7659, England, UK
Journal of Information and Computing Science
Vol. 2, No. 2, 2007, *pp.* 137-144

Analysis of active PVDF-paper/fabric speaker for Active Noise Control

Tilak Dias, Ravindra Monaragala and Manuchehr Soleimani

William Lee Innovation Centre, School of Materials, The University of Manchester, Manchester, UK

(*Received 4 October 2006, accepted 4 February 2007*)

Abstract. Knitted fabrics can be made to vibrate as a speaker by laminating it with Polyvinylidene fluoride (PVDF) strips to form curved arch laminated at the ends. The PVDF strip would vibrate due to the piezo electric effect when a 200 Vrms voltage is applied.A two-dimensional analytical prediction model is created based on the principles of stress and strain behaviour of a curved laminated arch. The model predicts the radial displacement of the arch with respect to the frequency of the excitation voltage of the PVDF strips. The Sound Propagation Level (SPL) at a particular point from the arch surface is modelled as a simple sound source in an infinite baffle. The acoustic pressure is linked with the radial displacement in the model. Simulation is carried out on structural properties of the fabric to discern the resonance region of the speaker vital for Active Noise Control to reduce automotive interior noise.

1. Introduction

Automotive interior noise may be undesirable for both the passengers and the driver and may make a long journey quite uncomfortable. Moreover noise contributes to driver fatigue which is a major cause of road accidents and thus reducing it is a valuable contribution to road safety [1]. Automotive interior noise is generally in the range between 100 to 4000 Hz and is more predominant between 100 to 600 Hz. This particular region can be reduced using Active Noise Control. This technique creates an 180^0 shifted version of the original noise source through a Noise Controller in real time. When this secondary source combines with the original version, reduction of noise is achieved in that particular space which is known as the quiet zone [2].

Thereby in this context, this work analyses and simulates the acoustic response of a curved Polyvinyldihyde (PVDF) based paper/knitted fabric speaker which can be used as the secondary source in an ANC system and can be used with the automotive upholstery.

2. Dynamic Displacement Model

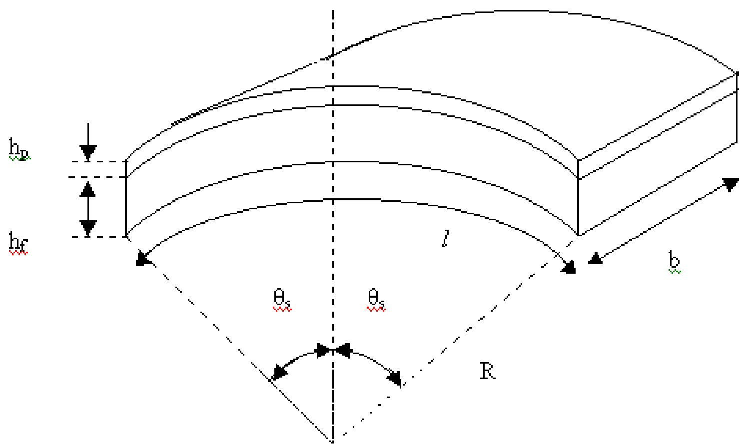

Figure 1 PVDF strip on knitted fabric modelled as a curved laminated shell.

This section presents the dynamic radial displacement of a curved PVDF strip laminated onto a fabric substrate. Subsequently the radial displacement is utilised to analyse the acoustic response of the structure. A PVDF strip on a knitted fabric can be modelled as a curved shell, laminated on to the surface of the fabric, Figure 1. The laminated structure is set in a curve with a subtended angle of $2\theta_s$.

The dimensions of the structure can be given as in Figure 1as, h_p the PVDF strip thickness, h_f the fabric thickness, *l* the length of the arc, *b* the width of the shell and R is the radius of curvature of the shell (to the mid plane)

2.1. Force and moments of the shell

The force and moment resultants are the integrals of the stresses over the shell thickness. Considering there are two laminates comprising the shell this can be written as [3]

$$\left[N.M\right] = b\left(\sum_{k=1}^{2}\int_{h_{k-1}}^{h_{k+1}}[1, z_k]Y_k \varepsilon dz_k\right) \tag{1}$$

where Y_k is the Young's Modulus of the k th layer, z_k the distance to the surface of the kth layer from the mid plane and ε: the strain which is given as, from [3] as,

$$\varepsilon = \frac{1}{(1+z_k)/R}\left(\varepsilon^0 + z_k \kappa\right) \tag{2}$$

where the mid plane strain (ε^0) and curvature (κ) are given as,

$$\varepsilon^0 = \frac{\partial v}{\partial x} + \frac{w}{R}, \kappa = -\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial v}{\partial x} \tag{3}$$

and v and w are the tangential and radial displacements of the segment respectively.

Substituting equation (2) into (1), neglecting the *z/R* term and carrying out the integration over the thickness of the shell from layer to layer, yields the following result for the force and moment.

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & E \end{bmatrix}\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \tag{4}$$

where

$$A = b\sum_{k=1}^{2} Y_k (z_k - z_{k-1}), B = \frac{1}{2}b\sum_{k=1}^{2} Y_k (z_k{}^2 - z_{k-1}{}^2), E = \frac{1}{3}b\sum_{k=1}^{2} Y_k (z_k{}^3 - z_{k-1}{}^3) \tag{5}$$

2.2. The equations of motion of the curved shell

Consider a differential element of the shell of mid surface of unit length *dx* (Figure 2).

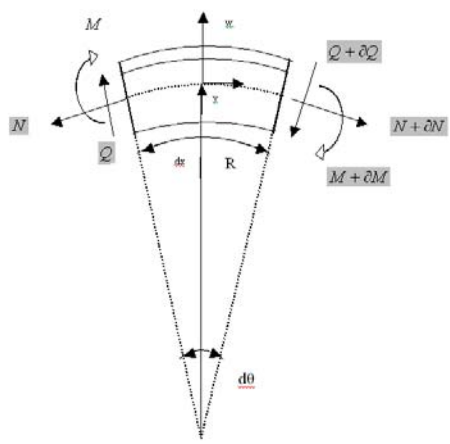

Figure 2 Differential curved element of shell.

The equation of motion may be obtained by considering the external and internal forces in the radial and tangential directions of the section. Only in-plane loading by the piezoelectric effect on the thickness direction and vibrations are considered here, as it is this motion that generates sound pressure [4],[5].The external force may be considered as periodic.

The equations of motion therefore become [3]:

$$\frac{\partial N}{\partial x}+\frac{Q}{R}=m\frac{\partial^2 v}{\partial t^2} \tag{6a}$$

$$-\frac{N}{R}+\frac{\partial Q}{\partial x}=m\frac{\partial^2 w}{\partial t^2}+\frac{F_w \sin \omega t}{R} \tag{6b}$$

$$\frac{\partial M}{\partial x}-Q=0 \tag{6c}$$

where, N is the normal force, Q the shear force, M the bending moment, m the mass per unit length of the segment, ω the angular frequency, F_w the external force due to the piezoelectric action, which is given as,

$$F_w = Y_p A d_{31} E \tag{7}$$

where, Y_p the Young's modulus of the PVDF strip, A the cross sectional area of the PVDF strip given as $h_p b$, d_{31} the piezoelectric coefficient in the thickness direction, E is the applied RMS electric field in the thickness direction of the strip with $E=\frac{V_{pk}}{\left(2\sqrt{2}\right)h_p}$, where V_{pk} is the peak to peak voltage of the applied electrical signal.

Solving equation (6c) for Q and substituting this into equation (6a) and (6b) yields,

$$\frac{\partial^2 N}{\partial x}+\frac{1}{R}\frac{\partial M}{\partial x}=m\frac{\partial^2 v}{\partial t^2}\ ,\frac{-N}{R}+\frac{\partial^2 M}{\partial x^2}=m\frac{\partial^2 w}{\partial t^2}-\frac{F_w \sin \omega t}{R} \tag{8}$$

Equation (8) is then multiplyed by –1 to which , equations (3) and (4) into (8) is substituted. The resultant equation is then arranged with the use of the D operator for $\frac{\partial}{\partial x}$, the equations of motion of the tangential and radial midplane displacements of the shell may be expressed as follows:

$$\left(A+\frac{2B}{R}+\frac{E}{R^2}\right)D^2 v+\left(\frac{A}{R}+\frac{B}{R^2}\right)Dw-\left(B+\frac{E}{R}\right)D^3 w=m\frac{\partial^2 v}{\partial t^2}$$

$$\left(\frac{A}{R}+\frac{B}{R^2}\right)Dv-\left(B+\frac{E}{R}\right)D^3 v+\left(\frac{A}{R^2}\right)w-\left(\frac{2B}{R}\right)D^2 w+ED^4 w=-m\frac{\partial^2 w}{\partial t^2}+\frac{F_w \sin \omega t}{R} \tag{9}$$

The solution is obtained using the undetermined coefficient method. For this purpose the particular solution for *v a*nd *w* is considered as:

$$w=W\sin(\omega t)\ \text{and}\ v=V\sin(\omega t) \tag{10}$$

Replacing them in equation (9) and arranging them in matrix form yields:

$$F\begin{pmatrix} V \\ W \end{pmatrix}=\begin{pmatrix} \frac{F_w}{R} \\ o \end{pmatrix} \tag{11}$$

where,

$$F = \begin{pmatrix} \left(\left(\frac{A}{R}+\frac{B}{R^2}\right)D - \left(B+\frac{E}{R}\right)D^3\right) & \left(ED^4 + \left(\frac{A}{R^2}\right) - \left(\frac{2B}{R}\right)D^2 - m\omega^2\right) \\ \left(\left(A+\frac{2B}{R}+\frac{E}{R^2}\right)D^2 + m\omega^2\right) & \left(\frac{A}{R}+\frac{B}{R^2}\right)D - \left(B+\frac{E}{R}\right)D^3 \end{pmatrix}$$

The solution for W can then be given as [6] ,

$$W = \frac{F_w}{\frac{A}{R} - Rm\omega^2} \tag{12}$$

From Figure 3 and equation (5) the value for A can be given as,

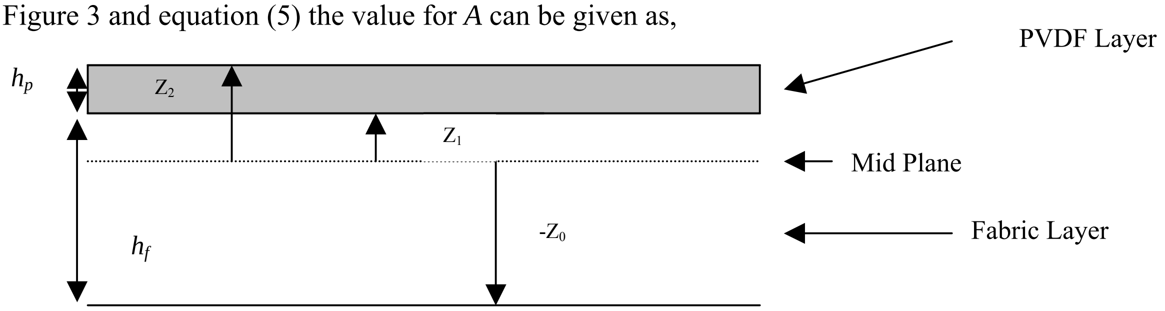

Figure 3. Thickness dimensions of the fabric and PVDF layers of the shell

$$A = bY_p h_p + bY_f h_f) \tag{13}$$

Substituting (13) in (12) and using (11) gives the radial displacements of the mid plane of the fabric-PVDF curved shell as,

$$w = \left(\frac{F_w}{\frac{b(Y_p h_p + Y_f h_f)}{R} - Rm\omega^2} \right) \sin \omega t \tag{14}$$

Considering the mass per unit length of the segment in terms of the respective densities of the individual layers comprising the shell, m can can be expressed in terms of the layer height and width (b) as,

$$m = \rho_f h_f b + \rho_p h_p b \tag{15}$$

where, ρ_f the fabric density, h_f the fabric thickness, ρ_p the PVDF layer density, h_p the PVDF layer thickness.

Substituting equations (15) equation (14), the radial displacements in terms of layer heights and densities and in its' complex form gives,

$$w = \left(\frac{F_w}{b\left[\frac{(Y_p h_p + Y_f h_f)}{R} - R(\rho_f h_f + \rho_p h_p)\omega^2 \right]} \right) e^{j\omega t} \tag{16}$$

3. Acoustic Response Model

To depict the on-axis acoustic pressure, a simple source model is considered in this section [6].

3.1. Complex acoustic strength of the arch

The dimensions of the vibrating surface are small compared to the wavelengths generated. 100 Hz is equal to 3.42 m and 1000 Hz is equal to 342 cm. The fabric vibrating area considered is in the region of $6.84 \cdot 10^{-3}$ m^2; thus the vibrating region of the fabric can be considered as a simple source [5].

The arc formed by the shell can be defined by, *n* infinitesimal segments, as shown in Figure 4.

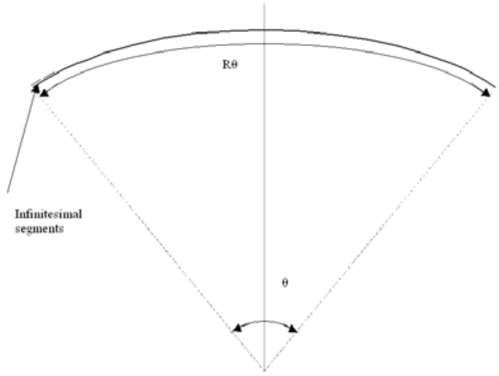

Figure 4 Arc of shell depicted as segments of infinitesimal length

Thus the length of each segment is $\frac{R\theta}{n}$ and the width is *b*. Assuming that each segment is vibrating in phase to each other, the instantaneous velocity at any *i* segment of the vibrating surface can be shown as follows,

$$u = \frac{\partial w}{\partial t} \tag{17}$$

This would yield from (16),

$$u = \left(\frac{j\omega F_w}{b\left[\frac{(Y_p h_p + Y_f h_f)}{R} - R(\rho_f h_f + \rho_p h_p)\omega^2\right]} \right) \tag{18}$$

This vibration causes the a radial displacement of a volume of fluid (air) at the rate of [5].

$$Q_i = uds \tag{19}$$

where, *ds* is the surface area of the segment and Q_i is the complex source strength of the segment. Therefore, the total source strength of the arch can be shown to be the integral of the source strength of a single segment over the surface area of the arch.

$$Q = \int_S uds \tag{20}$$

where, *S* is the surface area of the arch which yields,

$$Q = j\omega R\theta b \tag{21}$$

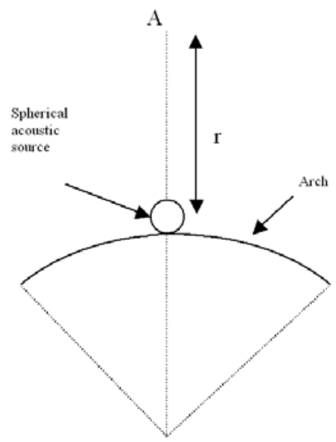

Figure 5 acoustic pressure from the vibrating arch

Therefore the complex acoustic pressure at point A, which is a distance *r* from the surface of the arch along its axis, as seen in Figure 5 may be expressed (omitting the time harmonic component) as [5].

$$P_A = \frac{j\rho_0 \omega Q}{4\pi r} \tag{22}$$

where, ρ_0 the density of air, Q the strength of the hemispherical source and r is the distance from the acoustic source to point A.

Substituting (21) into (22) and replacing *u* by (18), the modulus of the pressure at point A can be approximated as,

$$|P| = \left(\frac{\omega^2 \rho_0 E b h_p d_{31} R \theta}{4\pi r \left[\frac{(Y_p h_p + Y_f h_f)}{R} - R(\rho_f h_f + \rho_p h_p)\omega^2 \right]} \right) \tag{23}$$

Thus, the Sound Propagation Level (SPL) at point *A* may be given as,

$$SPL_A = 20 \log \left(\frac{|P|}{P_{ref}} \right) \tag{24}$$

The reference standard for airborne sounds is 10-12 W/m^2, which is approximately the intensity of a 1000 Hz pure tone (sine wave) that is barely audible to a person with unimpaired hearing. This value gives a pressure of 20 μPa [5]. This value is often used as a reference for sound pressure levels in air.

4. Mathematical Simulation of the Sound Propagation Level (SPL) of the PVDF-fabric arch

In this section a simulation has been done on the effect of the SPL of the arch by the mechanical properties of the base fabric, based on the equation (24) .The PVDF –fabric arch length and width were considered as 95 mm and 40 mm respectively.

For the simulation the mechanical properties of the rest of the layers of the arch has been considered from Table 1. The arch was kept at a subtended angle of 80^0.

Table 1. Mechanical Details of the layers used in the PVDF-Fabric arch

Layer	Young's Modulus (MPa)	Density (kgm^{-3})	Thickness (m)
PVDF	5900	1780	28e-6
Adhesive Layer	549.4573	872	0.2e-3

4.1.

4.2. Effect of base layer stiffness on SPL of PVDF-fabric arch

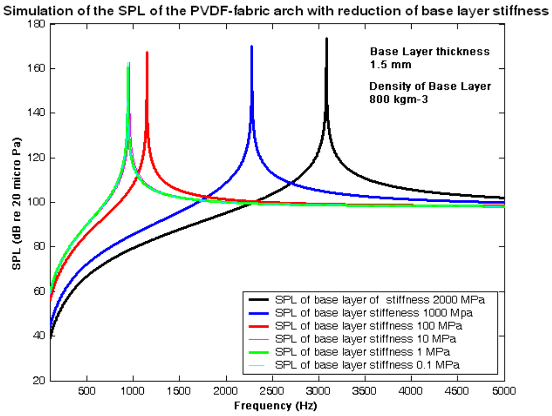

Figure 6. The simulation of the SPL of the PVDF-fabric arch with the reduction of the stiffness of its base layer whilst keeping the thickness and density of that layer constant

In this section the effect on the on the SPL of the arch by reducing the stiffness of the base layer whilst keeping its thickness and density constant, is simulated. The simulated result is shown in Figure 6. The thickness of the fabric was taken as 1.5 mm and its density as 800 kgm^{-3}.

It can be seen that till 10 MPa the reduction of the base layer stiffness shifts the resonant peak towards the lower frequency region of the spectrum. Further this accompanies an increase in the SPL towards the resonant peak. However below 10 MPa there is no change in the SPL and the result remains the same.

4.3. Effect of base layer thickness on the SPL of the arch

The SPL of the PVDF-fabric arch is simulated in this section by increasing the thickness of the base layer whilst keeping its density and stiffness constant. The stiffness of the base layer has been considered as 10 MPa and density 800 kgm^{-3}. The simulated result is shown in Figure 7.

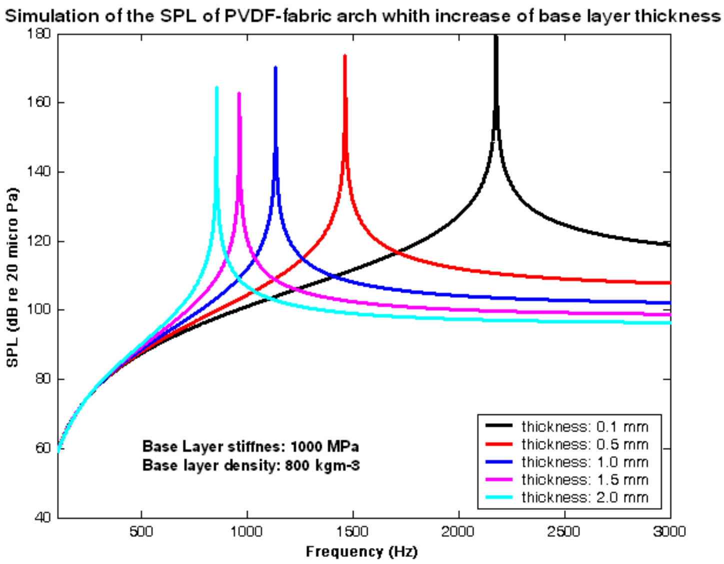

Figure 7. The simulation of the SPL of the PVDF-fabric arch with the increase thickness of its base layer whilst keeping its stiffness and density constant

It can be observed from Figure 7 that as the thickness of the base layer increases the resonant region progresses towards the lower frequency end of the spectrum. However there is an overall reduction in the total SPL of the arch.

4.4. Effect of base layer density on the SPL of the PVDF-fabric arch

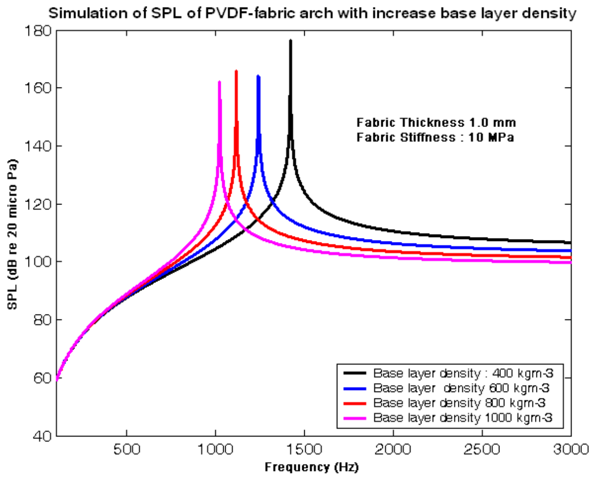

Figure 8. The simulation of the SPL of the PVDF-fabric arch with the increase of density in its base layer whilst keeping its stiffness and stiffness constant

The SPL of the PVDF –fabric arch has been simulated in this section with increased density of the base layer of the arch whilst keeping its thickness and stiffness constant. The stiffness of the base layer has been considered at 10 MPa and thickness of 1.0 mm. The simulated results are shown in Figure 8.

It can be observed from Figure 8 that as the density of the base layer increases there is progression of the resonant region of the SPL of the arch towards the lower end of the spectrum. However this shift is less than the preceding parameters. Thus the effect of this parameter on the SPL of the arch is less than the preceding parameters.

5. Conclusion

The simulation of the acoustic response of a PVDF –fabric laminated curved shell presents that with the following points.

a. Reduction of the fabric stiffness till 10 Mpa shifts the resonant frequency of the acoustic response towards the lower frequency region.

b. Increase of fabric thickness shifts the resonant frequency of the acoustic response towards the lower frequency region. However this further results in a reduction of acoustic pressure.

c. Increase of fabric density shifts the resonant frequency of the acoustic response towards the lower frequency region. However this variation is lesser than the preceding parameters.

The base layer of the PVDF arch is considered as a membrane without any perforation in this analysis. However when it comes to the use of fabrics in the base layer, due to its inherent porous nature, deviates it from an ideal membrane structure. The use of covered elastomeric yarn in a tightly knitted fabric causes the pores in the knitted structure to be closed. This is due to the nylon cover of the yarn. Thus future work to validate the simulation hypothesis will be conducted with the use of knitted fabrics from elastomeric yarn as the base fabric.

6. References

[1] W Fung and M. Hardcastle. *Textiles in Automotive Engineering*. Cambridge: Woodhead publishing limited. 2001.

[2] Sen M Kuo and D.R. Morgan. *Active noise control systems : algorithms and DSP implementations*. Chichester: John Willey and Sons Inc. 1996.

[3] M.S. Qatu. Theories and Analyss of Thin and Moderately Thick Laminated Composite Curved Beams. *International Journal of Solid Structures*. 1993, **30**(20): 2743-2756.

[4] R. Haberman. *Elementary Applied partial Differential Equations* (3rd Edition ed). New Jersey: Prentice Hall. 1998.

[5] L. E. Kinsler and A. B. Coppens. *Fundamentals of Acoustics*. (3 ed). New York: John Wiley & Sons,Inc. 1982.

[6] Kelly C Bailo, D. E. Brei, and K. Grosh. Investigation of Curved Polymeric Piezoelectric Active Diaphragms. *Journal of Vibration and Acoustics*. 2003, **125**: 145-154.