

Modified Projective Synchronization of a New Hyperchaotic System

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Abstract. This paper investigates the modified projective synchronization (MPS) of a new hyperchaotic system. The different nonlinear feedback controllers are designed by an active control method for synchronization of two hyperchaotic systems with the same or different structures. In addition, the MPS of the new hyperchaotic system with unknown parameters including the unknown coefficients of nonlinear terms is studied by using adaptive control. Numerical simulations are presented to show the effective of the proposed hyperchaos synchronization scheme.

Keywords: hyperchaotic system; MPS; active control; adaptive control

1. Introduction

Synchronization and controlling chaotic dynamical systems have recently attracted a great deal of attention since the early work on the synchronizing of chaos of Pecora and Carrol [1] and on the controlling of chaos of by Ott et al [2] was published in 1990. Up to now, various schemes of synchrony such as complete synchronization [3], phase synchronization [4], lag synchronization [5], generalized synchronization [6], anti-synchronization [7] etc., have been described and studied.

In recent year, projective synchronization, which has been first reported by Mainieri and Rehacek [8] in partially linear systems and developed by many authors [9-10], is the most noticeable one. More recently, a new synchronization method called 'Modified projective synchronization' is proposed in [11] where the chaotic systems can synchronize up to a constant scaling matrix. Modified projective synchronization in two chaotic systems with unknown parameters is realized by using adaptive control [12-14]. Furthermore, the PS has been used in the research of secure communication [15] due to the unpredictability of the scaling factor.

This paper addresses MPS of a new hyperchaotic system. MPS not only between two identical hyperchaotic systems but also between two different hyperchaotic systems are realized based on active control theory. Furthermore, we also present an effective scheme for MPS in two hyperchaotic systems with uncertainties rendered by the unknown coefficients of nonlinear terms, however, the current study mainly take into account chaotic system with uncertain linear terms coefficients [12-14].

2. Systems description

Recently, we constructed a new hyperchaotic system [16], which is described by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 + cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 = x_1^2 - hx_3 \\ \dot{x}_4 = -dx_1 \end{cases} \quad (1)$$

in which a, b, c, d and h are constant parameters, and x_1, x_2, x_3 and x_4 are the state variables. When parameters $a=20, b=1, c=10.6, d=3.7$ and $h=2.8$, system (1) has two positive Lyapunov exponents $\lambda_1=0.8298$ and

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$\lambda_2=0.1154$, and is hyperchaotic.

Recently, Qi et al.[17] developed a new four-dimensional (4D) continuous autonomous chaotic system, in which each equation in the system contains a 2-term cross product. Bifurcation analysis further shows that the new hyperchaotic system has very rich bifurcations in different directions and extremely complicated dynamics.

The hyperchaotic system is given by

$$\begin{cases} \dot{y}_1 = l(y_2 - y_1) + y_2 y_3 \\ \dot{y}_2 = n(y_2 + y_1) - y_1 y_3 \\ \dot{y}_3 = -m y_3 - p y_4 + y_1 y_2 \\ \dot{y}_4 = -q y_4 + g y_3 + y_1 y_3 \end{cases} \quad (2)$$

where l, n, m, p, q and g are constant parameters, and y_1, y_2, y_3 and y_4 are the state variables. When parameters $l=42.5, n=24, m=13, q=20, p=50, g=40$, system (2) shows hyperchaotic behavior, as shown in Fig.1.

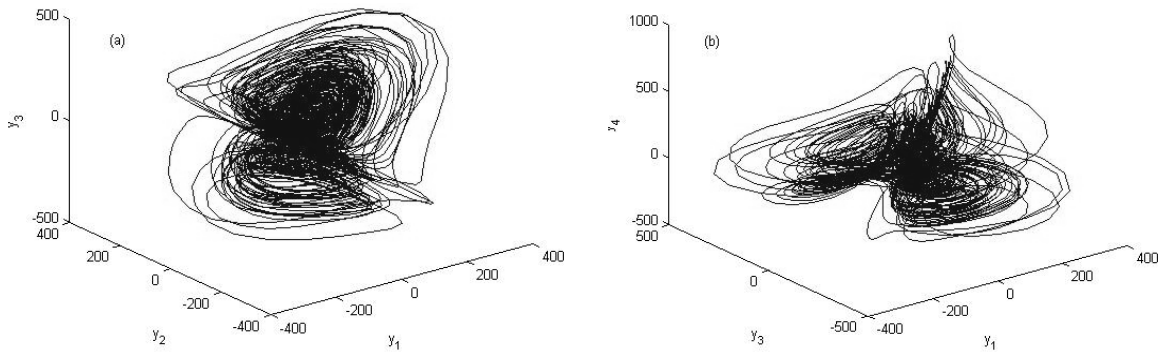


Fig.1. Hyperchaotic attractors (a) y_1 - y_2 - y_3 space; (b) y_1 - y_2 - y_4 space

3. Modified Projective synchronization of the new hyperchaotic system

In this section, based on active control theory, modified projective synchronizations not only between two identical hyperchaotic systems but also between two different hyperchaotic systems are achieved.

3.1. MPS of two identical hyperchaotic systems

In this subsection, by using active theory, we obtain the condition for MPS between two identical hyperchaotic systems. We choose system (1) as the drive system.

And the response system with control input reads

$$\begin{cases} \dot{z}_1 = a(z_2 - z_1) + u_1 \\ \dot{z}_2 = b z_1 + c z_2 - z_1 z_3 + z_4 + u_2 \\ \dot{z}_3 = z_1^2 - h z_3 + u_3 \\ \dot{z}_4 = -d z_1 + u_4 \end{cases} \quad (3)$$

where $u_i (i=1,2,3,4)$ are the nonlinear control laws such that two chaotic systems can be synchronized with a scaling factor α . Define the error signals as $e_i = x_i - \alpha_i z_i (i=1,2,3,4)$.

We have the following error dynamics:

$$\begin{cases} \dot{e}_1 = a(x_2 - x_1) - \alpha_1 a(z_2 - z_1) - \alpha_1 u_1 \\ \dot{e}_2 = b x_1 + c x_2 - x_1 x_3 + x_4 - \alpha_2 b z_1 - \alpha_2 c z_2 + \alpha_2 z_1 z_3 - \alpha_2 z_4 - \alpha_2 u_2 \\ \dot{e}_3 = x_1^2 - h x_3 - \alpha_3 z_1^2 + \alpha_3 h z_3 - \alpha_3 u_3 \\ \dot{e}_4 = -d x_1 + \alpha_4 d z_1 - \alpha_4 u_4 \end{cases} \quad (4)$$

For two identical chaotic systems without ($u_i = 0$), if the initial condition of two systems is different, the

trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control laws.

According to the clue of active control, the control functions u_i ($i=1, 2, 3, 4$) can be designed

$$\begin{cases} u_1 = \frac{1}{\alpha_1} [a(\alpha_2 - \alpha_1)z_2 - v_1] \\ u_2 = \frac{1}{\alpha_2} [bz_1(\alpha_1 - \alpha_2) + (\alpha_4 - \alpha_2)z_4 - x_1x_2 - v_2] + z_1z_2 \\ u_3 = \frac{1}{\alpha_3} (x_1^2 - v_3) - z_1^2 \\ u_4 = \frac{1}{\alpha_4} [dz_1(\alpha_4 - \alpha_1) - v_4] \end{cases} \quad (5)$$

Then the error system (4) becomes

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + v_1 \\ \dot{e}_2 = be_1 + ce_2 + e_4 + v_2 \\ \dot{e}_3 = -he_3 + v_3 \\ \dot{e}_4 = -de_1 + v_4 \end{cases} \quad (6)$$

The error system (4) to be controlled is a linear system with control input v_1, v_2, v_3 and v_4 as the function of the error states e_1, e_2, e_3 and e_4 . As long as these feedbacks stabilize the system, e_1, e_2, e_3 and e_4 converge to zero as time $t \rightarrow \infty$. This implies that two different hyperchaotic systems are synchronized with feedback control. There are many possible choices for the control v_1, v_2, v_3 and v_4 . We choose

$$\begin{cases} v_1 = -ae_2 \\ v_2 = -be_1 - (1+c)e_2 - e_4 \\ v_3 = 0 \\ v_4 = de_1 - e_4 \end{cases} \quad (7)$$

Then the error dynamical system is

$$\dot{e}_1 = -ae_1, \dot{e}_2 = -e_2, \dot{e}_3 = -he_3, \dot{e}_4 = -e_4 \quad (8)$$

Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$$

The time derivation of the Lyapunov function along the trajectory is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = -[ae_1^2 + e_2^2 + he_3^2 + e_4^2]$$

Since the Lyapunov function V is positive definite and its derivative \dot{V} is negative definite in the neighborhood of the zero solution for the system (6). In light of the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. This implies that the two identical hyperchaotic systems are synchronized.

Numerical simulations are given to verify the effectiveness of the controllers (5). Choose the following scaling factors $\alpha_1 = -2, \alpha_2 = 2, \alpha_3 = 0.4, \alpha_4 = -0.3$. The fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. We assume that the initial condition, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (2, 2.5, 4, 3.5)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1, 1, -3, -3)$, are employed, respectively. MPS of the system (1) and (3) via control laws (5) are shown in Fig.2 and Fig.3. Fig.2 displays the time response of the modified projective synchronization errors $e_1, e_2, e_3, e_4 \rightarrow 0$, as $t \rightarrow \infty$ implying that all the state variables tend to be synchronized in a proportional. Fig.3 depicts the projection of the synchronized attractors of the drive system (1) (dotted line) and the response system (3) (solid line), which illustrates a projective modified

synchronization with $\alpha_1 = -2$, $\alpha_2 = 2$, $\alpha_3 = 0.4$, $\alpha_4 = -0.3$.

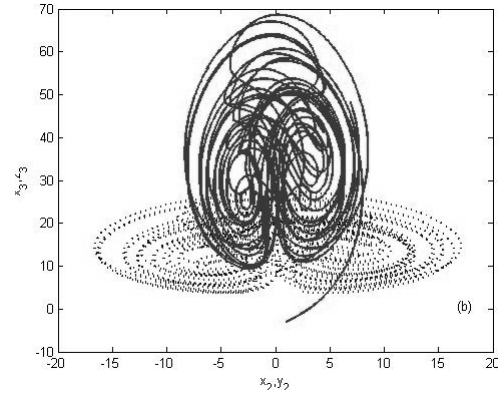
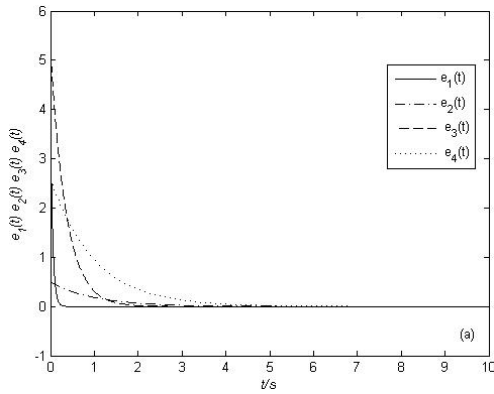


Fig2. Error signals between drive and response systems Fig3. Chaotic attractors when $\alpha_1 = -2$, $\alpha_2 = 2$, $\alpha_3 = 0.4$, $\alpha_4 = -0.3$

3.2. MPS of two different hyperchaotic systems

In this subsection, by using active theory, we obtain the condition for MPS between two different hyperchaotic systems. We choose system (2) and system (3) as the drive system and the respond system, respectively.

Similarly, we obtain the following error dynamical system

$$\begin{cases} \dot{e}_1 = l(y_2 - y_1) + y_2 y_3 - \alpha_1 a(z_2 - z_1) - \alpha_1 u_1 \\ \dot{e}_2 = n(y_1 + y_2) - y_1 y_3 - \alpha_2 b z_1 - \alpha_2 c z_2 + \alpha_2 z_1 z_3 - \alpha_2 z_4 - \alpha_2 u_2 \\ \dot{e}_3 = -m y_3 - p y_4 + y_1 y_2 - \alpha_3 z_1^2 + \alpha_3 h z_3 - \alpha_3 u_3 \\ \dot{e}_4 = -q y_4 + g y_3 + y_1 y_3 + \alpha_4 d z_1 - \alpha_4 u_4 \end{cases} \quad (9)$$

To guarantee the error dynamical system (9) converge to the origin asymptotically, we choose the active control functions u_i ($i=1, 2, 3, 4$) as follows:

$$\begin{cases} u_1 = \frac{1}{\alpha_1} [(l\alpha_2 - a\alpha_1)z_2 + y_2 y_3 - v_1] + (a - l)z_1 \\ u_2 = \frac{1}{\alpha_2} [z_1(n\alpha_1 - b\alpha_2) - y_1 y_3 - v_2] + (n - c)z_2 - z_4 + z_1 z_3 \\ u_3 = \frac{1}{\alpha_3} (-p\alpha_4 z_4 + y_1 y_2 - v_3) + (h - m)z_3 - z_1^2 \\ u_4 = \frac{1}{\alpha_4} (g\alpha_3 z_3 + y_1 y_3 - v_4) - q z_4 + d z_1 \end{cases} \quad (10)$$

This leads to

$$\begin{cases} \dot{e}_1 = l(e_2 - e_1) + v_1 \\ \dot{e}_2 = n(e_1 + e_2) + v_2 \\ \dot{e}_3 = -m e_3 - p e_4 + v_3 \\ \dot{e}_4 = g e_3 - q e_4 + v_4 \end{cases} \quad (11)$$

According to the original method of active control, v_i ($i=1, 2, 3, 4$) are taken as

$$\begin{cases} v_1 = -l e_2 \\ v_2 = -n e_1 - (n + 1) e_2 \\ v_3 = p e_4 \\ v_4 = -g e_3 \end{cases} \quad (12)$$

Hence the error dynamical system (9) becomes

$$\dot{e}_1 = -le_1, \dot{e}_2 = -e_2, \dot{e}_3 = -me_3, \dot{e}_4 = -qe_4 \quad (13)$$

Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$$

The time derivation of the Lyapunov function along the trajectory is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = -[le_1^2 + e_2^2 + me_3^2 + qe_4^2]$$

According to the Lyapunov stability theory, the error dynamical system (9) is asymptotically stable at the origin. Therefore, MPS between the system (2) and (3) is achieved with the controllers (10).

To verify and demonstrate the effectiveness of the proposed methods, we consider the numerical simulation. Choose the following scaling factors $\alpha = (-2, 3, 0.8, -3)$. Let the parameters $a=20, b=1, c=10.6, d=3.7$ and $h=2.8$, thus the response system (3) is hyperchaotic, choose parameters $l=42.5, n=24, m=13, q=20, p=50$ and $g=40$, then the drive system (2) is also hyperchaotic. Considered the systems given in (2) and (3), the initial values of the drive system and response system are taken as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.1, -0.2, 0.3, -0.4), (z_1(0), z_2(0), z_3(0), z_4(0)) = (-0.1, 0.2, -0.3, 0.4)$, respectively. We can observe that the drive system (2) and the response system (3) achieve MPS immediately (see Fig.4) after the control is activated although the initial condition are different. The state variables of the system (2) x_i ($i=1,2,3,4$) are in proportion to that of the system (3) y_i ($i=1,2,3,4$) with different scaling factors $-2, 3, 0.8, -3$, respectively. As a result of the large compression and stretch associated to the different proportions, the shape of the response system attractor become remarkably different from that of the drive system attractor. See Fig.5.

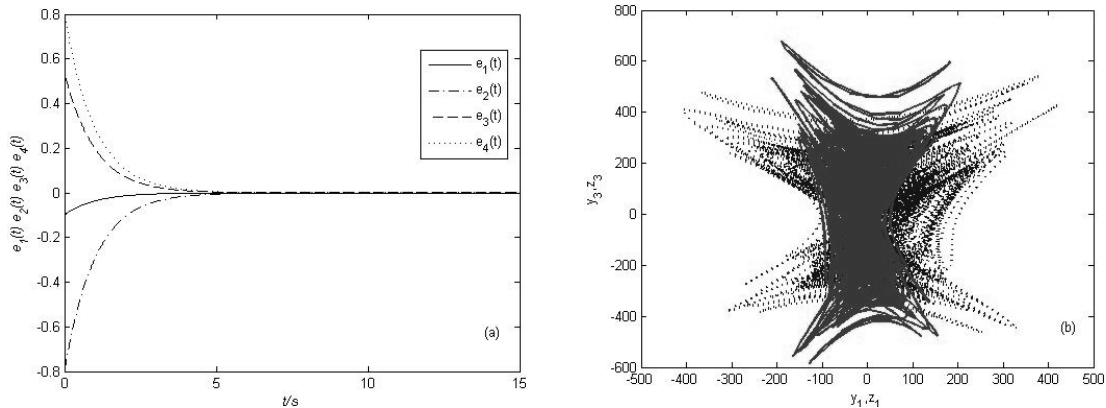


Fig4. Error signals between drive and response systems Fig5. Chaotic attractors when $\alpha_1 = -2, \alpha_2 = 3, \alpha_3 = 0.8, \alpha_4 = -3$

4. MPS between two uncertain hyperchaotic systems

In [11-14], projective synchronization between chaotic systems with unknown parameters is achieved, when the coefficients of linear terms are unknown. However, for any physical system, it is more important to know the nonlinear terms. Therefore, in this section, we study modified projective synchronization in the new hyperchaotic system with unknown coefficients of nonlinear terms.

For the hyperchaotic system (1), assume the parameters a, b, c, d, h and the coefficients of two nonlinear terms (denote them as m, n) are unknown. So the system (1) can be written as

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 + cx_2 + mx_1x_3 + x_4 \\ \dot{x}_3 = nx_1^2 - hx_3 \\ \dot{x}_4 = -dx_1 \end{cases} \quad (14)$$

The response system with control has the following form

$$\begin{cases} \dot{z}_1 = a(z_2 - z_1) + u_1 \\ \dot{z}_2 = bz_1 + cz_2 + mz_1z_3 + z_4 + u_2 \\ \dot{z}_3 = nz_1^2 - hz_3 + u_3 \\ \dot{z}_4 = -dz_1 + u_4 \end{cases} \quad (15)$$

Define the error vectors as $e_i = x_i - \alpha_i y_i$ ($i=1, 2, 3$), the error system is

$$\begin{cases} \dot{e}_1 = a[e_2 - e_1 + (\alpha_2 - \alpha_1)z_2] - \alpha_1 u_1 \\ \dot{e}_2 = b[e_1 + (\alpha_1 - \alpha_2)z_1] + ce_2 + m(x_1x_3 - \alpha_2z_1z_3) + e_4 + (\alpha_4 - \alpha_2)z_4 - \alpha_2 u_2 \\ \dot{e}_3 = -he_3 + n(x_1^2 - \alpha_3z_1^2) - \alpha_3 u_3 \\ \dot{e}_4 = -d[e_1 + (\alpha_1 - \alpha_4)z_1] - \alpha_4 u_4 \end{cases} \quad (16)$$

The following control laws and update laws for system (16) are designed

$$\begin{cases} u_1 = \frac{1}{\alpha_1} [\hat{a}(e_2 - e_1 + (\alpha_2 - \alpha_1)z_2) + k_1 e_1] \\ u_2 = \frac{1}{\alpha_2} [\hat{b}(e_1 + (\alpha_1 - \alpha_2)z_1) + \hat{c}e_2 + \hat{m}(x_1x_3 - \alpha_2z_1z_3) + e_4 + (\alpha_4 - \alpha_2)z_4 + k_2 e_2] \\ u_3 = \frac{1}{\alpha_3} [-\hat{h}e_3 + \hat{n}(x_1^2 - \alpha_3z_1^2) + k_3 e_3] \\ u_4 = \frac{1}{\alpha_4} [-\hat{d}(e_1 + (\alpha_1 - \alpha_4)z_1) + k_4 e_4] \end{cases} \quad (17)$$

and

$$\begin{cases} \dot{\hat{a}} = (e_2 - e_1 + (\alpha_2 - \alpha_1)z_2)e_1 - k_5 e_a \\ \dot{\hat{b}} = (e_1 + (\alpha_1 - \alpha_2)z_1)e_2 - k_6 e_b \\ \dot{\hat{c}} = e_2^2 - k_7 e_c \\ \dot{\hat{d}} = -(e_1 + (\alpha_1 - \alpha_4)z_1)e_4 - k_8 e_d \\ \dot{\hat{h}} = -e_3^2 - k_9 e_h \\ \dot{\hat{m}} = (x_1x_3 - \alpha_2z_1z_3)e_2 - k_{10} e_m \\ \dot{\hat{n}} = (x_1^2 - \alpha_3z_1^2)e_3 - k_{11} e_n \end{cases} \quad (18)$$

where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{c} = \hat{c} - c, \tilde{d} = \hat{d} - d, \tilde{h} = \hat{h} - h, \tilde{m} = \hat{m} - m, \tilde{n} = \hat{n} - n, \hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{h}, \hat{m}$ and \hat{n} are estimated parameters of unknown parameters a, b, c, d, h, m, n , respectively. The control gain $k_i > 0$ ($i = 1, 2, \dots, 11$).

Then we obtain the following theorem.

Theorem1: For given nonzero scalar α_i ($i = 1, 2, 3, 4$), MPS between two systems (14) and (15) will occur by the adaptive control laws (17) and update laws (18).

Proof. Define a Lyapunov function

$$V = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_h^2 + e_m^2 + e_n^2)$$

The time derivative of the Lyapunov function along the trajectory of error system (16) is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c + e_d\dot{e}_d + e_h\dot{e}_h + e_m\dot{e}_m + e_n\dot{e}_n$$

$$\begin{aligned}
&= e_1(a(e_2 - e_1 + (\alpha_2 - \alpha_1)z_2) - \alpha_1 u_1) \\
&\quad + e_2(b(e_1 + (\alpha_1 - \alpha_2)z_1) + ce_2 + m(x_1 x_3 - \alpha_2 z_1 z_3) + e_4 + (\alpha_4 - \alpha_2)z_4 - \alpha_2 u_2) \\
&\quad + e_3(-he_3 + n(x_1^2 - \alpha_3 z_1^2) - \alpha_3 u_3) + e_4(-d(e_1 + (\alpha_1 - \alpha_4)z_1) - \alpha_4 u_4) \\
&\quad + e_a \dot{e}_a + e_b \dot{e}_b + e_c \dot{e}_c + e_d \dot{e}_d + e_h \dot{e}_h + e_m \dot{e}_m + e_n \dot{e}_n
\end{aligned} \tag{19}$$

By substituting Eqs.(17) and(18) into Eq.(19), we have

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_h^2 - k_{10} e_m^2 - k_{11} e_n^2 = -\mathbf{e}^T \mathbf{P} \mathbf{e}$$

where $\mathbf{e} = [e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d, e_h, e_m, e_n]^T$, $\mathbf{P} = \text{diag}\{k_1, k_2, \dots, k_{11}\}$.

Since $\dot{V} \leq 0$, we have $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d, e_h, e_m, e_n \rightarrow 0$ as $t \rightarrow \infty$, i.e. $\lim_{t \rightarrow \infty} \|\mathbf{e}\| = 0$. This completes the proof.

In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. We assume that the control gain $k_i = 1$ ($i = 1, 2, \dots, 11$), the initial values of the drive system and response system are taken as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (2, 2, 2, 2)$, $(z_1(0), z_2(0), z_3(0), z_4(0)) = (-2, -2, -2, -2)$, respectively, and the scaling factor $\alpha = (3, 4, -2, 2)$. The unknown parameters are chosen as $(a, b, c, d, h, m, n) = (20, 1, 10.6, 3.7, 2.8, -1, 1)$, in simulations so that the new system exhibits a chaotic behavior. MPS of the system (14) and (15) via adaptive control laws (17) and (18) are shown in Fig.6 and 7. Fig.6. displays the synchronization errors of system (14) and (15). Fig.7 shows that the estimations $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{h}(t), \hat{m}(t), \hat{n}(t)$ of the unknown parameters converge to $a=20, b=1, c=10.6, d=3.7, h=2.8, m=-1$ and $n=1$, as $t \rightarrow \infty$.

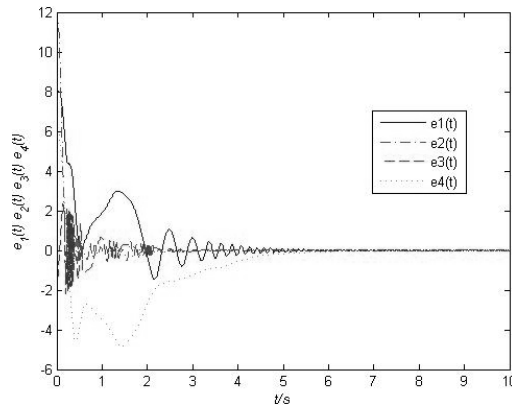


Fig6. Error signals between drive and response systems

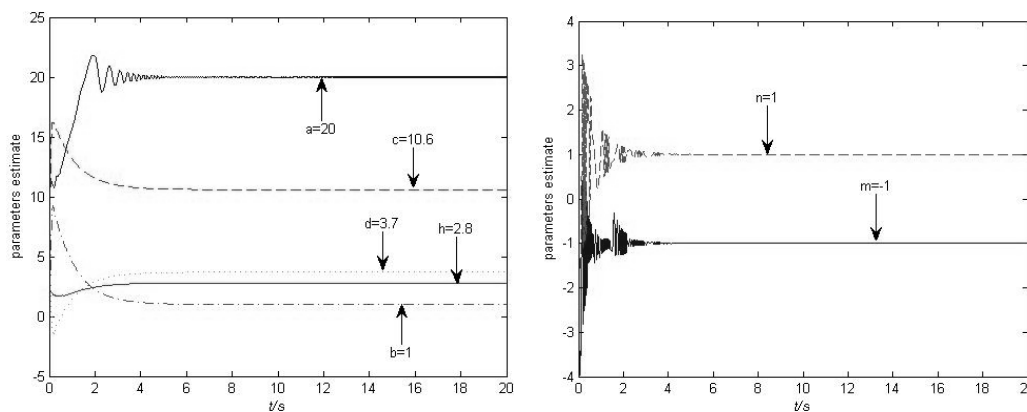


Fig.7 Estimated values for unknown parameters

5. Conclusion

In this paper, based on active control theory, we achieve modified projective synchronization of a new hyperchaotic system. In addition, the MPS of the new hyperchaotic system with uncertainties including the coefficients of nonlinear terms is obtained via adaptive control. Numerical simulations show the effectiveness of the analytical results.

6. References:

- [1] L. M. Pecora, T. L. Carroll. Synchronization in chaotic systems. *Phys. Rev. Lett.* 1990, **64**: 821-824.
- [2] Ott E, Grebogi C, Yorke JA. Controlling chaos. *Phys. Rev. Lett.* 1990, **64**: 1196-1199.
- [3] H.N.Agiza. Chaos synchronization of Lü dynamical system. *Nonlinear Anal.* . 2004, **58**: 11-20.
- [4] M.G.Rosenblum, A.S.Pikovsky, J.kurths. Phase synchronization of chaotic oscillators. *Phs.Rev.Lett.* 1996, **76**: 1804-1807.
- [5] Y Chen, XX Chen, SS Gu. Lag synchronization of structurally nonequivalent chaotic systems with time delay. *Nonlinear Anal.* 2007, **66**: 1929-1937.
- [6] YW Wang, ZH Guan. Generalized synchronization of continuous chaotic system. *Chaos Solitons Fractals.* 2006, **27**: 97-101.
- [7] ZL Wang. Anti-synchronization in two non-identical hyperchaotic systems with known or unknown parameters. *Commun Nonlinear Sci Numer Simulat.* inpress
- [8] R.Mainieri, J.Rehacek. Projective synchronization in three-dimensioned chaotic systems. *Phys.Rev.Lett.* . 1999, **82** : 3042-3045.
- [9] DL Xu,W.L.Ong, ZG Li. Criteria for the occurrence of projective synchronization in chaotic systems of arbitrary dimension. *Phys. Lett.* 2002, **A 305**: 167-172.
- [10] GL Wen, DL Xu. Observer-based control for full-state projective synchronization of a general class of chaotic maps in any dimension. *Phys. Lett.* 2004, **A 333** : 420-425.
- [11] GH Li. Modified projective synchronization of chaotic system. *Chaos Solitons Fractals.* 2007, **32**: 1786-1790.
- [12] J.H.Park. Adaptive controller design for modified projective synchronization of Genesio-Tesi chaotic system with uncertain parameters. *Chaos Solitons Fractals.* 2007, **34**: 1154-1159.
- [13] J.H.Park. Adaptive modified projective synchronization of a unified chaotic system with an uncertain parameter. *Chaos Solitons Fractals.* 2007, **34**: 1552-1559.
- [14] J.H.Park. Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters. *J. Comput. Appl. Math.* 2008, **213**: 288-293.
- [15] CC Wang, JP Su. A new adaptive structure control for chaotic synchronization and secure communication. *Chaos Solitons Fractals.* 2004, **20**: 967-977.
- [16] GL Cai, S Zheng, LX Tian. A new hyperchaotic system and its linear feedback control. *Chin. Phys.* 2008, **17**(11) in press.
- [17] GY Qi, ZQ Chen, ZZ Yuan. On a new asymmetric chaotic system. *Chaos, Solitons Fractals.* 2008, **37**: 409-423.