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Controlling hyperchaos in a novel hyperchaotic system

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Abstract. The control problem of a novel hyperchaotic system is investigated. The linear, speed, nonlinear, nonlinear doubly-periodic function feedback controls are used to suppress hyperchaos to unstable equilibrium. Limit cases of doubly-periodic function are considered and the hyperbolic function and trigonometric function feedback control laws are derived. The Routh－Hurwitz criterion is applied to study the conditions of the asymptotic stability of the controlled hyperchaotic system. Based on Matlab program, numerical simulations are presented to demonstrate the effectiveness of the proposed controllers.

Keywords: Hyperchaos control; New hyperchaotic system; Feedback control; Routh-Hurwiz criterion

1. Introduction

Since Lorenz found the first classical chaotic attractor in 1963 [1], chaos, as a very interesting nonlinear phenomenon has been intensively studied in science and engineering. Chaotic system is a very complex nonlinear dynamical system and its response possesses some characteristics, such as excessive sensitivity to initial conditions, broad spectral of Fourier transform, and fractal properties of the motion in phase space [2]. Due to the theoretical and practical applications of chaos in economics, ecology, lasers, plasma technologies, mechanical and chemical engineering, human brain dynamics [3], heart beat regulation [4], telecommunications, and micro-electro-mechanical systems [5], etc. Controlling these complex chaotic dynamics for engineering applications has emerged as a new and attractive field and has developed many profound theories and methodologies. Nowadays, many different techniques and methods have been proposed to achieve chaos control, such as OGY method [6], time-delay feedback method [7], Lyapunov method [8], impulsive control method [9], sliding method control 10], differential geometric method [11], H_∞ control [12], adaptive control method [13], chaos suppression method [14], and so on. Among them, the feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation.

However, most of the works mentioned so far involved mainly with low-dimensional chaos systems with only one positive Lyapunov exponent. Hyperchaotic system, possessing more than one positive Lyapunov exponents, has more complex behaviors and abundant dynamics than chaotic system. How to realize control and synchronization of hyperchaotic systems is an interesting and challenging work. Many attempts have been made to control hyperchaos and achieve synchronization of hyperchaos in [+]hyperchaotic systems [15－16]. Recently, Yan [17] suppressed a new hyperchaotic Chen system to unstable equilibrium by using feedback control method. Cai and Wang [18] constructed a new hyperchaotic system by introducing an additional state variable into the third-order chaotic system. The four-dimensional autonomous hyperchaotic system is described by

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz + u \\ \dot{z} = y^2 - hz \\ \dot{u} = -kx \end{cases} \tag{1}$$

where a, b, c, k, h are constant parameters. When parameters a=27.5, b=3, c=19.3, h=2.9 and k=3.3, system (1) is hyperchaotic, Its Lyapunov exponents can be obtained

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LE_1=1.6170, LE_2=0.1123, LE_3=0, LE_4= -12.8245.

The Lyapunov dimension of this system (1) is

$$D_L = j + \frac{1}{|LE_{j+1}|}\sum_{i=1}^{j} LE_i = 3 + \frac{1.6170 + 0.1123}{12.8245} = 3.1348$$

Obviously, the Lyapunov dimension of this new hyperchaotic system is also fractional dimension. The new system (1) shows hyperchaotic behavior, as depicted in Fig.1(a) ~ (b).The system(1) has only one equilibrium E(0,0,0,0,), and the equilibrium is an unstable saddle node under these parameters[18].

In this paper, we will control hyperchaos in the new hyperchaotic system (1). The linear feedback control, speed feedback control, nonlinear feedback control, nonlinear doubly-periodic function feedback control, nonlinear hyperbolic function feedback control and trigonometric function feedback control are used to suppress hyperchaos to unstable equilibrium. Moreover, numerical simulations are applied to verify the effectiveness of chosen controllers.

(a) x-y-z plane (b) y-z-u plane

Fig. 1: Phase portraits of hyperchaotic system (1), with a=27.5, b=3, c=19.3, h=2.9 and k=3.3

2. Controlling hyperchaotic attractor to equilibrium *E*(0,0,0,0)

In this section, we will control hyperchaotic system to equilibrium E(0,0,0,0,) by using different nonlinear feedback methods. Suppose that the control system is the following form:

$$\begin{cases} \dot{x} = a(y-x) + u_1 \\ \dot{y} = bx + cy - xz + u + u_2 \\ \dot{z} = y^2 - hz + u_3 \\ \dot{u} = -kx + u_4 \end{cases} \tag{2}$$

In which u_i (i=1,2,3,4,) are external control inputs. By choosing the suitable input functions u_i (i=1,2,3,4,), the hyperchaotic trajectory (x ,y, z, u) of system (2) will be dragged to E(0,0,0,0,).

2.1. Linear feedback control

We assume that ui (i=1,2,3,4,) are of the linear forms u1= -k1x, u2= -k2y, u3= -k3z, u4= -k4u,where ki(i=1,2,3,4) are feedback coefficients. The system (2) is rewritten as

$$\begin{cases} \dot{x} = a(y-x) - k_1 x \\ \dot{y} = bx + cy - xz + u - k_2 y \\ \dot{z} = y^2 - hz - k_3 z \\ \dot{u} = -kx - k_4 u \end{cases} \tag{3}$$

Whose Jacobi matrix is

$$J = \begin{pmatrix} -a-k_1 & a & 0 & 0 \\ b & c-k_2 & 0 & 1 \\ 0 & 0 & -h-k_3 & 0 \\ -k & 0 & 0 & -k_4 \end{pmatrix} \tag{4}$$

The characteristic equation of matrix (4) is

$$(\lambda+h+k_3)(\lambda^3+b_1\lambda^2+b_2\lambda+b_3)=0$$

where

$$b_1 = a-c+k_1+k_2+k_4,$$
$$b_2 = ak_2-ck_1+k_1k_2-ab-ac-ck_4+ak_4+k_2k_4-k_1k_4,$$
$$b_3 = ab+ak_2k_4-ack_4-ck_1k_4-abk_4+k_1k_2k_4.$$

According to Routh－Hurwitz criterion, the real parts λ are negative if and only if

$$h+k_3>0,\ b_1>0,\ b_3>0,\ b_1b_2>b_3. \tag{5}$$

Thus, when a=27.5, b=3, c=19.3, h=2.9 and k=3.3, and k_1, k_2, k_3, k_4 satisfy (5), the controlled hyperchaotic system (3) is asymptotically stable at equilibrium E(0,0,0,0,).

2.2. Speed feedback control

We assume that $u_1=u_2=u_3=0$ and u_4 is the speed form $u_4=-k_1\dot{y}$, where k_1 is a speed feedback coefficient. The controlled hyperchaotic system (2) is rewritten as

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx+cy-xz+u \\ \dot{z} = y^2-hz \\ \dot{u} = -kx-k_1(bx+cy-xz+u) \end{cases} \tag{6}$$

The Jacobi matrix of system (6) is

$$J = \begin{pmatrix} -a & a & 0 & 0 \\ b & c & 0 & 1 \\ 0 & 0 & -h & 0 \\ -k-k_1b & -k_1c & 0 & -k_1 \end{pmatrix} \tag{7}$$

The characteristic equation of matrix (7) is

$$(\lambda+h)(\lambda^3+b_1\lambda^2+b_2\lambda+b_3)=0$$

where

$$b_1 = a-c+k_1, b_2 = ak_1-ab, b_3 = ak.$$

When a=27.5, b=3, c=19.3, h=2.9 and k=3.3, it is easy to see that b_3=90.75>0. According to the Routh－Hurwitz criterion,

$$b_1>0, b_1b_2-b_3>0$$

i.e.,

$$k_1>3.2872 \tag{8}$$

The real parts λ of Jacobi matrix (7) with the equilibrium E(0,0,0,0,) are all negative. Therefore, when a=27.5, b=3, c=19.3, h=2.9 and k=3.3, and k_1 satisfy (8), the controlled hyperchaotic system (6) is asymptotically stable at equilibrium E(0,0,0,0,).

2.3. Nonlinear feedback control

We assume that $u_2=-\hat{k}_1y, u_4=-\hat{k}_2u$ and $u_1=u_3=0$, where $\dot{\hat{k}}_1=\beta_1 y^2$, $\dot{\hat{k}}_2=\beta_2 u^2$, ($\beta_1$, β_2 is positive feedback gain), the speed of hyperchaotic sate is controlled to the equilibrium E will varies when values of β_1, β_2 is altered.

The controlled hyperchaotic system (2) is rewritten as

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz + u - \hat{k}_1 y \\ \dot{z} = y^2 - hz \\ \dot{u} = -kx - \hat{k}_2 u \end{cases} \tag{9}$$

Then, Lyaponov function is constructed as follows:

$$V = \frac{1}{2}(x^2 + y^2 + z^2 + u^2) + \frac{1}{2\beta_1}(\hat{k}_1 - k^*)^2 + \frac{1}{2\beta_2}(\hat{k}_2 - k^*)^2$$

where k^* is a parameter to be determined. V for a derivation is:

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + u\dot{u} + (\hat{k}_1 - k^*)y^2 + (\hat{k}_2 - k^*)u^2 \\ &= -ax^2 - (k^* - c)y^2 - hz^2 - k^*u^2 + (a+b)xy - kxu + uy \\ &= -(x,y,z,u)\begin{pmatrix} a & -\frac{a+b}{2} & 0 & \frac{k}{2} \\ -\frac{a+b}{2} & k^* - c & 0 & -\frac{1}{2} \\ 0 & 0 & h & 0 \\ \frac{k}{2} & -\frac{1}{2} & 0 & k^* \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = -x^{\mathrm{T}}Px\cdot \end{aligned}$$

To make the system (9) asymptotically stable, the matrix P should be positive definite, based matrix theory, when P satisfy

$$\begin{cases} a > 0 \\ a(k^* - c) - (\frac{a+b}{2})^2 > 0 \\ h > 0 \\ ak^* - (ac + \frac{k^2}{4} + \frac{(a+b)^2}{4})k^* + \frac{k^2}{4}c - \frac{1}{4}a > 0 \end{cases}$$

Even if when

$$k^* > \max\left\{\frac{(a+b)^2 + 4ac}{4}, \frac{4ac + k^2 + (a+b)^2 + \sqrt{\left(4ac + k^2 + (a+b)^2\right)^2 - 16(ack^2 - a^2)}}{8a}\right\},$$

P is a positive definite matrix, thus $\dot{V}$ is negative semi-definite, however, we can't get immediately the controlled system (9) is asymptotically stable at equilibrium E(0,0,0,0,).

In fact, $\dot{V} \le 0, x, y, z, u \in L_\infty, \hat{k}_1 - k^*, \hat{k}_2 - k^* \in L_\infty$, for this reason , $\dot{x}, \dot{y}, \dot{z}, \dot{u} \in L_\infty$, As a result of $\dot{V} = -x^{\mathrm{T}}Px$, we arrive at

$$\int_0^t \lambda_{\min}(P)\|X\|^2 d_t \le \int_0^t x^{\mathrm{T}}Pxd_t \le \int_0^t -\dot{V}d_t = V(0) - V(t) \le V(0),$$

where $\lambda_{\min}(P)$ is the minimum eigenvalue of definite matrix P, and so $x, y, z, u \in L_2$, according to the Barbalat Lemma, $\lim_{t\to\infty}\|X(t)\| = 0$ .even x, y, z, u will be asymptotically converged to zero, therefore, the controlled hyperchaotic system (9) is asymptotically converged to equilibrium E(0,0,0,0,).

2.4. Nonlinear doubly-periodic function, hyperbolic function and trigonometric function feedback control

We assume that $u_1 = u_2 = u_3 = 0$ and u_4 is the nonlinear doubly-periodic function form $u_4 = -k_1 sn(-x+y;m)$, where k_1 is a feedback coefficient, $-k_1 sn(-x+y;m)$ is Jacobi elliptic sine function, and m(0<m<1) is the modulus of Jacobi elliptic function. Thus the controlled hyperchaotic system (2) is rewritten as

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz + u \\ \dot{z} = y^2 - hz \\ \dot{u} = -kx - k_1 sn(-x+y;m) \end{cases} \tag{10}$$

The Jacobi matrix of system (10) is

$$J = \begin{pmatrix} -a & a & 0 & 0 \\ b & b & 0 & 1 \\ 0 & 0 & -h & 0 \\ -k+k_1 & -k_1 & 0 & 0 \end{pmatrix} \tag{11}$$

The characteristic equation of matrix (11) is

$$(\lambda + h)(\lambda^3 + (a-c)\lambda^2 + (k_1 - ac - ab)\lambda + ak_1) = 0$$

When a=27.5, b=3, c=19.3, h=2.9 and k=3.3, obviously, a-c=8.2>0, ak_1=90.75>0.

If

$$(a-c)(k_1 - ac - ab) > k_1 a,$$

i.e.,

$$k_1 > 626.8, \tag{12}$$

the Jacobi matrix (11) has four negative real part eigenvalues.

The Jacobi elliptic function will degenerate to hyperbolic function and trigonometric function when the modulus reduced to be unity and zero. We will consider two kinds of limit cases in system (8).

As $m \to 1$, the system (10) degenerates to

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz + u \\ \dot{z} = y^2 - hz \\ \dot{u} = -kx - k_1 \tanh(-x+y) \end{cases} \tag{13}$$

i.e., u_4 is the hyperbolic function form $u_4 = -k_1 \tanh(-x+y)$, where k_1 is a feedback coefficient.

As $m \to 0$, the system (10) degenerates to

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = bx + cy - xz + u \\ \dot{z} = y^2 - hz \\ \dot{u} = -kx - k_1 \sin(-x+y) \end{cases} \tag{14}$$

i.e., u_4 is the trigonometric function form $u_4 = -k_1 \sin(-x+y)$, where k_1 is a feedback coefficient.

It is easy to see that the systems (10), (13), (14) possess the same Jacobi matrix. Therefore, when a=27.5, b=3, c=19.3, h=2.9, k=3.3, and k_1 satisfy (12), the controlled hyperchaotic systems (10), (13), (14) are asymptotically stable at equilibrium E(0,0,0,0,).

3. Numerical results

To verify the effectiveness and feasibility of the control approach, by using Matlab program, the numerical simulations have been completed. In the simulations, we choose the parameters a=27.5, b=3, c=19.3, h=2.9 and k=3.3,

(1) Linear feedback control

The initial states of the controlled system (3) are selected as (-0.6, -0.6, -0.5, 0.5) and the corresponding feedback coefficients are given by $k_1 = 25, k_2 = 25, k_3 = 10, k_4 = 0$, The behaviors of the state variable (x; y; z; u) of the controlled hyperchaotic system (3) with time are displayed in Fig.2.

(2) Speed feedback control

The initial values of the system (6) are selected as (1.0, 0.2, 10.0, -10) and the corresponding feedback coefficient is given by k_1=30. The behaviors of the state variable (x; y; z; u) of the controlled hyperchaotic system (6) with time are illustrated in Fig. 3.

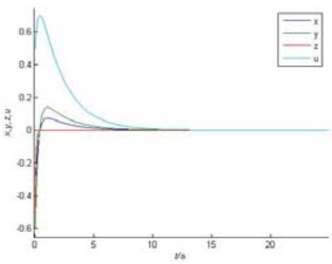


Fig. 2: The state of system (3).

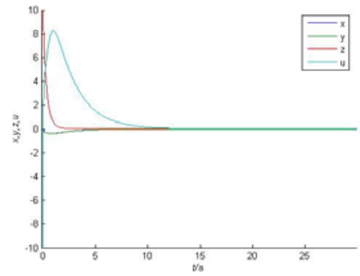
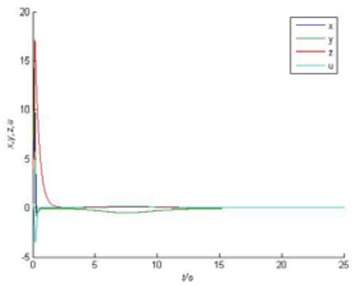


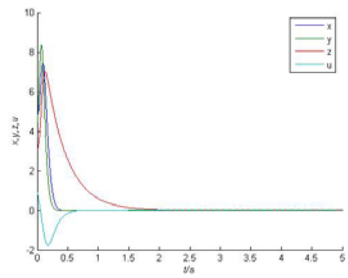
Fig. 3: The state of system (6).

(3) Nonlinear feedback control.

The initial states of the controlled system (9) are selected as (5, 4, 3, 1,) and the corresponding feedback gain coefficients are given by $\beta_1 = 1, \beta_2 = 4$ and $\beta_1 = 5, \beta_2 = 8$, The behaviors of the state variable (x; y; z; u) of the controlled hyperchaotic system (9) with time are displayed in Fig. 4.(a)~(b).



(a)β1=1,β2=4



(b) β_1 =5, β_2 =8

Fig. 4: The evolution of the controlled system (9) for the feedback control gain β1, β2

(4) Nonlinear doubly-periodic function, hyperbolic function and trigonometric function feedback control.

The initial values of the system (10), (13), (14) are selected as the same values (-0.3, -0.1, 5.0, 10.0) and the corresponding feedback coefficient is given by k_1=900. Figs. 5 – 7 show the simulation results.

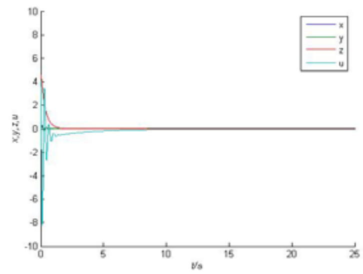


Fig. 5: The state of system (10) (m=0:8).

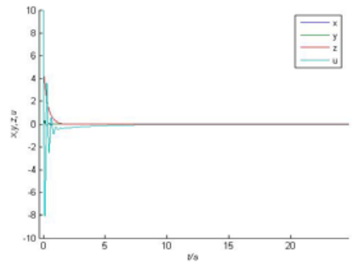


Fig. 6: The state of system (13).

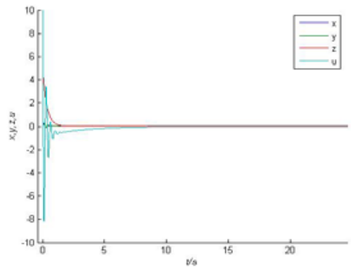


Fig. 7: The state of system (14).

4. Conclusion

The control problem of a new hyperchaotic system is investigated. The linear feedback control, speed feedback control, nonlinear feedback control and nonlinear doubly-periodic function feedback control are used to suppress hyperchaos to unstable equilibrium $E(0,0,0,0,)$. Limit cases of doubly-periodic function are considered and hyperbolic function; trigonometric function feedback control laws are derived. The Routh–Hurwitz criterion is used to study the conditions of the asymptotic stability of the controlled hyperchaotic system. Furthermore, numerical simulations are presented to verify the effectiveness of the proposed controllers. These control methods are also used to control other chaos or hyperchaotic systems.

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