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Uniqueness of positive solutions for a class of *p*-Laplacion problems

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Abstract: We consider uniqueness of positive radial solutions to the system.

$$\begin{cases} -\Delta_p u = \lambda f(v) & in\ \Omega \\ -\Delta_q v = \mu g(u) & in\,\Omega \\ u = v = 0 & on\,\partial\Omega \end{cases} \tag{1}$$

Where $\Delta_p u = div\left(|\nabla u|^{p-2}\nabla u\right)$, $\Delta_q v = div\left(|\nabla v|^{q-2}\nabla v\right)$, $p, q > 1, \Omega$ is the open unit ball in R^N, $N \geq 2$ and $\partial\Omega$ is its boundary.

Keywords: uniqueness, laplacion problems, positive solutions.

1. Introduction

We consider the boundary value problem

$$\begin{cases} -\Delta_p u = \lambda f(v) & in\ \Omega \\ -\Delta_q v = \mu g(u) & in\,\Omega \\ u = v = 0 & on\,\partial\Omega \end{cases} \tag{1}$$

Where Ω is the open unit ball in $R^N, N \geq 2$, with a smooth boundary $\partial\Omega, \lambda, \mu$ are positive parameters and f, g are smooth functions. We shall establish existence and nonexistence of positive radial solutions for (1). Dalmasso [2] investigated the existence and uniqueness of positive radial solutions of the boundary value problem

$$\begin{cases} -\Delta u = \lambda f(v) & in\ B \\ -\Delta v = \mu g(u) & in\,B \\ u = v = 0 & on\,\partial B \end{cases} \tag{2}$$

Where B is the open unit ball in R^N, $N \geq 2$, $f, g : [0, \infty) \to [0, \infty)$. He obtained some results in the sublinear case when f, g are nondecreasing and there exist positive numbers p, q with $pq < 1$ such that

$$\frac{f(x)}{x^p} \text{ and } \frac{g(x)}{x^q} \text{ are nonincreasing on } R^+. \tag{H}$$

D.D. Hai [3] considered system (2) and extended (H) to hold only for large x. According to a result of Troy [6] , positive solutions of the boundary value problem

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$$\begin{cases} -\Delta u = \lambda a(|x|) f(v) & in\ \Omega \\ -\Delta v = \mu b(|x|) g(u) & in\ \Omega \\ u = v = 0 & on\ \partial\Omega \end{cases} \tag{3}$$

are radially symmetric, where Ω is the open unit ball in $R^N(N \geq 2)$, $f, g : R^+ \times R^+ \to R^+$ and $a, b : R^+ \to (0, \infty)$ are continuous function, $R^+ = [0, \infty)$. Yulian An [6] investigated uniqueness of positive solutions of the system

$$\begin{cases} \left(r^{(N-1)}u'\right)' = -a(r) r^{(N-1)} f(u(r), v(r)) & r \in (0,1) \\ \left(r^{(N-1)}v'\right)' = -b(r) r^{(N-1)} g(u(r), v(r)) & r \in (0,1) \\ u'(0) = v'(0) = u(1) = v(1) = 0. \end{cases} \tag{4}$$

M. Chhetri and P. Girg [5] investigated nonexistence of nonnegative solutions to the BVP:

$$\begin{cases} -\Delta_p u = \lambda f(u) & in\ B \\ u = 0 & on\ \partial B \end{cases} \tag{5}$$

Where $p > 1, \lambda > 0$ and B is the open unite ball in R^N. We consider the system (1) and make the follwing assumptions:

1.1. **(H.1)** $f, g : R^+ -> R^+$ are continuous, nondecreasing and C^1 on $(0, +\infty)$ and

$$\limsup_{x \to 0^+} x f'(x) < \infty, \quad \limsup_{x \to 0^+} x g'(x) < \infty$$

1.2. **(H.2)** There exist $m \in (p-1, p^*)$ such that $\lim_{x->\infty} \frac{f(x)}{x^m} > 0$ and $\lim \inf_{x->x^+} \frac{f(x)}{x^m} > 0$ where $p^* = \frac{Np}{N-p}$ for $p < N$ and $p^* = +\infty$ for $p \geq N$ is the critical exponent.

1.3. **(H.3**) There exist $n \in (q-1, q^*)$ such that $\lim_{x \to \infty} \frac{g(x)}{x^n} > 0$ and $\liminf_{x \to 0^+} \frac{g(x)}{x^n} > 0$ where $q^* = \frac{Nq}{N-q}$ for $q < N$ and $q^* = +\infty$ for $q \geq N$ is the critical exponent and for $m_1 > m, n_1 > n, \frac{f(x)}{x^{m_1}}$ and $\frac{g(x)}{x^{n_1}}$ are nonincreasing for x large.

Due to [3], studying nonnegative solution of (1) is equivalent to studying positive solution of

$$\begin{cases} -\left(r^{N-1}\phi_p(u')\right)' = \lambda r^{N-1} f(v(r)) & r \in (0,1) \\ -\left(r^{N-1}\phi_p(v')\right)' = \mu r^{N-1} g(u(r)) & r \in (0,1) \\ u'(0) = v'(0) = u(1) = v(1) = 0 \end{cases} \tag{6}$$

where $\phi_p(s) = |s|^{p-2}s, \phi_q(s) = |s|^{q-2}s$ for $s \neq 0$ and $\phi_p(0) = \phi_q(0) = 0$. Our main result is:

1.4. Theorem 1. Let (H.1)-(H.3) hold.

Then there exist a positive number η, such that the system (6) has at most one positive solution for $\min\left(\lambda\mu^{m(q'-1)}, \mu\lambda^{n(p'-1)}\right) \geq \eta$.

1.5. Remark 2. For reader's convenience we list some of the properties of the function $\phi_p : R \to R$, defined above, that are relevant in this paper. Namely,

(i) ϕ_p is an odd increasing homeomorphism of R onto itself;

(ii) the inverse mapping of ϕ_p, denoted by $(\phi_p)_{-1}$, is given by $(\phi_p)_{-1} = \phi_{p'}$

where $\frac{1}{p} + \frac{1}{p'} = 1$.

1.6. Lemma 3. Let h be continuous on R^+ and C^1 on $(0,+\infty)$ such that $\limsup_{x \to 0^+} xh < \infty$.

Let M, ε, r be positive numbers with $\varepsilon < 1$. Then there exists a positive number C such that $|h(\gamma x) - \gamma^T h(x)| \leq C(1-\gamma)$ for $\varepsilon \leq \gamma < 1$ and $0 \leq x \leq M$.

Poof. Let $0 \leq x \leq M$. Define $H(\gamma) = h(\gamma x) - \gamma^r h(x), \varepsilon \leq \gamma < 1$. By the mean value theorem, there e exists $c \in (\gamma, 1)$ such that

$$\begin{aligned} |H(\gamma)| &= |H(\gamma) - H(1)| = (1-\gamma)\left|xh'(cx) - rc^{r-1}h(x)\right| \\ &\leq C(1-\gamma), \end{aligned}$$

where

$$C = \frac{\sup\{|yh'(y)| : 0 < y \leq M\}}{\varepsilon} + r\max(\varepsilon^{r-1}, 1)\sup\{|h(y)| : y \leq M\}.$$

Lemma 4. Let (u, v) be a positive solution of the system (6). Then there exist positive constants $M_i, i = 1, 2, 3, 4$ and M independent of u, v such that

$$M_1\left(\lambda\mu^{m(q'-1)}\right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}}(1-r) \leq u(r) \leq M_2\left(\lambda\mu^{m(q'-1)}\right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}}(1-r)$$

$$M_3\left(\mu\lambda^{n(p'-1)}\right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}}(1-r) \leq v(r) \leq M_4\left(\mu\lambda^{n(p'-1)}\right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}}(1-r)$$

for $\min\left(\lambda\mu^{m(q'-1)}, \mu\lambda^{n(p'-1)}\right) \geq M$ and $0 < r < 1$.

Proof. Let (u, v) be a positive solution of the system (6). By integrating two equations in (6) we obtain respectively,

$$\phi_p(u'(r)) = -\frac{1}{r^{N-1}}\int_0^r \lambda\tau^{(N-1)} f(v)\, d\tau ds \tag{7}$$

$$\phi_q(v'(r)) = -\frac{1}{r^{N-1}}\int_0^r \mu\tau^{(N-1)} g(u)\, d\tau ds \tag{8}$$

It is clear that

$$u(r) = \int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{p'-1} \left(\int_0^s \lambda\tau^{(N-1)} f(v)\, d\tau\right)^{p'-1} ds. \tag{9}$$

$$v(r) = \int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u)\, d\tau\right)^{q'-1} ds. \tag{10}$$

From now on, we shall denote by $C_i, i=1,2,....,$ positive constants independent of u, v, λ, μ.

$$u\left(\frac{1}{2}\right) \geq \int_{\frac{1}{2}}^{1}\left(\frac{1}{s^{N-1}}\right)^{p'-1}\left(\int_0^{\frac{1}{2}}\lambda\tau^{(N-1)}f(v)\,d\tau\right)^{(p'-1)}ds$$

$$\geq \left(\frac{\lambda}{2}f\left(v\left(\frac{1}{2}\right)\right)\right)^{(p'-1)}\left(\int_0^{\frac{1}{2}}\tau^{(N-1)}d\tau\right)^{(p'-1)} \tag{11}$$

$$=\left(\frac{\lambda f\left(v(\frac{1}{2})\right)}{N2^{(N+1)}}\right)^{p'-1}.$$

Similary,

$$v\left(\frac{1}{2}\right) \geq \left(\frac{\mu g\left(u\left(\frac{1}{2}\right)\right)}{N2^{(N+1)}}\right)^{q'-1}. \tag{12}$$

By (H.2) and (H.3) , we have

$$f(v) \geq K_1 v^m \tag{13}$$

$$g(u) \geq K_2 u^n. \tag{14}$$

Both (13) , (14) together with (11) , (12) give us

$$u\left(\frac{1}{2}\right) \geq \left(\frac{K_1}{N2^{(N+1)}}\right)^{p'-1}\lambda^{p'-1}\left(v\left(\frac{1}{2}\right)\right)^{m(p'-1)}, \tag{15}$$

$$v\left(\frac{1}{2}\right) \geq \left(\frac{K_2}{N2^{(N+1)}}\right)^{q'-1}\mu^{q'-1}\left(u\left(\frac{1}{2}\right)\right)^{n(q'-1)}. \tag{16}$$

It is clear that

$$u\left(\frac{1}{2}\right) \geq C_1\left(\lambda\mu^{m(q'-1)}\right)^{\frac{p'-1}{1-mn(q'-1)(p'-1)}}, \tag{17}$$

$$v\left(\frac{1}{2}\right) \geq C_2\left(\mu\lambda^{n(p'-1)}\right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}}. \tag{18}$$

It follows from (13), (14), (17) , (18) that for $r \geq \frac{1}{2}$,

$$
\begin{aligned}
-u'(r) &= \left(\frac{\lambda}{r^{(N-1)}}\right)^{p'-1}\left(\int_0^r s^{(N-1)} f(v(s))ds\right)^{(p'-1)} \\
&\ge \lambda^{p'-1}\left(\int_0^{\frac{1}{2}} s^{(N-1)} f(v(s))ds\right)^{p'-1} \\
&\ge \left(\frac{\lambda f\left(v\left(\frac{1}{2}\right)\right)}{N2^N}\right)^{p'-1} \ge \left(\frac{K_1}{N2^N}\right)^{p'-1}\lambda^{p'-1}\left(v\left(\frac{1}{2}\right)\right)^{m(p'-1)} \\
&= C_3\left(\lambda^{p'-1}\mu^{m(q'-1)(p'-1)}\right)^{\frac{1}{1-mn(q'-1)(p'-1)}},
\end{aligned}
$$

and after integrating ,

$$u(r) \ge C_3\left(\lambda\mu^{m(q'-1)}\right)^{\frac{p'-1}{1-mn(q'-1)(p'-1)}}(1-r), \qquad r \ge \frac{1}{2}. \tag{19}$$

In a similar manner,

$$v(r) \ge C_4\left(\mu\lambda^{n(p'-1)}\right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}}(1-r), \qquad r \ge \frac{1}{2}. \tag{20}$$

Since u, v are decreasing , this implies that there exist positive constants M_1, M_2 independent of u, v such that the left-side inequalities for u, v in theorem 1 hold. From (9) , (10) , we have

$$|u|_0 \le \left(\lambda f\left(|v|_0\right)\right)^{p'-1}, \; |v|_0 \le \left(\mu g\left(|u|_0\right)\right)^{q'-1} \tag{21}$$

where $|.|_0$ denotes the sup-norm. By the conditions (H.2) , (H.3) we have from (21) that

$$
\begin{aligned}
|u|_0 &\le \left(\lambda f\left(|v|_0\right)\right)^{p'-1} \le C_5\lambda^{p'-1}\left(|v|_0\right)^{m(p'-1)}, \\
|v|_0 &\le \left(\mu g\left(|u|_0\right)\right)^{q'-1} \le C_6\mu^{q'-1}\left(|u|_0\right)^{n(q'-1)}.
\end{aligned}
$$

It is clear that

$$|u|_0 \le C_7\left(\lambda\mu^{m(q'-1)}\right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}},$$

similarly,

$$|v|_0 \le C_8\left(\mu\lambda^{n(p'-1)}\right)^{\frac{(q'-1)}{1-mn(q'-1)(p'-1)}}.$$

Using this in the equation for u' gives

$$-u'(r) \le \left(\lambda f\left(|v|_0\right)\right)^{(p'-1)} \le C_9\left(\lambda\mu^{m(q'-1)}\right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}},$$

In a similar manner, we have the upper estimate for $v(r)$. This complete the proof of Lemma 4.

Proof of Theorm 1. Let (u, v) and (u_1, v_1) be positive solution of (6) and let min $(\lambda\mu^{m(q'-1)}, \mu\lambda^{p'-1})$ be large enough so that Lemma 4 applies . By Lemma 4,

$$\frac{M_1}{M_2}u_1 \le u \le \frac{M_2}{M_1}u_1 \quad on \quad (0,1).$$

Let $\alpha = \sup\{c > 0 : u \ge c u_1 in (0,1)$. Then clearly $\alpha_0 < \alpha < \infty$ and $u \ge \alpha u_1$ in $(0,1)$, where $\alpha_0 = \frac{M_1}{M_2}$. We claim that $\alpha \ge 1$. Suppose to the contrary that $\alpha < 1$. Since

$$\left(r^{N-1}\phi_p(u')\right)' = -\lambda r^{N-1} f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u) d\tau\right)^{q'-1} ds$$

$$\left(r^{N-1}\phi_p(\alpha u_1')\right)' = -\lambda r^{N-1} \alpha f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u) d\tau\right)^{q'-1} ds\right),$$

it follows that

$$\left[r^{N-1}\left(\phi_p(u') - \phi_p(\alpha u_1')\right)\right]' \le -\lambda r^{N-1}\left[f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(\alpha u_1) d\tau\right)^{q'-1} ds\right) - \alpha f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds\right)\right] \tag{22}$$

Let $n_1 > n_2 > n, m_1 > m_2 > m.$ We claim that

$$\left(\int_0^s \tau^{(N-1)} g(\alpha u_1) d\tau\right)^{q'-1} > \alpha^{n_1(q'-1)} \left(\int_0^s \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1}, s \ge 0. \tag{23}$$

Since $\alpha \ge \alpha_0$ and $\frac{g(x)}{x^{n_2}}$ is nonincreasing for $x >> 1$,

$$g(\alpha x) \ge \alpha^{n_2} g(x) \text{for } x >> 1.$$

Let $\frac{1}{2} < T < 1$. By Lemma 4,

$$u_1(s) \ge M_1(1-T)\left(\lambda\mu^{m(q'-1)}\right)^{\frac{(p''-1)}{1-mn(q'-1)(p'-1)}} >> 1, \ s \le T,$$

and therefore

$$\left(\int_0^s \tau^{(N-1)}\left(g(\alpha u_1) - \alpha^{n_1} g(u_1)\right) d\tau\right)^{q'-1} \ge \left(\alpha^{n_2} - \alpha^{n_1}\right)^{(q'-1)} \left(\int_0^s \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} \ge 0 , s \le T.$$

For $s > T$,

$$\begin{aligned}
&\left(\int_0^s \tau^{(N-1)}\left(g(\alpha u_1) - \alpha^{n_1} g(u_1)\right) d\tau\right)^{q'-1} \\
&= \left(\int_0^T \tau^{(N-1)}\left(g(\alpha u_1) - \alpha^{n_1} g(u_1)\right) d\tau\right)^{q'-1} + \left(\int_T^s \tau^{(N-1)}\left(g(\alpha u_1) - \alpha^{n_1} g(u_1)\right) d\tau\right)^{q'-1} \\
&\ge \left(\alpha^{n_2} - \alpha^{n_1}\right)^{q'-1} \left(\int_0^T \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} - \left(C(1-T)(1-\alpha)\right)^{q'-1},
\end{aligned}$$

where we have used Lemma 3 with $h = g$. Since

$$\left(\int_0^T \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} \geq \left(\int_0^{\frac{1}{2}} \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} \geq \left(\frac{g\left(u_1\left(\frac{1}{2}\right)\right)}{N2^N}\right)^{q'-1}$$

and since there exist a positive number $k > 0$ such that

$$\alpha^{n_2} - \alpha^{n_1} \geq k(1-\alpha) \text{ for } \alpha_0 \leq \alpha \leq 1,$$

It follow that

$$\left(\int_0^s \tau^{(N-1)} \left(g(\alpha u_1) - \alpha^{n_1} g(u_1)\right) d\tau\right)^{q'-1} > 0 \text{ , } s > T$$

if T is sufficiently close to 1. This proves the claim.

Inserting (23) into (22) and integrating gives

$$z^{N-1}\left(\phi_p(u') - \phi_p(\alpha u_1')\right)(z) \leq -\lambda \int_0^z B(\alpha, r) dr,$$

where

$$B(\alpha, r) = r^{N-1}\left[f\left(\alpha^{n_1(q'-1)} \int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds\right.\right.$$
$$\left.\left. - \alpha f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds\right)\right]$$

Using (13) , (14) and Lemma 4, we obtain for $r \leq T$,

$$\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds \geq \int_T^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^T \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1}$$
$$\geq \left(\frac{\mu T^N (1-T)}{N}\right)^{q'-1} \left(g(u_1(T))\right)^{q'-1}$$
$$\geq \left(\frac{\mu T^N (1-T)}{N}\right)^{q'-1} (K_2)^{q'-1} (u_1(T))^{n(q'-1)}$$
$$\geq c_1(T) \left(\lambda \mu^{m(q'-1)}\right)^{\frac{p'-1}{1-mn(q'-1)(p'-1)}} >> 1$$

where $c_1(T) = \left(\frac{\mu T^N K_2}{N}\right)^{q'-1} M_1^{n(q'-1)} (1-T)^{(n+1)(q'-1)}$. Since $\frac{f(x)}{x^{m_1}}$ is nonincreasing for $x >> 1$,

$$f(\alpha^{n_1} x) \geq \alpha^{n_1 m_1} f(x) \text{ for } x >> 1$$

and therefore

$$B(\alpha, r) \geq r^{N-1} \left(\alpha^{n_1 m_1 (q'-1)} - \alpha\right) f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)^{q'-1} \left(\int_0^s \mu\tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds\right)$$
$$\geq c_2(T) r^{N-1} \left(\lambda \mu^{m(q'-1)}\right)^{\frac{m(p'-1)}{1-mn(q'-1)(p'-1)}} (1-\alpha) > 0, r \leq T \tag{24}$$

where $c_2(T) = K_1(c_1(T))^m k_0$ and k_0 is a positive constant such that

$\alpha^{n_1 m_1 (q'-1)} - \alpha \geq k_0(1-\alpha)$ for $\alpha_0 \leq \alpha \leq 1$.

This shows that

$$z^{N-1}\left(\phi_p(u') - \phi_p(\alpha u_1')\right)(z) < 0, \qquad 0 < z \leq T.$$

For $z > T$, we have by lemma 3 and (24) that

$$\int_0^z B(\alpha, r)dr \geq \int_0^{\frac{1}{2}} B(\alpha, r)dr + \int_T^z B(\alpha, r)dr$$

$$\geq \frac{c_2\left(\frac{1}{2}\right)}{N2^N}\left(\lambda\mu^{m(q'-1)}\right)^{\frac{m(p'-1)}{1-mn(q'-1)(p'-1)}}(1-\alpha) - \left(C(1-T)(1-\alpha)\right)^{(q'-1)} > 0$$

for large $\lambda\mu^{m(q'-1)}$ and $\mu\lambda^{n(p'-1)}$ and T sufficiently close to 1. Hence

$$\left(\phi_p(u') - \phi_p(\alpha u_1')\right)(z) < 0 \ , \ 0 < z \leq 1,$$

from which it follows that there exists $\tilde{\alpha} > \alpha$ such that $u \geq \tilde{\alpha} u_1$ in $(0,1)$, a contradiction.

Thus $\alpha \geq 1$ and so $u = u_1$ in $(0,1)$. Using the formulas for v and v_1 it follows that $v = v_1$ in $(0,1)$, completing the proof of Theorem1.

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