not allow serious yielding to take place. Therefore the leaf spring is supposed to be made of linear hardened material of which the axial force is about 3% of P_e .The yielding is mainly due to the bending moment. The aforesaid finite element formulas can directly be applied to it. The simple diagram of computation is shown in fig.1.

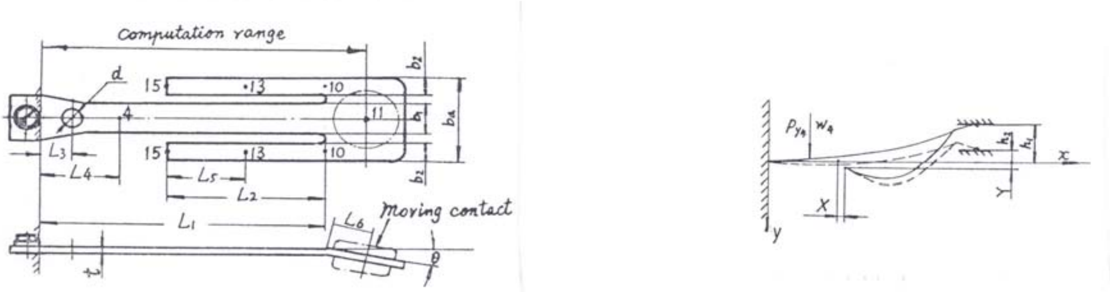


Fig1 Simple diagram of computation

3.2 Seeking solution model of the installment process

In order to accurately analogize the complex nonlinear mechanics features, we divide the boundary-value problem into two stages, installation and work.

In the stage of installation, we break the installation process into stages of solution, of which the corresponding boundary-value problems are listed in Table 1, where the equation given is the over-all equation under the global coordinates, $K={}_t^tK_L + {}_t^tK_{NL}$, $\gamma={}^t\gamma$, e_{xi} or e_{yi} denoting the unit vector action on Node i. The unknown load is expressed by an unknown constant C multiplied a unit vector. So the value of C obtained means the unknown load obtained. Installment process of getting solution model is showed in table 1.

3.3 Seeking solution model of the working process

At the beginning of the work, a pressure P_{y4} is brought to bear on Node 4 and the touch-point pressure $CP_{O\ is}$ gradually diminished to zero. The upper unilateral constraint is relieved and the switch is in a critical state before bounding. Its corresponding work pressure is AF and the corresponding W_4 is the action distance PT. When the switch is bouncing positive, its bouncing ways is not one. In this paper we have chosen three different bouncing ways, which are expressed as i, ii, iii in Table 2. This stage is written as E_6, the relieving force $RF = 6p_{y4}$, the differential distance MD=$6_{W_4} - 5_{W_4}$ (the i-type bounding way).

After the touch point bounces to the lower touch point h_2, an over distance OT is applied to Node 4. Node 11 is constrained unilaterally from below only, yet it cannot possibly move upward. During this stage (E_7), we can consider it as being constrained by the Y-direction displacement and thereby incurs the followings: a lower touch-point pressure CP_1= ${}^7P_{y11}$, the total work pressure TF= ${}^7P_{y4}$, the maximum distributed stress being ${}^7\sigma$ whose maximum value is written as $\sigma_{\max}$. The ${}^7P_{y4}$ is discharged until CP_1=0.The unilateral constraint from below is relieved. The switch bounces back to h1, P_{y4} being finally discharged to zero and the switch returns to the installed state. This process can be divided into three stages, namely, E_8, E_9, and E10. $E_4 \rightarrow E_{10}$ corresponds to a stress cycle.

In order to make a quantitative study of the life expectancy, we define:

$$\text{The mean cyclical stress } S_m = ({}^7\sigma + {}^4\sigma)/2 \quad (15)$$

$$\text{The cyclical stress amplitude } S_a = {}^7\sigma - {}^4\sigma \quad (16)$$

$$\text{The cyclical stress } \overline{S} = \sqrt{S_a(S_a + S_m)} \quad (17)$$

By comparing the value of S_a at various points, we can obtain S_a max and the corresponding $\overline{S}_{\max}$. In the V-chapter of this paper we shall derive the relationship of $\overline{S}_{\max}$ ~N, which can be used to make a quantitative computation of the fatigue life. The working process solution model is shown in table 2.

Table1. Installation stages and corresponding boundary-value problems (Being the proportional increment load factor)

Stage	Controlling conditions	Solution Equations	Main Items to be obtained	Constraint conditions
E1	To node 15 is applied the known isplacement Y	$\begin{cases} KU^{(i+1)} = ce_{r15} + r + r_w^{(i)} \\ W_{15} = \lambda Y \end{cases}$	P_{y15}	Node 15 is a moveable hinge joint and node 11 free
E2	When the value of Y keeps the value, the node 15 is applied the X-direction displacement	$\begin{cases} KU^{(i+1)} = ce_{r15} + r + r_w^{(i)} \\ U_{15} = \lambda X \end{cases}$	P_{x15} , W_{11} $P_{x15} \sim U_{15}$ curve $P_{x15} \sim W_{11}$ curve	Node 15 is constrained in the Y-direction, but is a movable hinge joint in the Z-direction. 11 is free
E3	To node 13 is applied the installation pressure P_{y13} to lower 11 to h1	$\begin{cases} KU^{(i+1)} = ce_{r13} + r + r_w^{(i)} \\ W_{11} = \lambda({}^2W_{11} - h_1 - {}^0W_{11}) \end{cases}$	$P_{y13}, W_{13}, P_{y13} \sim W_{13}$ curve, $P_{y13} \sim W_{11}$ curve	Node 15 is a fixed hinge joint and node is free
E4	P_{y13} at node 13 is discharged to zero. Node 11 is forced upon the upper touch h_1	$\begin{cases} KU^{(i+1)} = ce_{y11} - \lambda\,{}^3P_{Y13} \\ \quad + r + r_w^{(i)} \\ W_{11} = 0 \end{cases}$	$P_{y11}, CP_0 = {}^4P_{y11}$ (The upper-touch-point reassure), ${}^4\sigma$ (the installation stress)	Node 15 is a fixed hinge joint, node 11 is constrained in the Y-direction

Table2. Work process and its boundary-value problems (Node 15 being a fixed hinge point)

Stage		Controlling conditions	Solution Equations	Main Items to be obtained	Constraints
E5		Node 11 remains at h1.py4 is being applied to make $CP_0 = 0$	$\begin{cases} KU^{(i+1)} = ce_{r15} + r + r_w^{(i)} \\ W_{15} = \lambda Y \end{cases}$ $\begin{cases} KU^{(i+1)} = ce_{y4} + r + r_w^{(i)} \\ W_{11} = 0 \end{cases}$	$P_{y4} - W_4 curve$ ${}^5P_{y4} = AF$ ${}^5W_4 = PT$	Node 11 is constrained in the y-direction to the end of this stage, the constrained is relieved
E6	I	Node 11 is free. P_{y4} is still applied to it. Node 11 goes from $h_1 \sim h_2$	$\begin{cases} KU^{(i+1)} = ce_{y4} + r + r_w^{(i)} \\ W_{11} = \lambda(h_1 - h_2) \end{cases}$	P_{y4} : W_4 curve ${}^6P_{y4} = RF$ ${}^6W_4 - {}^5W_4 = MD$	Node 11 is free.
	II	Displacement increment at node 4 is zero. P_{y4} is being still applied and node 11 passing from $h_1 \to h_2$	$\begin{cases} KU^{(i+1)} = c_1e_{y4} + c_2e_{y11} + r + r_w^{(i)} \\ W_{11} = \lambda(h_1 - h_2) \\ W_4 = 0 \end{cases}$	$P_{y4} - W_4$ curve ${}^6P_{y4} = RF$ $MD = 0$	Node 4 is constrained in the y direction. Node 11 is free.
	III	P_{y4} at node 4 remains unchanged, node 11 is passing from $h_1 \sim h_2$	$\begin{cases} KU^{(i+1)} = ce_{y11} + r + r_w^{(i)} \\ W_{11} = \lambda(h_1 - h_2) \end{cases}$	P_{y11}, W_4	Node 11 is free.
E7		Node 11 remains at h_2 . An over distance OT is being applied to Node 4.	$\begin{cases} KU^{(i+1)} = ce_{y11} + r + r_w^{(i)} \\ W_4 = \lambda OT \\ W_{11} = 0 \end{cases}$	P_{y4} : $W_4 curve$ P_{y4} : $W_{11} curve$ ${}^7P_{y11} = cP_1$, ${}^7P_{y4} = TF$	Node 11 is constrained in the y-direction
$E_8 \sim E_{10}$		They are the reverse process of E_7, E_6 and E_5 , and ${}^8P_{y4} = RF$, ${}^8W_4 - {}^5W_4 = MD$ From $E_5 \sim E_{10}$ can be derived the curves of $P_{y4} \sim W_4$, $P_{y11} \sim W_4$, $P_{y4} \sim W_{11}$, etc.			

4. Building model of a general unconventional problem

In the boundary-value problems listed in table I, II, the items on the right-hand side contain unknown load factors, whose number is the same as that of known displacements is the controlling condition. This kind of problem is to reverse obtain load from the known displacements. Thus we call it from the conventional problem, which is to obtain the unknown displacements from the known load items. The 'unconventional' boundary-value problems listed in table 1, 2 can be generalized as the following math form ($\gamma, \gamma_w^{(i)}$ being include in p_0):

$$KU^{(i+1)} = P_0 + \sum_{k=1}^{m} C_k P_k \tag{18}$$

$$BU^{(i+1)} = C \tag{19}$$

Where the known quantities are matrix K_{nxn}, B_{mxn}, column vectors $P_{kn\times1}$, k=0,1,2…m, $C_{m\times1}$ the unknown quantities, the unknown quantities column vector $U_{n\times1}$, and the scalar quantity C_k, k=1,2…m and m<<n.

5. Superposition solution for a general unconventional problem

First of all, we should solve m+1 n-order liner equation series:

$$KU^{0(i+1)} = P_0^{(i)} \tag{20}$$

$$KU^{k} = P_k \qquad k = 0,1,2\text{L}\ \ m \tag{21}$$

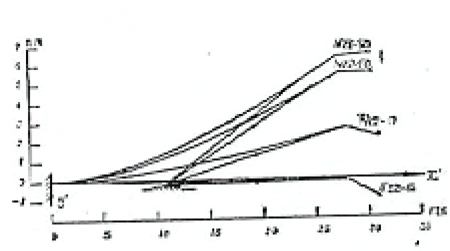
Then, according to the principle of super-position, from can be interpreted as:

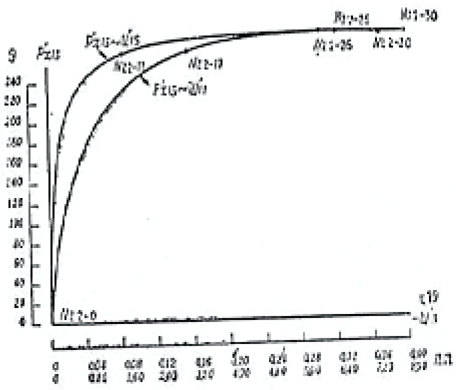
$$U^{(i+1)} = U^{o(i+1)} + \sum_{k=1}^{m} C_k U^{k} \tag{22}$$

Finally, by substituting equation (22) Into equation (19), we can derive the m-order equation series about C_k .in solving for the unknown factor C_k. We can obtain the unknown load, and again by substituting the result into equation (22) we can obtain the unknown displacement. The superposition method used in solving 'unconventional' problems, as compared with that used solving the conventional problems, has only added m-times reverse substitutions and m-order equation series about C_k , maintain meanwhile the fine characteristics of matrix K is definite-positive, symmetric, scattered and belt-like.

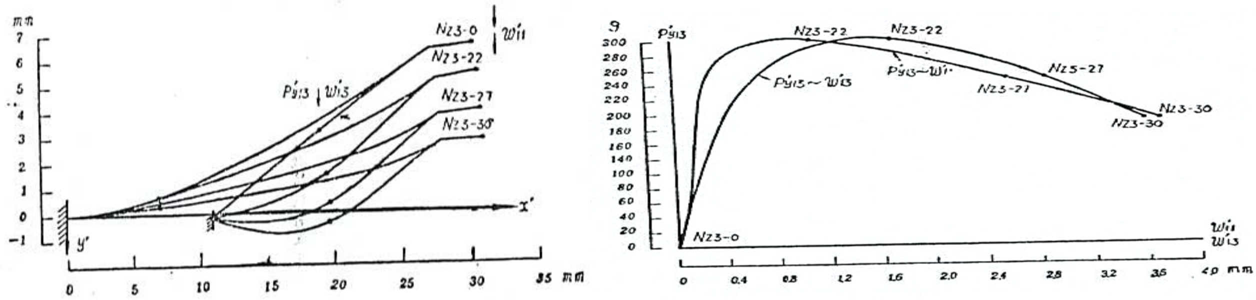
6. Typical computing result

According to the model, we can get the computing result for the AK type microswitch's static process from the superposition solution in section 4. The result can be showed in figure 2 and table 3.

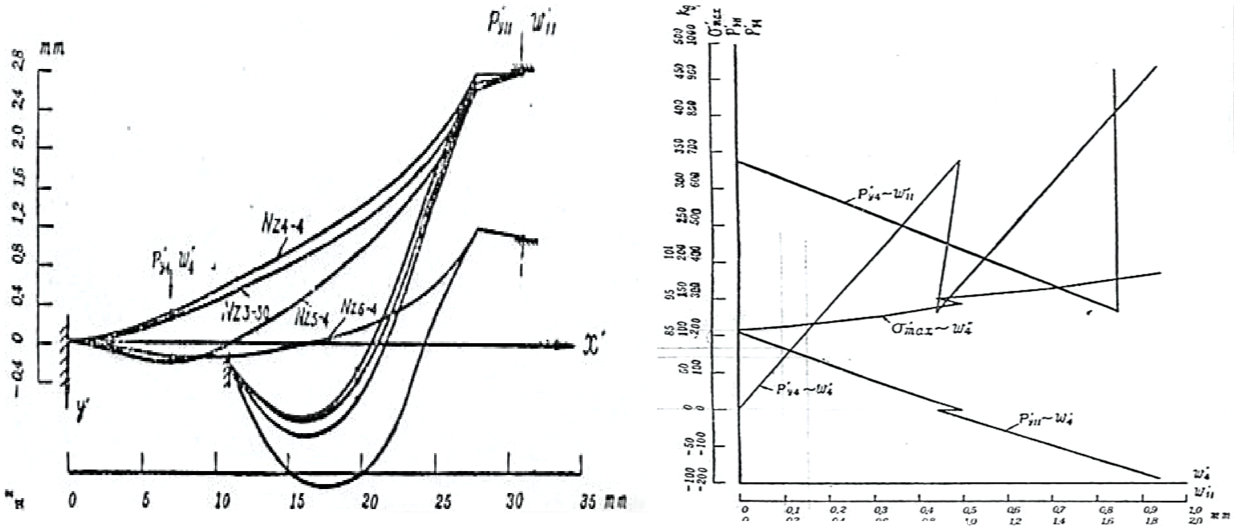




A E_2*stage*



B E_3*stage*



C work process $E_5 \to E_6 \to E_7 \to E_8 \to E_9 \to E_{10}$

Fig 2 Diagrams of the load-displacement curves and deformation

Table 3A Variant's areas

Level	z	y	h1	h2	b	t
Lowe	0.30	0.10	2.40	1.10	0.70	0.17
Upper	0.40	0.20	2.80	1.70	0.90	0.21
Zero	0.35	0.15	1.40	1.40	0.80	0.19

Table 3AB The Computing and Test's Result (Zero-Level)

Parameters / Values	AF	RF	PT	MD	CP_0
Computing	505	264	0.49	0.05	81
Test	520	254	0.51	0.06	79
Error %	2.97	-3.79	-4.07	20.0	-2.47

Table 3C The Mean error of 32 groups computing and test's results

Parameters / Mean error	AF	RF	PT	MD	CP_0
%	1.3	4.0	-9.5	30.0	4.6

7. Conclusion

In this paper, we have given one kind of unconventional model and its superposition solutions. The computation's real example for the static process of AK type microswitch and the corresponding test's result are given as well. From the experimental result, a constructed model and its solution are stable and feasible . We can point out its stable degree and precision meet the requirement in practice.

8. Reference

[1] J. Li. Cad for the Mechanical Properties of the Switches for Low-Voltage Electrical Appliances and its Applications. Computational Structural Mechanics and Applications. 1985, **2** (1).

[2] Z. Yang and J. Li. The Cad for the Microswitch. Caddm, Beijing, 1987, 574-581.

[3] J. Li. Analysis of the Static and Dynamic of Microswitch's Transfer Process. Ic-Cemca'86, 251-261.

[4] J. Li, W. Ni, L. Jiao. An Application of the Finite Element Model of the Thermal Process of Contacts in Precise Electrical Components. Icecm'92, 81-84.

[5] J. Li, The Application of the Finite Element Model of Electric-Contract Thermal Process to Precise Mechano-Electronic Components. Applied Mathematics-a Journal of Chinese Universities. 1995 **10A**(1).

[6] J. Li, T. Chen, Z. Yang. The Syntheized Model and Computer Simulation About Static-Dynamic-Electrical Contract Process of Microswitch. Chinese Journal of Computational Physics. 2001 **13**(4).

[7] Z. Yn, J. Li, W. Ni. A Three-Stage Design for Ak-Type Microswitch (In Steady Case). Icecm'92, 95-98.