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A Random Network Coding-based ARQ Scheme and Performance Analysis for Wireless Broadcast

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Abstract. A wireless broadcast retransmission scheme based on the random network coding is put forward for the reliable broadcast in wireless broadcast system. With the strategy to allow the sender to combine and retransmit lost packets by using the random linear codes, the scheme can effectively reduce the number of retransmissions and improve the throughput efficiency. Based on the burst error channel modeled by a two-state Markov chain, analytic solutions are derived for the throughput efficiency of three schemes including the traditional Selective-Repeat ARQ (SR-ARQ), the XOR network coding-based ARQ (XOR-ARQ) and the random network coding-based ARQ (RNC-ARQ).The theoretical and simulation results show that the throughput efficiency of RNC-ARQ is considerably better than the SR-ARQ, while it can achieve the upper bound of throughput performance of the XOR-ARQ under the same channel conditions.

Keywords: broadcast; retransmission; throughput; random network coding

1. Introduction

Reliable wireless broadcast has gained significant interest with the emerging services such as IPTV and Video on Demand (VoD) in cellular networks and Worldwide Interoperability for Microwave Access (WiMAX) networks. Whereas multimedia data can tolerate residual errors to some extent, the file distribution application must be performed error-free. Automatic Repeat reQuest (ARQ) is the most common method for guaranteeing reliable communications due to its simple implementation and robustness. In general, ARQ schemes are normally classified in three basic types: Stop-and-Wait (SW), Go-Back-N (GB(N)) and Selective-Repeat (SR). But the traditional ARQ schemes are designed for the point-to-point communication. With node number increasing under broadcast scenario, the throughput performance of the classic ARQ schemes decreases rapidly.

Recently, Network Coding (NC), that allows intermediate nodes to combine packets before forwarding, has been demonstrated as an effective approach to improve the network performance for wireless broadcast [1-4]. In [1], Eryilmaz *et al.* demonstrate that broadcast using network coding outperforms traditional scheduling strategies in terms of delay in lossy networks. [2]-[4] focus on throughput and reliability gains obtained from XOR network coding. The core idea of these approaches is to allow the base station to retransmit an innovative coded packet produced by XORing a set of distinct lost packets across different receivers. Then, receivers may use the previously received packets to decode and recover new information from each coded packet. The approaches from [2]-[4] suffer from drawbacks from two aspects. On one hand, they require the base station to perform scheduling algorithm for each lost packet, and the worst complexity of the algorithm is $O(K \cdot 2^M)$ [5].On the other hand, some retransmission packets can only be retransmitted as original versions due to no appropriate packet to combine. Obviously, that would decrease the throughput of the network.

In this paper, a new wireless broadcast retransmission scheme based on the random network coding is proposed for the reliable wireless broadcast. Differing from the approach in [3] to use XOR coding, the base station combines all lost packets to a single one by random network coding for retransmission. Compared to [3], our scheme can greatly simplify the coding process as well as decrease the number of retransmissions. Note that most of the existing researches to date have focused on throughput gains of network coding under

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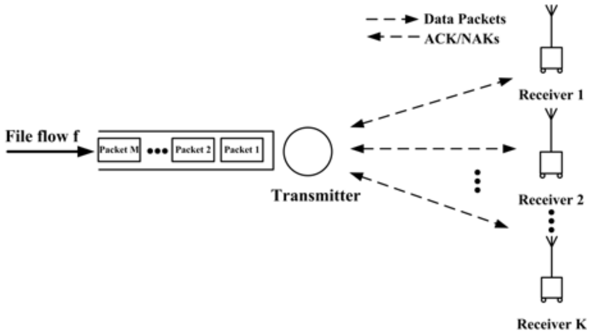
the binary symmetric channel [2-4]. We investigate the throughput performance of the proposed scheme under the burst error channel, which is modeled by a two-state Markov chain. Our contributions include (a) some analytic solutions for the throughput efficiency of the traditional SR-ARQ, the XOR network coding-based ARQ (XOR-ARQ) and the random network coding-based ARQ (RNC-ARQ) (b) some results on throughput performance influenced by different channel conditions.

The rest of the paper is organized as follows. In Section 2, the considered system model and assumptions are introduced. In Section 3, a wireless broadcast retransmission scheme based on the random network coding is elaborated. Analytic solutions of throughput efficiency for the RNC-ARQ, XOR-ARQ and SR-ARQ under the burst error channel are derived in Section 4. Results of the computer simulation for three schemes are discussed in Section 5. Our conclusions as well as future work are discussed in Section 6.

2. System Model

The considered broadcast system consists of K+1 stations, one being the base station and other K (K>1) being receivers（see Fig.1）. A given file flow f composed of M packets needs to be broadcasted to K receivers. It is assumed that transmissions occur in time slots and only one packet can be delivered per slot. All receivers use positive and negative acknowledgements (ACK/NAKs) to feed back. For simplicity, all the ACK/NAKs are assumed instantaneous and never lost. In addition, receivers are assumed to have the ability of perfect error-detect and infinite memory resource.



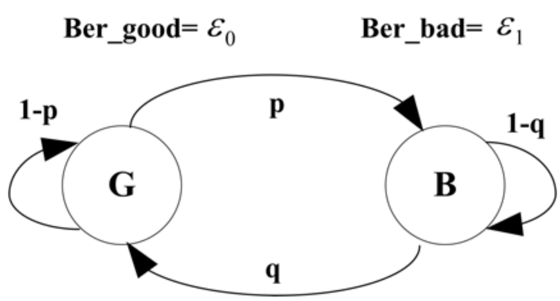


Fig.1 Wireless broadcast system model. Fig.2 Burst error channel modeled by a two-state Markov chain

There are K burst error channels between the base station and K receivers, each of which is modeled by a two-state Markov chain as shown in Fig.2. The channel can be in one of two states: a good state G and a bad state B. In state G, the bit error rate (BER) is ε_0. And in state B, the BER is ε_1 ($\varepsilon_1 >> \varepsilon_0$). Let p denotes the transmission probability from state G to state B, and q denotes the transmission probability from state B to state G. Following the approach of [6], the kth burst error channel between the sender and the kth receiver can be described by the average bit error rate $\overline{\varepsilon}_k$, the average burst length $\overline{b}_k$ and the stable probability for bad state $P_{1,k}$. All parameters can be determined from (1)-(3).To facilitate the analysis in Section 4, we assume that $\overline{\varepsilon}_k = \overline{\varepsilon}$， $\overline{b}_k = \overline{b}$， $P_{1,k} = P_1$， $k = 1,2,...K$ 。

$$\overline{\varepsilon} = \frac{p}{p+q}\varepsilon_0 + \frac{q}{p+q}\varepsilon_1 \quad (1)$$

$$\overline{b} = \frac{1}{q} \quad (2)$$

$$P_1 = \frac{p}{p+q} \quad (3)$$

3. Random Network Coding-Based ARQ

3.1. RNC-ARQ Overview

The transmission in the random network coding-based ARQ incorporates regular broadcast phase and retransmission phase:

1) *Regular broadcast phase:* Assume the base station broadcasts data packets to K receivers and each

packet is sent in a time slot of fixed duration. If the receiver has received a corrupted packet, it sends the NAK immediately and then discards corrupted one. Instead of retransmitting the lost packet immediately, the sender maintains a reception list to record all lost packets for each receiver. Until *M* packets have been sent, the sender enters the retransmission phase.

2) *Retransmission phase:* Consider a set of *m* lost packets $\{X_1, X_2, ..., X_m\}$ need to be retransmitted. The sender encodes all lost packets X_i by random linear codes [7], resulting a new coded packet as

$$Y = g_1 X_1 + g_2 X_2 + \cdots + g_m X_m \quad (4)$$

where coding coefficients $g_i (1 \le i \le m)$ are random elements of a selected finite field F_q. Let *l* be the maximum number of lost packets for one receiver. In order to send *m* original packets, at least *l* coded packets must be formed and transmitted. Note that the coefficients g_i must be sent together with the composite packet *Y* for decoding. Consequently, each receiver can recover the original packets $X_1, X_2, ..., X_m$ by solving linear equations as

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix} = \begin{pmatrix} g_{11} \cdots g_{1m} \\ \vdots \ \ddots \ \vdots \\ g_{m1} \cdots g_{mm} \end{pmatrix}^{-1} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} \quad (5)$$

A necessary condition here is that *m* sets of coefficients $g_1, g_2, ..., g_m$ must be independent. When the finite field F_q is sufficiently large, e.g. $q = 28$, the probability of decoding is over 99.6% [8]. If the composite packet has been lost during the retransmission, it will be retransmitted until all required receivers have successfully received. When all lost packets on the list are accepted by *K* receivers, the sender returns to the regular broadcast phase.

3.2. A simple example for RNC-ARQ

Fig. 3 shows a reception list of the sender in the case for four receivers and ten packets, where lost packets are denoted by the crosses. Assumption that the retransmission packets are never lost, we compare the number of retransmissions needed by the SR-ARQ, XOR-ARQ and RNC-ARQ schemes. For the scheme SR-ARQ, there are seven original packets 1, 2, 3, 6, 7, 8, 9 needed to be retransmitted. In the scheme XOR-ARQ, five combined packets are required to transmit again, that are $1 \oplus 9, 2 \oplus 3, 6, 7, 8$. If using the RNC-ARQ scheme, only four composite packets Y_i need to be retransmitted, where $Y_i = g_{i1} X_1 + g_{i2} X_2 + g_{i3} X_3 + g_{i6} X_6 + g_{i8} X_8 + g_{i9} X_9$, $i = 1, 2, 3, 4$. In contrast to SR-ARQ, the XOR-ARQ scheme can decrease the retransmission number by 28.5%. And the RNC-ARQ scheme can decrease the retransmission number over 42.9% under the same conditions. That proves the significant advantages of the RNC-ARQ scheme over others.

R1	1	X	3	4	5	X	7	X	X	10
R2	1	2	X	4	5	6	X	X	X	10
R3	1	X	3	4	5	X	X	8	X	10
R4	X	2	X	4	5	X	7	X	9	10

Fig. 3 *K*=4, *M*=10, a reception list of the sender

4. Throughput Analysis

In this section, we first model the receiver's decoding process by the simple Markov chain. Next, analytic solutions are derived for the throughput efficiency of the SR-ARQ, XOR-ARQ and RNC-ARQ schemes. The flow of the SR-ARQ and XOR-ARQ schemes are omitted here, details can be referred in [3]. We define the throughput efficiency $\eta = E_p / E[T]$, where $E[T]$ denotes the average number of transmission and retransmission required for one packet to be successfully received by all *K* receivers, and E_p denotes the ratio of the payload to the packet size. The throughput gain is then defined as $G = \eta_A / \eta_B$.

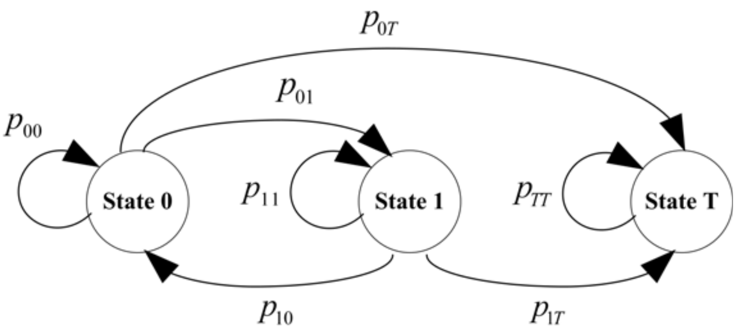


Fig.4 State transition diagram for the kth receiver's reception

Fig.4 gives the reception processing for the *k*th receiver, that modeled by a three-state Markov chain. In Fig.4, states 0 and 1 mean that the receiver fails to receive a packet, while the packet has been sent in the channel state *G* and state *B*, respectively. State T is an absorbing state, which corresponds to a successful reception. Thus, the one-step transition probability matrix can be easily expressed as

$$\mathbf{P}=\begin{bmatrix} p_{00} & p_{01} & p_{0T} \\ p_{10} & p_{11} & p_{1T} \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

where $p_{00}=(1-(1-\varepsilon_0)^L)(1-p)$, $p_{01}=(1-(1-\varepsilon_1)^L)p$, $p_{0T}=1-p_{00}-p_{01}$, $p_{10}=(1-(1-\varepsilon_0)^L)q$, $p_{11}=(1-(1-\varepsilon_1)^L)(1-q)$, $p_{1T}=1-p_{10}-p_{11}$. In above transition probabilities, *L* is the packet size. Other parameters $\varepsilon_0,\varepsilon_1,p,q$ can be referred in Section II.

Define random variable X_k , $k=1,2,\ldots,K,$ to be the number of transmission and retransmissions required for one packet to be successfully accepted by the *k*th receiver. And let $P(X_k\le i)$ denotes the successful reception probability for the *k*th receiver after *i* times transmissions. To facilitate the analysis, the packet size for SR-ARQ, XOR-ARQ and RNC-ARQ schemes are assumed to be the same, and the coefficient vectors are randomly selected from Galois Field (2^8). Thus the decoding failure caused by linearly dependent coefficients can be neglected.

Theorem 1: The throughput efficiency of the SR-ARQ scheme with *K* receivers under the burst error channel is

$$\eta_{SR-ARQ}=\frac{E_p}{1+\sum_{i=1}^{\infty}(1-P(X_k\le i)^K)} \qquad (7)$$

where

$$P(X_k\le i)=\begin{cases}(1-P_1)(1-\varepsilon_0)^L+P_1(1-\varepsilon_1)^L & i=1\\ (1-P_1)(1-\varepsilon_0)^L+P_1(1-\varepsilon_1)^L+(1-P_1)(1-(1-\varepsilon_0)^L)\mathbf{P}_{0T}^{(i-1)}+P_{\ 1}(1-(1-\varepsilon_1)^L)\mathbf{P}_{1T}^{(i-1)} & i>1\end{cases}$$

Proof: After one transmission, the successful reception probability for the *k*th receiver can be computed as

$$P(X_k\le 1)=(1-P_1)(1-\varepsilon_0)^L+P_1(1-\varepsilon_1)^L \qquad (8)$$

For the transmission number *i* >1, the successful reception probability for the *k*th receiver is

$$P(X_k\le i)=(1-P_1)(1-\varepsilon_0)^L+P_1(1-\varepsilon_1)^L+(1-P_1)(1-(1-\varepsilon_0)^L)\mathbf{P}_{0T}^{(i-1)}+P_1(1-(1-\varepsilon_1)^L)\mathbf{P}_{1T}^{(i-1)} \qquad (9)$$

where the $\mathbf{P}_{0T}^{(i-1)}$ and $\mathbf{P}_{1T}^{(i-1)}$ denote the *i*-1 step transition probability from state 0 and 1 to state T, respectively. The X_k , $k=1,2,\ldots,K,$ are i.i.d random variables because the broadcast channel from the sender to each individual receiver is assumed to be independent identically distributed. Then the expected number of transmissions to deliver a successful packet to all the receivers is

$$E[T]=\sum_{i=1}^{\infty}i\times(\Pr\left\{\max_{k\in\{1,2,\ldots K\}}(X_k)=i\right\})=1+\sum_{i=1}^{\infty}(1-P(X_k\le i)^K) \qquad (10)$$

Theorem 2: The upper bound of throughput efficiency for XOR-ARQ scheme with *K* receivers and sufficiently large *M* under the burst error channel is

$$\eta^{*}_{XOR-ARQ} = \frac{E_p}{1+(1-(1-P_1)(1-\varepsilon_0)^L - P_1(1-\varepsilon_1)^L)(1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K))} \tag{11}$$

Proof: As the XOR-ARQ scheme provided in [3], the throughput efficiency depends on how many combined packets can be generated from lost packets. When the number of packets M broadcasted in the regular phase is sufficiently large, we consider that the numbers of lost packets at each receiver are equal. Next, the ideal case for generating the minimum composite packets is considered. Note that each combined packet is generated by XOR *K* lost packets from different receivers and each receiver can recover one lost packet from the combined packet in one retransmission. Therefore, the minimum number of the composite packets is

$$N = M(1-(1-P_1)(1-\varepsilon_0)^L - P_1(1-\varepsilon_1)^L) \tag{12}$$

The average number of transmissions required to successfully deliver all *M* packets to all receivers equals to

$$n = M + N(1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K)) \tag{13}$$

Thus, the average number of transmissions required to successfully deliver one packet to all the receivers equals to

$$E[T] = 1+(1-(1-P_1)(1-\varepsilon_0)^L - P_1(1-\varepsilon_1)^L)(1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K)) \tag{14}$$

Theorem 3: The throughput efficiency of the scheme RNC-ARQ with *K* receivers and sufficiently large *M* under burst error channel is

$$\eta_{RNC-ARQ} = \frac{E_p}{1+(1-(1-P_1)(1-\varepsilon_0)^L - P_1(1-\varepsilon_1)^L)(1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K))} \tag{15}$$

and it can achieve the upper bound of the performance of the scheme XOR-ARQ.

Proof: According to the proof in Theorem 2, when the number of packets *M* broadcasted in the regular broadcast phase is sufficiently large, the numbers of lost packets at each receiver are considered to be the same. When use the random network coding to combine the lost packets, the number of composite packets is equal to the value of the XOR-ARQ scheme in the ideal case. It can be computed by the equation (12). Therefore, the throughput efficiency of RNC-ARQ can achieve the upper bound of the throughput performance for the scheme XOR-ARQ under the same channel conditions.

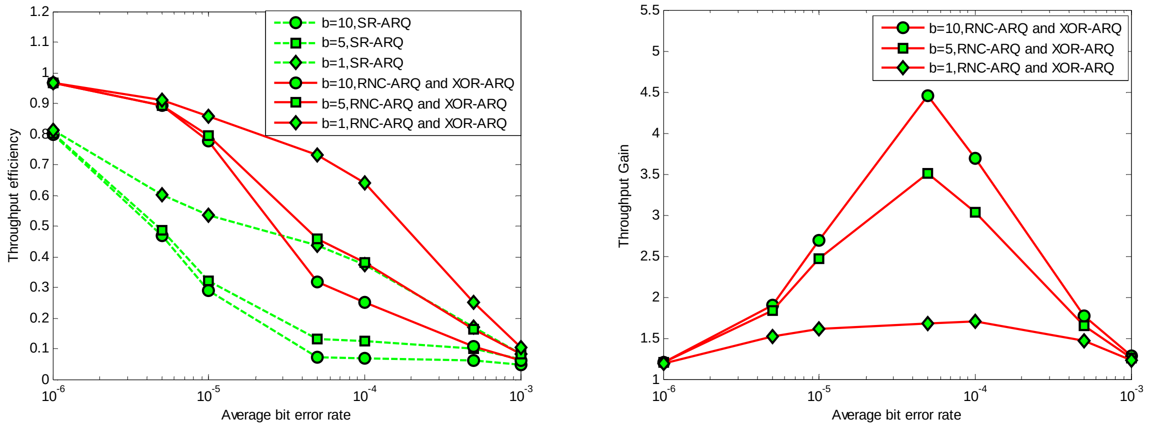
Theorem 4: The throughput gain of the scheme RNC-ARQ and XOR-ARQ with *K* receivers and sufficiently large *M* under the burst error channel are derived as

$$G_{RNC-ARQ} = G^{*}_{XOR-ARQ} = \frac{\eta_{RNC-ARQ}}{\eta_{ARQ}} = \frac{1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K)}{1+(1-(1-p_1)p_{gs} - p_1 p_{bs})(1+\sum_{i=1}^{\infty}(1-P(X_k \le i)^K))} \tag{16}$$

Proof: According to the definition of the throughput gain and the Theorem 1, 2 and 3, it is easy to derive the equation (15).

5. Simulation Results

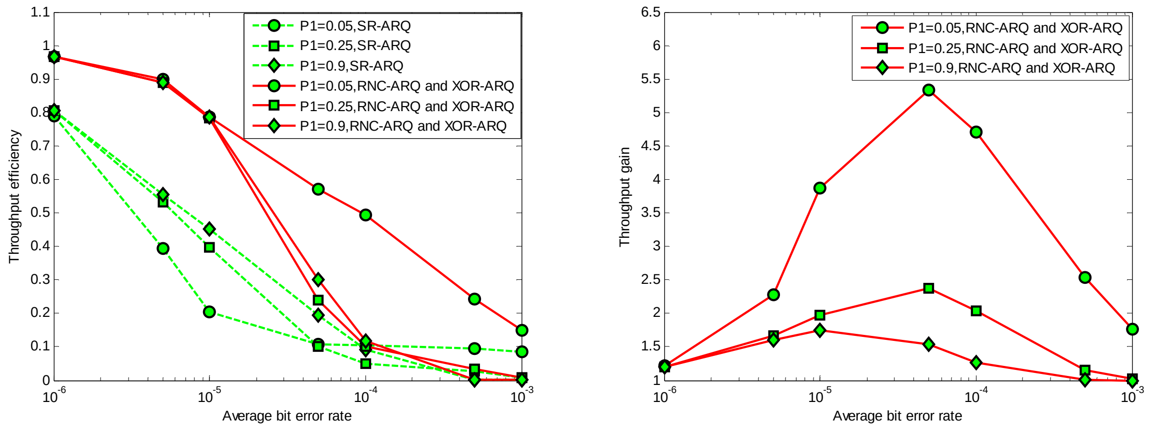
In this section, the simulation is provided for the throughput efficiency of the SR-ARQ, XOR-ARQ and RNC-ARQ scheme under typical burst error channel with different parameters $\bar{\varepsilon}$, $\bar{b}$, P_1 and *K*. The average bit error rate $\bar{\varepsilon}$ varies from 10(-6) to 10(-3). And the packet size is set to 1500 bytes and the payload is set to 1470 bytes. Thus the E_p is equal to 0.98.



(a) (b)

Fig. 5(a) Throughput efficiency of three schemes with P_1=0.1, K=20 and various values of $\bar{b}$

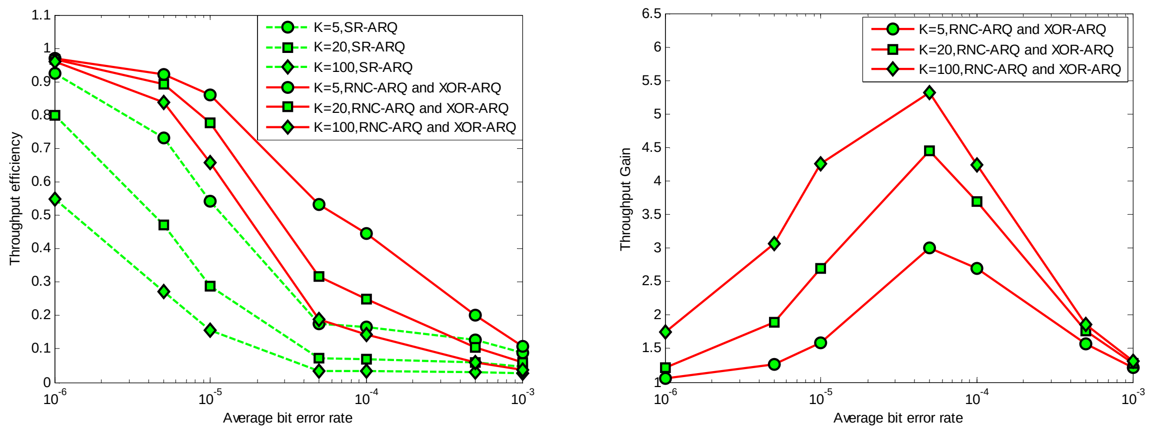
Fig. 5(b) Throughput gain of the scheme RNC-ARQ and XOR-ARQ with P_1=0.1, K=20 and various values of $\bar{b}$



(a) (b)

Fig.6 (a) Throughput efficiency of three schemes with $\bar{b}=10, K=20,$ and various values of P_1

Fig.6 (b) Throughput gain of the scheme RNC-ARQ and XOR-ARQ with $\bar{b}=10, K=20,$ and various values of P_1



(a) (b)

Fig.7 (a) Throughput efficiency of three schemes with $\bar{b}=10, P_1=0.1,$ and various values of K

Fig.7 (b) Throughput gain of the scheme RNC-ARQ and XOR-ARQ with $\bar{b}=10, P_1=0.1,$ and various values of K

Fig.5 (a) shows the throughput efficiency varies as a function of the average bit error rate $\bar{\varepsilon}$ and the average burst length $\bar{b}$. It indicates that the throughput efficiency of the scheme RNC-ARQ is always better than the scheme SR-ARQ as the value of $\bar{\varepsilon}$ increases. As $\bar{\varepsilon}$ becomes large, the performance of RNC-ARQ degrades rapidly with the $\bar{b}$ increasing. Fig.5 (b) illustrates the throughput gains of the RNC-ARQ and XOR-ARQ over the SR-ARQ for different $\bar{b}$. As seen, for some average BER region, the throughput gain of proposed RNC-ARQ scheme can be 4.5 times higher than SR-ARQ scheme, which proves the advantage of our scheme. Fig.6 (a) gives the curve of throughput efficiency versus the average BER $\bar{\varepsilon}$ with various probabilities P_1 for channel in bad state. As seen, for small values of $\bar{\varepsilon}$, the throughput efficiency of the RNC-ARQ is not sensitive to P_1. As the average BER $\bar{\varepsilon}$ gets large, the throughput efficiency degrades but slowly for the small P_1. This is because the channel errors are concentrated in fewer packets for small P_1, and that results in fewer retransmissions. Fig.6 (b) shows the maximum throughput gain achieved by RNC-ARQ can be 5.5 times higher than SR-ARQ scheme. Fig.7 (a) and Fig.7 (b) show that the proposed RNC-ARQ scheme is more effective for the large number of receivers.

From above examples, we observe that the RNC-ARQ scheme performs considerably better than the corresponding SR-ARQ scheme, especially in environments with a large number of receivers and concentrated channel errors.

6. Conclusions

In this paper, a broadcast retransmission scheme based on the random network coding has been proposed for wireless broadcast system such as cellular systems and WiMAX networks. Based on the burst error channel modeled by a two-state Markov chain, the analytical results are derived for the throughput efficiency for three schemes including the SR-ARQ, XOR-ARQ and RNC-ARQ. The theoretical and simulation results show that the proposed RNC-ARQ scheme always outperforms the traditional SR-ARQ in terms of the throughput efficiency for a typical range of channel conditions, while it can achieve the upper bound of the throughput for XOR-ARQ scheme. Future work will consider the joint optimization of the network coding and channel coding to further improve the throughput performance for wireless broadcast.

7. Acknowledgements

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