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On the Positive and Negative Solutions of p-Laplacian BVP with Neumann Boundary Conditions

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Abstract. In the paper, we consider the following Neumann boundary value problem

$$\begin{cases} -(\varphi_p(u'(x)))' = u(x)|u(x)|^p - \lambda |u(x)|^q, & x \in (0,1), \\ u'(0) = 0 = u'(1), \end{cases}$$

Where $\lambda \in \mathbb{R}$, p and q are parameters such that $p \in (1,\infty), q \in (0,\infty)$ and $p > q$. We study the positive and negative solution of this problem, by using a quadrature method, we obtain our results. Also we provide some properties of the solutions are obtained in details.

Keywords and phrases: positive and negative solution, Interior critical points, Quadrature method, Neumann boundary condition, p-Laplacian problem. AMS subject classification: 34B15, 34B18

1. Introduction

Consider the nonlinear two point boundary value problem

$$-(\varphi_p(u'(x)))' = u(x)|u(x)|^p - \lambda |u(x)|^q, \qquad x \in (0,1), \tag{1}$$

$$u'(0) = 0 = u'(1), \tag{2}$$

where $\lambda \in \mathbb{R}, p > 1$ and $q > 0$ are parameters and $\varphi_p(x) = |x|^{p-2} x$ foe all $x \neq 0$ and $\varphi_p(0) = 0$ where $(\varphi_p(u'))'$ is the one dimensional p-Laplacian operator. We study the positive and negative solutions of this problem with respect to a parameter ρ (that is the value of the solutions at zero, i.e. $u(0) = \rho$). Also by using a quadrature method, we obtain our results. In [9] the problem $-(\varphi_p(u'(x)))' = |u(x)|^p u(x) - \lambda$ on $(0,1)$, with Dirichlet boundary value conditions have been studied by Ramaswamy for the case Laplacian and in [1] the same problem with the same boundary value condition have been extended by Addou to the general quasilinear case $p-$ Laplacian with $p > 1$. In [2] and [7] for semipositone problems with $p-$Laplacian operator, existence and multiplicity results have been established with Neumann boundary value conditions and Dirichlet boundary value conditions, respectively. In [5], for semipositone and positone problems have been studied by Anuradha, Maya and shivaji by using a quadrature method with Neumann-Robin boundary conditions and Laplacian operator. In [8] for semipositone problems, existence and multiplicity results have been established with Laplacian operator and Neumann boundary value conditions. Also, in [3] and [6] for semipositone problems with problems Laplacian operator have been studied for solution curves with Dirichlet boundary value conditions.

This paper is organized as follows. In section 2, we first state some remarks and then our main results and finally in section 3, we provide the proof of our main results that contain several Lemmas.

2. Main Results

By a solution of $(1)-(2)$ we mean a function $u \in C^1([0,1])$ for which $\varphi_p(u') \in C^1([0,1])$ and both the equation and the boundary value conditions are satisfied

Remark 1 If u is a solution to $(1)-(2)$ at λ, then $-u$ is a solution to $(1)-(2)$ at $-\lambda$.

Remark 2 Every solution u of $(1)-(2)$ is symmetric a bout any interior critical points such that for any point $x_0 \in (0,1)$ where $u'(x_0)=0,$ we have $u(x_0-z)=u(x_0+z)$ for all $z\in[0,\min\{x_0,1-x_0\}].$

In fact, let $w_1(z)=u(x_0-z)$ and $w_2(z)=u(x_0+z)$, then it is clear that both w_1 and w_2 satisfy the IVP

$$\begin{cases} -(\varphi_p(w'(x)))' = w(x)|w(x)|^p - \lambda |w(x)|^q, \\ w(0)=u(x_0), \\ w'(0)=0. \end{cases}$$

Hence, by uniqueness theorem for ODE, one can conclude result.

Remark 3 If u is a solution to $(1)-(2)$ then $u(1-x)$ is also a solution to $(1)-(2)$.

Definition 4 Let u be a solution to problem $(1)-(2)$ and k be the number of interior critical points of u then define the following sets:

$A_k = \{u : u$ is decreasing at the beginning and end of $(0,1)\}$,
$B_k = \{u : u$ is decreasing at the beginning and increasing at the end of $(0,1)\}$,
$C_k = \{u : u$ is increasing at the beginning and decreasing at the end of $(0,1)\}$,
$D_k = \{u : u$ is increasing at the beginning and end of $(0,1)\}$,

Theorem 1 Let $\rho\in\mathbb{R},$ then, **(a)**

(1) the problem $(1)-(2)$ has exactly one positive solution u with $u(0)=\rho$ at any $\lambda\in S_\rho$ where

$$S_\rho = \begin{cases} (0,\rho^{p-q+1})\bigcup(\rho^{p-q+1},+\infty), & \rho>0, \\ (0,\infty), & \rho=0, \\ (0,\infty), & \rho<0, \end{cases}$$

(2) the problem $(1)-(2)$ has no positive solution with $u(0)=\rho$ at any $\lambda\in S_\rho^c$

(b) the corresponding solution is defined by

$$\int_\rho^{u(x)} \{M(p,\rho,\lambda,s)\}^{-\frac{1}{p}} ds = \kappa_1 \{p'\}^{\frac{1}{p}} x, \qquad x\in(0,x_0),$$

$$\int_{u(x)}^{u(1)} \{M(p,\rho,\lambda,s)\}^{-\frac{1}{p}} ds = \kappa_2 \{p'\}^{\frac{1}{p}} (1-x), \qquad x\in(kx_0,1),$$

such that

$$\kappa_1 = \begin{cases} -, & \lambda\in(0,\dfrac{q+1}{p+2}\rho^{p-q+1}),\ \rho>0, \\ +, & \lambda\in(\dfrac{q+1}{p+2}\rho^{p-q+1},\infty),\ \rho>0, \\ +, & \lambda\in(0,\infty), \qquad \rho\le 0. \end{cases}$$

and κ_2 at any $\lambda\in S_\rho$ may be + or – and k be the number of interior critical of u where $k=0,1,2,...$ and if $k>0, x_0$ is the first interior critical point.

Theorem 2 Let u be a solution to $(1)-(2)$ at $\lambda\in S_\rho$ with $u(0)=\rho$ and k be the number of interior critical points that such $k=0,1,2,...$, then solution u

(a) For $q>1, p\in(2,\infty), \rho>0$ and $\lambda=\dfrac{q+1}{p+1}\rho^{p-q+1},$ is nonnegative and $u(0)=\rho=\|u\|_\infty$ such that belong to A_k or B_k .

(b) For $q>1, p\in(2,\infty), \rho>0$ and $\lambda\in(0,\dfrac{q+1}{p+1}\rho^{p-q+1}),$ is sign-changing and $u(0)=\rho=\|u\|_\infty$ such that belong to A_k or B_k .

(c) For $q>1, p\in(2,\infty), \rho>0$ and $\lambda\in(\frac{q+1}{p+1}\rho^{p-q+1},\infty), \lambda\neq\rho^{p-q+1}$, is positive and $\rho_0=\|u\|_\infty$ and $u(0)=\rho$ such that belong to C_k or D_k .

(d) For $q>1, p\in(2,\infty), \rho=0$ and $\lambda\in(0,\infty)$, is nonnegative and $\rho_0=\|u\|_\infty$ and $u(0)=\rho$ such that belong to C_k or D_k .

(e) For $\lambda\in(0,\infty)$ and $\rho<o$, is sign-changing and $\rho_0=\|u\|_\infty$ and $u(0)=\rho$ such that belong to C_k or D_k .

3. Proof

Let u be nontrivial solution to $(1)-(2)$ at λ with $u(0)=\rho$. Now multiplying (1) throughout by u' and

$$[u']^p = p'\{-\frac{u^2|u|^p}{p+2}+\frac{\lambda u|u|^q}{q+1}+c\} \tag{3}$$

where c is constant. Applying the condition $u(0)=\rho$ and $u'(0)=0$, we have

$$[u']^p = p'\{\frac{\rho^2|\rho|^p}{p+2}-\frac{u^2|u|^p}{p+2}+\frac{\lambda}{q+1}(u|u|^q-\rho|\rho|^q)\}, \qquad x\in(0,1). \tag{4}$$

Now, we define the function

$$s\mapsto M(p,\rho,\lambda,s)=\frac{\rho^2|\rho|^p}{p+2}-\frac{s^2|s|^p}{p+2}+\frac{\lambda}{q+1}(s|s|^q-\rho|\rho|^q)\ on\ IR, \tag{5}$$

Where $\lambda>0, \rho\in\mathbb{R}$, p and q that $p\in(1,\infty), q\in(0,\infty)$ such that $p>q$ are parameters. The following Lemma collects the variations of this function that immediately and we omit its proof.

Lemma 1: For all $\lambda\in\mathbb{R}^+$ and $\rho\in\mathbb{R}$,

(a) $\lim_{s\to\pm\infty} M(p,\rho,\lambda,s)=-\infty.$ 1

(b) The function $s\mapsto M(p,\rho,\lambda,s)$ is concave on $\mathbb{R}$

(c) The function $s\mapsto M(p,\rho,\lambda,s)$ is increasing on $(-\infty,\lambda^{\frac{1}{p-q+1}})$ and decreasing on $(\lambda^{\frac{1}{p-q+1}},+\infty)$, and if $\rho>o$, then

$$\max_{s\in\mathbb{R}} M(p,\rho,\lambda,s)=M(p,\rho,\lambda,\lambda^{\frac{1}{p-q+1}})\begin{cases}=0, & if\ \lambda=\rho^{p-q+1},\\ >0 & if\ \lambda\neq\rho^{p-q+1},\end{cases}$$

if $\rho<o$, then

$$\max_{s\in\mathbb{R}} M(p,\rho,\lambda,s)=M(p,\rho,\lambda,\lambda^{\frac{1}{p-q+1}})\begin{cases}=0, & if\ \lambda=(-\rho)^{p-q+1},\\ >0 & if\ \lambda\neq(-\rho)^{p-q+1},\end{cases}$$

if $\rho=o$, then

$$\max_{s\in\mathbb{R}} M(p,\rho,\lambda,s)=M(p,\rho,\lambda,\lambda^{\frac{1}{p-q+1}})>0.$$

(d) The y −intercept of the graph of $M(p,\rho,\lambda,.)$, i.e.

$$M(p,\rho,\lambda,0)=\frac{\rho^2|\rho|^p}{p+2}-\frac{\lambda}{q+1}\rho|\rho|^q\begin{cases}=0, & if\ \lambda=\frac{q+1}{p+2}\rho^{p-q+1},\rho>o,\\ >o, & if\ o<\lambda<\frac{q+1}{p+2}\rho^{p-q+1},\rho>0,\\ <0, & if\ \lambda>\frac{q+1}{p+2}\rho^{p-q+1},\rho>0,\\ >0, & if\ \rho<0,\\ =0, & if\ \rho=0.\end{cases}$$

(e) The function $M(p,\rho,\lambda,s)$ has two zeros ρ and ρ_0 such that

$$\begin{cases}\rho<\rho_0, & if\ \rho=0,\lambda\in(0,\infty),\\ \rho_0<\rho, & if\ \rho>0,\lambda=\frac{q+1}{p+2}\rho^{p-q+1},\\ \rho_0<0<\rho, & if\ \rho>0,0<\lambda<\frac{q+1}{p+2}\rho^{p-q+1},\\ 0<\rho<\rho_0, & if\ \rho>0,\lambda\in(\frac{q+1}{p+2}\rho^{p-q+1},+\infty),\lambda\neq\rho^{p-q+1},\\ \rho=\rho_0, & if\ \rho>0,\lambda=\rho^{p-q+1},\\ \rho<o<\rho_0, & if\ \rho<0.\end{cases}$$

Lemma 2 Let u be a nontrivial and positive solution to $(1)-(2)$ at λ with $u(0)=\rho$ and k be the number of interior critical point of u where $k=0,1,2,...$ and if $k>0,x_0$ is the first interior critical point, then

(a) The interior critical points of u are $x_0=\frac{1}{k+1}, x_1=2x_0, x_2=3x_0,...,x_{k-1}=kx_0$ and

$$\rho_0=\begin{cases}u(x_0), & if\ k>0,\\ u(1), & if\ k=0.\end{cases}$$

(b)

$$u\big|_{[0,1]}=\begin{cases}[\rho_0,\rho]\ or\,[\rho,\rho_0], & if\ \rho>0,\\ [0,(\frac{p+2}{q+1}\lambda)^{\frac{1}{p-q+1}}], & if\ \rho=0,\\ [\rho,\rho_0], & if\ \rho<0.\end{cases}$$

(c) If u is decreasing at the beginning of $(0,1)$ then:

$$\|u\|_\infty=\rho=u(0)=u(x_1)=u(x_3)=..., \tag{6}$$

$$\min_{x\in[0,1]}u(x)=\rho_0=u(x_0)=u(x_2)=u(x_4)=..., \tag{7}$$

and if u is increasing at the beginning of $(0,1)$ then:

$$\|u\|_\infty=\rho_0=u(x_0)=u(x_2)=u(x_4)=..., \tag{8}$$

$$\min_{x\in[0,1]}u(x)=\rho=u(0)=u(x_1)=u(x_3)=.... \tag{9}$$

Proof of Lemma 2.

(a) Let x_0 be the first interior critical point of u and $k>0$ be the number of interior critical points of u .

Thus the values of u for any $x \in (0, x_0)$ must be between $u(0) = \rho$ and $u(x_0)$. Now we show that $u(x_0) = \rho_0$. We know that $u'(x_0) = 0$, hence from (4) and (5), one can conclude that $M(p, \rho, \lambda, u(x_0)) = 0$, also from the Lemma $1(e), M(p, \rho, \lambda, u(0)) = 0$. On the other and $M(p, \rho, \lambda, u(x)) > 0$ for any $x \in (0, x_0)$. In fact, if there exists a real number $x_{00} \in (0, x_0)$ such that $M(p, \rho, \lambda, u(x_{00})) = 0$ then from (4), one can conclude that $u'(x_{00}) = 0$, i.e. $x_{00} \in (0, x_0)$ is an interior critical point of u and this is a contradiction, because x_0 is the first interior critical point of u in the interval $(0,1)$.

Now, from the Lemma $1(e)$, it follows that $u|_{[0,x_0]} = [\rho_0, \rho]$ or $[\rho, \rho_0]$, hence $u(x_0) = \rho_0$. But if $k = 0$, then the values of u for any $x \in (0,1)$ must be between $u(0)$ and $u(1)$. Hence by similar argument, one can show that $u(1) = \rho_0$

It is clear that $x_0 = \dfrac{1}{k+1}, k > 0$ and also by Remark 2, one can conclude that $2x_0, 3x_0, ..., kx_0$, are the rest interior critical points of u. The proof of part (a) follows.

We omit the proof part (b) and (c). Δ

Lemma 3 Let u be a nontrivial and positive solution $(1)-(2)$ at $\lambda \in S_\rho$ with $u(0) = \rho$ and k be the number of interior critical points of u where $k = 0,1,2,...$,then:

(a)

$$S_\rho = \begin{cases} (0, \rho^{p-q+1}) \bigcup (\rho^{p-q+1}, +\infty), & \rho > 0, \\ (0, \infty), & \rho = 0, \\ (0, \infty), & \rho < 0. \end{cases}$$

(b) The corresponding solution is defined by

$$\int_\rho^{u(x)} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \kappa_1 \{p'\}^{\frac{1}{p}} x, \qquad x \in (0, x_0),$$

$$\int_{u(x)}^{u(1)} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \kappa_2 \{p'\}^{\frac{1}{p}} (1-x), \qquad x \in (kx_0, 1),$$

Such that

$$\kappa_1 = \begin{cases} -, & if \;\; \lambda \in (0, \dfrac{q+1}{p+2} \rho^{p-q+1}), \; \rho > 0, \\ +, & if \;\; \lambda \in (\dfrac{q+1}{p+2} \rho^{p-q+1}, \infty), \; \rho > 0, \\ +, & if \;\; \lambda \in (0, \infty), \qquad \rho \leq 0. \end{cases}$$

κ_2 may be + or – for any $\lambda \in S_\rho$ and if $k > 0, x_0$ is the first interior critical point of u.

Proof of Lemma3 Let $\rho > 0$. By the Lemma 2 (b), $u(x) \in [\rho_0, \rho]$ or $[\rho, \rho_0]$ for any $x \in [0,1]$, and it, by the Lemma $1(e), 4$ and (5), yield that λ must belong to $(0, +\infty)$. Now we show that, $\lambda \neq \rho^{p-q+1}$. In fact, if $\lambda = \rho^{p-q+1}$, then $\rho = \rho_0$ (by the Lemma$1(e)$), hence by the Lemma 2(b), $u \equiv \rho$ and this a contradiction, because the solution u is nontrivial. Thus we conclude that $S_\rho = (0, \rho^{p-q+1}) \bigcup (\rho^{p-q+1}, +\infty)$.

Also by similar argument, one can show that if $\rho \leq 0$ then $S_\rho = (0, \infty)$.

(b) Note that since every solution of $(1)-(2)$ is symmetric about each of its interior critical points, thus it is enough to study solution on $[0, x_0]$ and $[kx_0, 1]$ where x_0 is the first interior critical point. If

$\lambda \in (0, \rho^{p-q+1})$, then by the Lemma 1(e), $\rho > \rho_0$ and so, by Lemma 2(c), $u(x_0) < u(0)$. Therefore u must be decreasing on $[0, x_0]$ and $\min_{x \in [0,1]} u(x) = \rho_0$. Hence from (3), we have

$$u'(x) = -\{p'\}^{\frac{1}{p}} \{M(p, \rho, \lambda, u(x))\}^{\frac{1}{p}}, \quad x \in (0, x_0). \tag{10}$$

Also if $\lambda \in (\rho^{p-q+1}, \infty)$, then by the Lemma 1(e), $\rho_0 > \rho$ and so, by the Lemma 2(c), $u(x_0) > u(0)$. Therefore u must be increasing on $[0, x_0]$ and $\|u\|_\infty = \rho_0$. Hence from (4), we have

$$u'(x) = +\{p'\}^{\frac{1}{p}} \{M(p, \rho, \lambda, u(x))\}^{\frac{1}{p}}, \quad x \in (0, x_0). \tag{11}$$

Also by (2), u may be increasing or decreasing on the interval $[kx_0, 1]$. Hence from (4),we have

$$u'(x) = \kappa_2 \{p'\}^{\frac{1}{p}} \{M(p, \rho, \lambda, u(x))\}^{\frac{1}{p}}, \quad x \in (kx_0, 1). \tag{12}$$

where $\kappa_2 = +$ or $-$.Now , integrating (10) and (11) on $(0, x)$, where $x \in (0, x_0)$, one can obtain

$$\int_{\rho}^{u(x)} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \kappa_1 \{p'\}^{\frac{1}{p}} x, \qquad x \in (0, x_0), \tag{13}$$

$$\int_{u(x)}^{u(1)} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \kappa_2 \{p'\}^{\frac{1}{p}} (1 - x), \qquad x \in (kx_0, 1), \tag{14}$$

where κ_1 and κ_2 have been defined in the Lemma 3(b). By substituting $x = x_0$ in (13), and using the fact that $u(x_0) = \rho_0$ (by the Lemma 2(c)), we get

$$\int_{\Omega_1} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \{p'\}^{\frac{1}{p}} x_0, \tag{15}$$

$$\int_{\Omega_2} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds = \{p'\}^{\frac{1}{p}} (1 - kx_0), \tag{16}$$

where

$$\Omega_1 = \begin{cases} (\rho_0, \rho), & if \;\; \lambda \in (0, \rho^{p-q+1}), \rho > 0, \\ (\rho, \rho_0), & if \;\; \lambda \in (\rho^{p-q+1}, \infty), \rho > 0, \\ (\rho, \rho_0), & if \;\; \lambda \in (0, \infty), \rho \le 0, \end{cases}$$

and

$$\Omega_2 = (\rho_0, \rho) \text{ or } (\rho, \rho_0).$$

Note that in (15) and (16) the integrals are convergent. In fact,

Claim 1 The integral $\int_{\Omega_i} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds \in (0, \infty)$, when $i = 1, 2$.

Proof of Claim 1. It is suffice to show that $\int_{\rho_0}^{\rho} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} ds \in (0, \infty)$. For this mean, by (5) and Lemma 1(e), one can conclude that

$$\lim_{s \to \rho} |s - \rho|^{\frac{1}{p}} \{M(p, \rho, \lambda, s)\}^{-\frac{1}{p}} = \frac{1}{\left| \lambda |\rho|^q - \rho |\rho|^p \right|^{\frac{1}{p}}} \in (0, \infty),$$

$$\lim_{s\to\rho_0}|s-\rho_0|^{\frac{1}{p}}\{M(p,\rho,\lambda,s)\}^{-\frac{1}{p}}=\frac{1}{\left|\lambda|\rho_0|^q-\rho_0|\rho_0|^p\right|^{\frac{1}{p}}}\in(0,\infty),$$

Also we know that the integrals $\int_{\rho_0}^{\rho}|s-\rho|^{-\frac{1}{p}}ds$ and $\int_{\rho_0}^{\rho}|s-\rho_0|^{-\frac{1}{p}}ds$ for $p>1$ are convergent. Thus one can conclude that the convergence of the integral $\int_{\rho_0}^{\rho}\{M(p,\rho,\lambda,s)\}^{-\frac{1}{p}}ds$ is a consequence of that of the integrals $\int_{\rho_0}^{\rho}|s-\rho|^{-\frac{1}{p}}ds$ and $\int_{\rho_0}^{\rho}|s-\rho_0|^{-\frac{1}{p}}ds$. Δ

Here the proof Lemma 3 is complete. Δ

Now theorem (1) follows. Δ

Proof of theorem 2(a) If $\rho>0$ and $\lambda=\frac{q+1}{p+2}\rho^{p-q+1}$ by the Lemma 1(e) $\rho>\min_{x\in[0,1]}u(x)=\rho_0$.

Hence u must be nonnegative solution. Also since $\rho>\rho_0$, u must at beginning of $(0,1)$ be decreasing at the beginning of $(0,1)$. So u belong to C_k or D_k. The proof of part (a) is follows. By similar argument parts (b),(c),(d) and (e) are followed.

Here the proof of Theorem 2 is complete. Δ

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