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A new rate-based congestion control scheme for wireless networks

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Abstract. We consider link error or physical channel error in wireless networks and propose a new rate-based congestion control scheme for wireless networks, which is based on the NUM framework developed for TCP-like congestion control model in wired networks. This scheme is distributed and can be implemented by changing the number of connections opened by one user and requires no modification to either the network infrastructure, or the network protocols. The scheme is proved to be global stability in the absence of round trip delay of each user and all trajectories can converge to the unique equilibrium point. The convergence rate is also studied and stochastic disturbance is analyzed. Furthermore, a sufficient condition is obtained under which the scheme is locally stable at the equilibrium point in the presence of delay based on the general Nyquist criterion of stability.

Keywords: wireless networks, congestion control, convergence, stability, Nyquist.

1. Introduction

Recently, there has been a flurry of research activity on decentralized end-to-end congestion control algorithms on wired networks. A widely used framework, introduced by Kelly etc.[1] and refined for TCP[2], is to associate a utility function with each flow and maximize the aggregate utility function which is subject to link capacity constraints-an optimization problem known as network utility maximization (NUM). Congestion control schemes can be viewed as algorithms to compute the optimum or some suboptimum solution to this maximization problem.

Congestion control schemes can be divided into two classes: primal algorithm and dual algorithm. In primal algorithms, the users adapt the resource rates dynamically based on the route prices, and the links select a static law to determine the link prices directly from the arrival rates at the links[1]. In dual algorithms, however, the links adapt the link prices dynamically based on the link rates, and the users select a static law to determine the source rates directly from the route prices and the source parameters[1],[3-5].

Owing to the NUM framework presented by Kelly, some modified primal algorithms are introduced for wired networks[6-8], which are formulated as distributed end-to-end rate-based congestion control problems. In parallel, intensive efforts have been devoted to designing congestion control algorithms capable of accommodating error-prone and time varying wireless links, which have been demonstrated to be well beyond normal TCP's reach[9]. While earlier approaches, such as I-TCP and Snoop-TCP[10], have mainly been engineered based on empirical techniques, recent efforts have been based on optimization and control theory approach[11-14]. For example, an optimal congestion control scheme in conjunction with power control was developed by Chiang[11] based on optimization theory for multi-hop wireless network, various performance metrics maximized in wireless network were studied by Radunovic and Boudec [12] and a hop-by-hop congestion control scheme was proposed by Yi and Shakkottai [13] specifically for wireless networks.

These congestion schemes developed for wireless networks often assume that packet loss is duo to

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congestion. While such assumption may suit wired networks comfortably, it is certainly not really true in wireless networks. It is well known that wireless channels are characterized by inherent time-varying capacities[15] that have been shown to significantly reduce the throughput of TCP[10]. And packet loss is caused not only by flow congestion, but also by link price error or physical channel error (or equivalently, reduced link bandwidth). This packet loss due to error on link has been studied by Chen and Zakhor[16], Adate and Chen[17], Lee and Chiang[18]. TCP-friendly rate control (TFRC) scheme in wireless network was considered by Chen and Zakhor[16], in which packet loss was mainly due to link price error, and an end-to-end rate control solution was proposed for wireless video streaming. Stability of a new congestion control scheme with link price error was analyzed and a sufficient condition for local stability with delay was obtained by Adate and Chen[17]. Furthermore, a rate-based congestion control scheme was brought forward and the convergence to the globally optimal rate-reliability tradeoff was proved by Lee and Chiang[18].

In this paper, we also consider link error in wireless networks, study a class of utility function and propose a new rate-based congestion control scheme for wireless networks, which is based on the recently developed congestion control model for TCP-like schemes in wired networks. We prove the global stability of this scheme in the absence of round trip delay, and obtain that all trajectories can converge to the unique equilibrium point. Furthermore, we study the convergence rate of this scheme to the equilibrium point and the robustness of convergence rate to inevitable disturbances in the real network environment. And we also consider the scheme with round trip delay and analyze its stability. We obtain a sufficient condition for local stability of this scheme in the presence of delay based on the general Nyquist criterion of stability. This condition is simple and decentralized.

2. Rate-Based Congestion Control Scheme

2.1. The model for the wired networks

First we review the model for the wired networks and the end-to-end rate-based congestion control scheme. Consider a network with a set J of links, each link $j \in J$ has a finite capacity C_j. The network is shared by a set R of users, i.e. sender-receiver pairs. The route of each user $r \in R$ is also denoted by r and consists of a subset of J. The connections of the network are described via a routing matrix $A=(A_{jr}, j \in J, r \in R)$, where $A_{jr}=1$ if source j is in user r's route and $A_{jr}=0$ otherwise. Every user is endowed with a sending rate $x_r(t)=0$ at time t. Kelly was the first one to interpret the flow control as the solution of the following concave maximization problem, dependent on the aggregate utility function for each user and cost function for every link:

$$\max \sum_{r\in R} w_r U_r(x_r(t)) - \sum_{j\in J}\int_0^{\sum_{s:j\in s} x_s(t)} p_j(z)dz \tag{1}$$

where $U_r(x_r(t))$, the utility function of user r, is continuously differentiable, strictly concave, and increasing; $w_r > 0$ is a weight that can be interpreted as pay per unit time ; $\int_0^{\sum_{s:j\in S} x_s(t)} P_j(z)dz$ is the cost incurred at link j; $P_j(z)$ is the price function and required to be non-negative, continuous, increasing and not identically zero. With these assumptions above, the objective function in (1) is strictly concave, thus the unique optimal point of this optimization problem can be easily obtained.

One common price function used in practice is the packet loss rate, which is zero if there is no congestion, and concavely increases otherwise:

$$p_j(z) = \frac{(z-C_j)^+}{z} \;, \tag{2}$$

where $u^+ = \max(u,0)$.The end-to-end packet loss rate for user r is then $1-\coprod_{j\in y}[1-p_j(\sum_{s:j\in S} x_S(t))]$ which is approximately equal to $\sum_{j\in r} p_j(\sum_{s:j\in S} x_S(t))]$, assuming $p_j(\sum_{s:j\in S} x_S(t))$ is small.

We are interested in the class of users' utility functions in the following form

$$U_r(x_r(t)) = -\frac{1}{\alpha}\frac{1}{x_r^{\alpha}(t)}, \tag{3}$$

If $\alpha = 0$, we assume the utility function $U_r(x_r(t)) = \log x_r(t)$. In particular, $\alpha = 1$ has been found useful for modeling the utility function of TCP algorithm[8]. This class of utility functions in (3) has been used extensively[2],[8],[19],[20]. Also, it has been used to carry out a tradeoff between system throughput and fairness among the users in a cellar network[21].

We consider the following scheme for this class of utility function, which is proposed by Kunniyur and Srikant[8]:

$$\frac{dx_r(t)}{dt} = \kappa_r(w_r - x_r^{\alpha+1}(t)\sum_{j:j\in r}\mu_j(t)), \tag{4}$$

$$\mu_j(t) = p_j(\sum_{s:j\in s} x_s(t)), \tag{5}$$

where K_r is a positive scale factor affecting the adaptation rate; $u_j(t)$ is the congestion price generated at link j.

With this primal scheme (4)(5), the unique, globally and asymptotically stable, equilibrium point of the network, denoted by $x = (x_r, r \in R)$, is given by

$$x_r = \left(\frac{w_r}{\sum_{j:j\in r} p_j(\sum_{s:j\in s} x_s)}\right)^{\frac{1}{\alpha+1}}. \tag{6}$$

In particular, when $\alpha = 1$, the equilibrium point x_r of scheme (4)(5) is similar to that of the primal algorithm given by Kelly[2]. This unique solution is also the optimal solution for the optimization problem in (1). The solution is desirable in the sense that the network's bottlenecks are fully utilized, and the total net utility is maximized.

2.2. The wireless networks case

One of the main differences between the wired networks and the wireless is the presence of physical channel errors in the latter case; in the setting of wireless case, these affect the packet loss rate, which in the wired case depends mainly on the congestion measure. We assume the price error on every link j is $r_j(\sum_{s:j\in s} x_s(t))$, then the new price function should be

$$v_j(t) = p_j(\sum_{s:j\in s} x_s(t)) + r_j(\sum_{s:j\in s} x_s(t)) \triangleq q_j(\sum_{s:j\in s} x_s(t)). \tag{7}$$

The link error can be positive, zero or negative, if error is zero, then the exact price is used in scheme (4)(5).

The utility maximization problem for wireless networks still has the form (1), as none of the utility or the cost is a function of error $r_j(t)$, $j \in J$. The primal scheme (4)(5) will adapt itself according to this new price function $q_j(t)$, which has the same structural properties as the old $p_j(t)$; the equilibrium point of the system will therefore change accordingly. From an optimization prospective, the new equilibrium point is a suboptimal solution for the optimization problem in (1). Exploiting duality arguments, this means that some of the bottleneck links may be under utilized.

We will solve this underutilization problem by dynamically adjusting ω_r based on $u_j(t)$ and $v_j(t)$, which physically corresponds to adjusting the number of connections the user has to the network. Assume w_r is adjusted according to the following new law:

$$w_r(t) = w_r\frac{\sum_{j:j\in r} v_j(t)}{\sum_{j:j\in r}\mu_j(t)}. \tag{8}$$

Then the new primal rate-based congestion control scheme for user r is given by:

$$\frac{dx_r(t)}{dt} = \kappa_r (w_r(t) - x_r^{\alpha+1}(t) \sum_{j:j\in r} \nu_j(t)), \tag{9}$$

where $\nu_j(t)$ has the form (7).

This scheme is an end-to-end application layer based scheme, since charging $w_r(t)$ can be implemented by changing the number of connections opened by one user. Therefore they require no modification to either the network infrastructure, e.g. route, or to the network protocols, e.g. TCP.

3. Global Stability and Convergence Rate

3.1. Global stability

We can obtain that, under the change (8) in $w_r(t)$, the equilibrium of scheme (9) is again x. Intuitively, seen from (8), if the error is large, i.e. $\nu_j(t) > \mu_j(t)$, it can be counterbalanced by an increase in $w_r(t)$. So we obtain the following theorem

Theorem 1. System (5)(7)(8)(9) is globally asymptotically stable, and all trajectories converge to the equilibrium point x in (6) that maximizes $V(x(t))$:

$$V(x(t)) = \sum_{r\in R} w_r U_r(x_r(t)) - \sum_{j\in J} \int_0^{\sum_{s:j\in s} x_s(t)} p_j(z)dz\,. \tag{10}$$

Proof: We have assumed that $w_r > 0$ for all $r \in R$ and $p_j \ge 0$ for all $j \in J$. These ensure that $V(x(t))$ is strictly concave on $x \ge 0$ with an interior maximum. The maximizing value of $x(t)$ is thus unique.

Observe that

$$\frac{\partial}{\partial x_r(t)} V(x(t)) = \frac{w_r}{x_r^{\alpha+1}(t)} - \sum_{j:j\in r} p_j(\sum_{s:j\in s} x_s(t))\,.$$

Setting these derivatives to zero identifies the maximum, that is, at the maximum, x_r satisfies

$$w_r = x_r^{\alpha+1} \sum_{j:j\in r} p_j(\sum_{s:j\in s} x_s)\,.$$

Further

$$\frac{d}{dt}V(x(t)) = \sum_{r\in R} \frac{\partial V(x(t))}{\partial x(t)} \frac{dx(t)}{dt} = \sum_{r\in R} \frac{1}{x_r^{\alpha+1}} \frac{\sum_{j:j\in r} \nu_j(t)}{\sum_{j:j\in r} \mu_j(t)} (w_r - x_r^{\alpha+1}(t) \sum_{j:j\in r} \mu_j(t))^2,$$

establishing that $V(x(t))$ is strictly increasing with t, unless $x(t) = x$, the unique $x(t)$ maximizing $V(x(t))$. Thus the theorem is obtained.

3.2. Convergence rate

Although global stability implies all the trajectories converge to the unique equilibrium, it does not indicate how fast they do so and upon what this depends. The analysis on the rate of convergence gives insights into the latter questions and hints at designing improved protocols. The real implementation of the proposed scheme depends on accurate measure on the packet loss rates. However, the real network environment always introduces noise to the measurements. By modeling the effects of all these disturbances as Brownian motion perturbations on the deterministic system, an analysis can help to understand robustness of the scheme to these inevitable disturbances.

Let $y_j(t) = \sum_{s:j\in s} x_s(t)$, $j \in J$ be the aggregate rate arriving at link j. We assume $p_j(y_j(t))$ and $q_j(y_j(t))$, the exact and inexact price at link $j \in J$, is differentiable at the equilibrium point $x = (x_r, r \in R)$ which satisfies (6) with derivative $p'_j(y_j(t))$ and $q'_j(y_j(t))$, respectively. Let

$$p_j = p_j(y_j(t)),\ q_j = q_j(y_j(t)),\ p'_j = p'_j(y_j(t)),$$

$$l_r = \kappa_r \sum_{j:j\in r} q_j \Big/ \sum_{j:j\in r} p_j,\ x_r(t) = x_r + l_r^{1/2} x_r^{(\alpha+1)/2}\xi_r(t),$$

at the equilibrium point, for simplicity.

We linearize the system (9) around the equilibrium point and obtain

$$\frac{d\xi_r(t)}{dt} = -((\alpha+1)l_r x_r^{\alpha} \sum_{j:j\in r} p_j)\xi_r(t) - l_r^{1/2} x_r^{(\alpha+1)/2} \sum_{j:j\in r} p'_j \cdot \sum_{s:j\in s} l_s^{1/2} x_s^{(\alpha+1)/2}\xi_s(t). \quad (11)$$

We can write this in matrix form as

$$\frac{d}{dt}\xi(t) = -((\alpha+1)^{1/2} L^{1/2} X^{\alpha/2} diag\{\sum_{j:j\in i} p_j\} X^{\alpha/2} L^{1/2}(\alpha+1)^{1/2} + L^{1/2} X^{(\alpha+1)/2} A^T P' A X^{(\alpha+1)/2} L^{1/2})\xi(t),$$

where

$$X = diag\{x_r, r\in R\},\ L = diag\{l_r, r\in R\},\ P' = diag\{p'_j, j\in J\},\ A = (A_{jr}, j\in J, r\in R).$$

Let

$$\mathrm{H}^T \Phi \mathrm{H} = (\alpha+1)^{1/2} L^{1/2} X^{\alpha/2} diag\{\sum_{j:j\in i} p_j\} X^{\alpha/2} L^{1/2}(\alpha+1)^{1/2} + L^{1/2} X^{(\alpha+1)/2} A^T P' A X^{(\alpha+1)/2} L^{1/2}, \quad (12)$$

where H is an orthogonal matrix, $H^T H = I$, and $\Phi = diag(\phi_r, r\in R)$ is the matrix of eigenvalues, necessarily positive, of the real, positive definite matrix (12). Then

$$\frac{d}{dt}\xi(t) = -\mathrm{H}^T \Phi \mathrm{H}\xi(t), \quad (13)$$

and thus the rate of convergence to the equilibrium point is determined by the smallest eigenvalue, ϕ_r, $r\in R$, of the matrix (12).

3.3. Stochastic analysis for this congestion control scheme

Now we consider the stochastic perturbation of the linearized equation (13). The perturbation can be caused by the random nature of packet loss. Let

$$d\xi(t) = -\mathrm{H}^T \Phi \mathrm{H}\xi(t)dt - FdB(t), \quad (14)$$

where F is an arbitrary $|R|\times|I|$ matrix and $B(t) = (B_i(t), i\in I)$ is a collection of independent standard Brownian motions, extended to $-\infty < t < +\infty$. The stationary solution to the system (14) is

$$\xi(t) = -\int_{-\infty}^{t} e^{-(t-\tau)\mathrm{H}^T\Phi\mathrm{H}} FdB(\tau). \quad (15)$$

The solution (15) is a linear transformation of the Gaussian process $(B(\tau), \tau < t)$; hence $\xi(t)$ has a multivariate normal distribution, $\xi(t)\square N(0,\sum\ )$, where

$$\Sigma = E[\xi(t)\xi(t)^T] = \int_{-\infty}^{0} e^{\tau\mathrm{H}^T\Phi\mathrm{H}} FF^T e^{\tau\mathrm{H}^T\Phi\mathrm{H}} d\tau = \mathrm{H}^T[\int_{-\infty}^{0} e^{\tau\Phi}\mathrm{H}FF^T\mathrm{H}^T e^{\tau\Phi} d\tau]\mathrm{H}.$$

Define $[HF;\Phi]H$ by

$$[\mathrm{H}F;\Phi]_{rs} = [\int_{-\infty}^{0} e^{\tau\Phi}\mathrm{H}FF^T\mathrm{H}^T e^{\tau\Phi} d\tau]_{rs} = \frac{[HFF^T H^T]_{rs}}{\phi_r + \phi_s},$$

Then

$$\sum\ = H^T[HF;\Phi]H.$$

4. Stability with Round Trip Delay

We now consider the scheme described above with round trip delay and study its stability. Given a route

r, for each link $j \in r$, we define a forward delay $d_{jr}^{->}$, and a return delay $d_{jr}^{<-}$. The forward delay is the delay incurred in communication from the sender of route r to the link; the return delay is the delay incurred in communication from the link back to the sender of route r. In the current Internet, each route is subject to a round trip delay. We model this delay by assuming each route has an associated delay D_r, such that $d_{jr}^{<-} + d_{jr}^{->} = D$ for each $j \in r$. The end-to-end rate-based congestion control scheme (9) with delay has the following form

$$\frac{dx_r(t)}{dt} = \kappa_r \frac{\sum_{j:j\in r} \nu_j(t - d_{jr}^{\leftarrow})}{\sum_{j:j\in r} \mu_j(t - d_{jr}^{\leftarrow})} (w_r - x_r^{\alpha+1}(t - D_r) \sum_{j:j\in r} \mu_j(t - d_{jr}^{\leftarrow})), \quad (16)$$

$$\mu_j(t) = p_j(\sum_{s:j\in s} x_s(t - d_{js}^{\rightarrow})), \quad (17)$$

$$\nu_j(t) = q_j(\sum_{s:j\in s} x_s(t - d_{js}^{\rightarrow})). \quad (18)$$

First we let $x(t) = x_r + l_r^{1/2} x_r^{\frac{\alpha+1}{2}} \xi_r(t)$, where $l_r = K_r \sum_{j:j\in r} q_j / \sum_{j:j\in r} p_j$, linearize the system (16) around the equilibrium point (6), and obtain the following system for user $r \in R$.

$$\frac{d\xi_r(t)}{dt} = -((\alpha+1) l_r x_r^{\alpha} \sum_{j:j\in r} p_j) \xi_r(t - D_r) - l_r^{1/2} x_r^{(\alpha+1)/2} \sum_{j:j\in r} p_j' \cdot \sum_{s:j\in s} l_s^{1/2} x_s^{(\alpha+1)/2} \xi_s(t - d_{js}^{\rightarrow} - d_{jr}^{\leftarrow}). \quad (19)$$

We analyze the stability of (19) and obtain the following theorem

Theorem 2. The continuous-time system (16)-(18) is locally stable at the equilibrium point if the following condition is satisfied for all $r \in R$

$$\kappa_r \frac{\sum_{j:j\in r} q_j}{\sum_{j:j\in r} p_j} ((\alpha+1) x_r^{\alpha} \sum_{j:j\in r} p_j + \sum_{j:j\in r} p_j' \sum_{s:j\in s} x_s^{\alpha+1}) < \frac{\pi}{2D_r}. \quad (20)$$

In order to prove this theorem, we need two lemmas presented by Tian and Yang[22] which are the following

Lemma 1. For any given natural number $m > 2$, let

$$G^r(j\omega) = k_r \exp(-jD_r\omega) / j\omega, r = 1, \cdots m,$$

where $k_r = \pi / (20 < D_r \in R)$ and $0 < D_r \in R$ are divers nonnegative delay constants. Then $kCo(0 \cup \{G^r(j\omega)\})^{r=1,\dots m\})}$ does not contain the point $(-1, j0)$ for any given real number $0 < k < 1$ and for all $\omega \in (-\infty, +\infty)$, where $Co\{.\}$ denotes the convex hull of set $\{.\}$.

Lemma 2. Let $Q = Q^* > 0$ and $T = diag(t_r, t_r \in C)$ be given. Then $\lambda(QT) \in \rho(Q) Co(0 \cup \{t_r\})$, where $\rho(.)$ denotes the matrix spectral radius.

Proof of the Theorem 2:

We assume the initial state of system (19) is zero, that is, $\xi(0) = 0$, take the Laplace transform, and obtain

$$s\xi_r(s) = -((\alpha+1) l_r x_r^{\alpha} \sum_{j:j\in r} p_j) \xi_r(s) \exp(-sD_r)$$
$$-l_r^{1/2} x_r^{(\alpha+1)/2} \sum_{j:j\in r} p_j' \exp(sd_{jr}^{\rightarrow}) \sum_{s:j\in s} l_s^{1/2} x_s^{(\alpha+1)/2} \exp(-sd_{js}^{\rightarrow}) \xi_s(s) \cdot \exp(-sD_r), \quad (21)$$

where $\xi_r(s) = L(\xi_r(t))$. Define the following matrices

$X = diag\{x_r, r \in R\}$, $P' = diag\{p_j', j \in J\}$, $L = diag\{l_r, r \in R\}$, $A(s) = \{A_{jr} \exp(sd_{jr}^{\rightarrow}), j \in J, r \in R\}$.

Then the system (21) can be rewritten as

$$s\xi(s) = -\mathrm{M}(s)diag\{\exp(-sD_r), r \in R\}\xi(s)\,, \tag{22}$$

where

$$\mathrm{M}(s) = (\alpha+1)^{1/2} L^{1/2} X^{\alpha/2} diag\{\sum_{j:j\in r} p_j\} X^{\alpha/2} L^{1/2} (\alpha+1)^{1/2} + L^{1/2} X^{(\alpha+1)/2} A^T(s) P' A(-s) X^{(\alpha+1)/2} L^{1/2}\,.$$

The characteristic equation of the above closed-loop system is

$$\det(I + \mathrm{M}(s)diag\{\exp(-sD_r)/s\}) = 0\,. \tag{23}$$

The system is local stability, if all the roots of the characteristic equation above have negative real parts. By the generalized Nyquist criterion[23], the system (16) is locally asymptotically stable, if the eigenloci of $M(j\omega)diag\exp(-j\omega Dr)/(j\omega)$ does not enclose the point $(-1, j0)$ for all $\omega \in (-\infty, +\infty)$.

Rewrite $M(j\omega)diag\{\exp(-sDr)/s\}$ as

$$\mathrm{M}(s)diag\{\exp(-sD_r)/s\} = \mathrm{M}(s)Kdiag\{k_r \exp(-sD_r)/s\}\,,$$

where $k_r = \pi/(2D_r)$, $K = diag\{k_r^{-1}\}$. Then

$$\begin{aligned}
&\rho(\mathrm{M}(j\omega)K) \\
&= \rho\left(\left((\alpha+1)^{1/2} L^{1/2} X^{\alpha/2} diag\{\sum_{j:j\in r} p_j\} X^{\alpha/2} L^{1/2} (\alpha+1)^{1/2} + L^{1/2} X^{(\alpha+1)/2} A^T(j\omega) P' A(-j\omega) X^{(\alpha+1)/2} L^{1/2}\right) K\right) \\
&= \rho\left(\left((\alpha+1) X^{\alpha} diag\{\sum_{j:j\in r} p_j\} L + A^T(j\omega) P' A(-j\omega) X^{\alpha+1} L\right) K\right) \\
&= \rho\left(\left((\alpha+1) X^{\alpha} diag\{\sum_{j:j\in r} p_j\} + A^T(j\omega) P' A(-j\omega) X^{\alpha+1}\right) LK\right).
\end{aligned}$$

From matrix theory, the spectral radius of any matrix is bounded by its maximum absolute row sum. Then we can get

$$\begin{aligned}
\rho(\mathrm{M}(j\omega)K) &\le \max_{r\in R}\left\{\left((\alpha+1)x_r^{\alpha}\sum_{j:j\in r} p_j + \sum_{j\in J}\sum_{s\in R}\left|A_{jr}\exp\left(j\omega\left(d_{jr}^{\rightarrow} - d_{js}^{\rightarrow}\right)\right)p_j' A_{js} x_s^{\alpha+1}\right|\right) l_r k_r^{-1}\right\} \\
&= \max_{r\in R}\left\{\left((\alpha+1)x_r^{\alpha}\sum_{j:j\in r} p_j + \sum_{j\in J} A_{jr} p_j' \sum_{s\in R} A_{js} x_s^{\alpha+1}\right) l_r \left(\frac{\pi}{2D_r}\right)^{-1}\right\} \\
&= \max_{r\in R}\left\{l_r\left((\alpha+1)x_r^{\alpha}\sum_{j:j\in r} p_j + \sum_{j:j\in r} p_j' \sum_{s:j\in s} x_s^{\alpha+1}\right)\left(\frac{\pi}{2D_r}\right)^{-1}\right\} < 1.
\end{aligned}$$

From Lemma 2, the following holds for all $\omega \in (-\infty, +\infty)$,

$$\lambda\left(\mathrm{M}(j\omega)diag\left\{\exp(-j\omega D_r)/(j\omega)\right\}\right) \in \rho\left(M(j\omega)K\right)\mathrm{Co}\left(0 \cup \left\{k_r \exp(-j\omega D_r)/(j\omega)\right\}\right),$$

where $\lambda(.)$ denotes the matrix eigenvalue.

From Lemma 1, we can know that $\rho(M(j\omega)K)Co(0\bigcup\{k_r \exp(-j\omega D_r)/(j\omega)\}$ does not contain the point $(-1, j0)$ when $\rho(M(j\omega)K)$, and thus the eigenloci $\lambda(M(j\omega)diag\{\exp(-j\omega Dr)/j\omega)\}$ of $M(j\omega)diag\exp(-j\omega Dr)/(j\omega)$ can not enclose the point $(-1, j0)$ for all $\omega \in (-\infty, +\infty)$. The theorem is obtained.

Remark 1. If the link price error $r_j(t) = 0$ on every link j, then the sufficient condition above reduces to

$$\kappa_r((\alpha+1)x_r^{\alpha}\sum_{j:j\in r} p_j + \sum_{j:j\in r} p'_j \sum_{s:j\in s} x_s^{\alpha+1}) < \frac{\pi}{2D_r},$$

which can be regarded as the sufficient condition for local stability of the following rate congestion control scheme

$$\frac{dx_r(t)}{dt} = \kappa_r(w_r - x_r^{\alpha+1}(t-D_r)\sum_{j:j\in r}\mu_j(t-d_{jr}^{\leftarrow})),$$

where

$$\mu_j(t) = p_j(\sum_{s:j\in s} x_s(t-d_{js}^{\rightarrow})).$$

Remark 2. If $\alpha = 0$, that is, the utility function $U_r(x_r(t)) = \log x_r(t)$, then the sufficient condition above reduces to

$$\kappa_r \frac{\sum_{j:j\in r} q_j}{\sum_{j:j\in r} p_j}(\sum_{j:j\in r} p_j + \sum_{j:j\in r} p'_j \sum_{s:j\in s} x_s) < \frac{\pi}{2D_r},$$

which has been obtained by Adate [17] and is the sufficient condition for local stability of the following rate congestion control scheme

$$\frac{dx_r(t)}{dt} = \kappa_r \frac{\sum_{j:j\in r} \nu_j(t-d_{jr}^{\leftarrow})}{\sum_{j:j\in r} \mu_j(t-d_{jr}^{\leftarrow})}(w_r - x_r(t-D_r)\sum_{j:j\in r}\mu_j(t-d_{jr}^{\leftarrow})),$$

where

$$\mu_j(t) = p_j(\sum_{s:j\in s} x_s(t-d_{js}^{\rightarrow})),$$

$$\nu_j(t) = q_j(\sum_{s:j\in s} x_s(t-d_{js}^{\rightarrow})).$$

Remark 3. If link price error $x_j(t) = 0$ on every link j and $\alpha = 0$, then the sufficient condition above reduces to

$$\kappa_r(\sum_{j:j\in r} p_j + \sum_{j:j\in r} p'_j \sum_{s:j\in s} x_s) < \frac{\pi}{2D_r},$$

which has been proposed as a conjecture by Massoulie[24] and is the sufficient condition for local stability of the following rate congestion control scheme

$$\frac{dx_r(t)}{dt} = \kappa_r(w_r - x_r(t-D_r)\sum_{j:j\in r}\mu_j(t-d_{jr}^{\leftarrow})),$$

where

$$\mu_j(t) = p_j(\sum_{s:j\in s} x_s(t-d_{js}^{\rightarrow})).$$

Remark 4. From the sufficient condition above, we can know that network stability can be guaranteed by simple, decentralized condition, i.e., in order to be stable, each end system needs only its own information, for example, round trip delay of each user.

5. Conclusions

In wireless networks, packet loss is caused not only by flow congestion, but also by link error or physical channel error. This packet loss due to errors on link has been studied in some literatures. We also consider these errors in wireless networks and propose a new rate-based congestion control scheme, based on the NUM framework developed for TCP-like congestion control model in wired networks. This scheme can be implemented by changing the number of connections opened by one user and requires no modification to either the network infrastructure, e.g. route, or the network protocols, e.g. TCP. This scheme is globally stable in the absence of round trip delay, and all trajectories described by this scheme can converge to the unique equilibrium point. Furthermore, we analyze the convergence rate of this scheme to the equilibrium point and study the robustness of this scheme to inevitable disturbances in the real network environment. And we consider the scheme with round trip delay and analyze its stability. We obtain a sufficient condition under which the scheme is locally stable at the equilibrium point in the presence of delay based on the general Nyquist criterion of stability. This condition is simple and decentralized, i.e., network stability can be guaranteed if each user only knows its only information.

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