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Multisection Technique to Solve Interval-valued Purchasing Inventory Models without Shortages

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Abstract. This paper investigates an interval valued economic order quantity (EOQ) problem without shortage. Since it is almost impossible to find an analytic method to solve the proposed model, an optimization algorithm is designed. First, a brief survey of the existing works on comparing and ranking any two interval numbers on the real line is presented. Finally, the effectiveness of the designed algorithm is illustrated by a numerical example.

Key words: Inventory, Interval Number, Demand, Production, Simulation

1. Introduction

The economic order quantity (EOQ) model is first introduced by F.Harris [4]. Inventory control is an important field in supply chain management, since it can help companies reach the goal of ensuring delivery, avoiding shortages, helping sales at competitive prices and so forth. A proper control of inventory can significantly enhance a company's profit. To control an inventory system, one cannot ignored demand monitoring since inventory is partially driven by demand, and as suggested by Lau and Lau [2] in many cases a small change in the demand pattern may result in a large change in optimal inventory decisions. A manager of a company has to investigate the factors that influence demand pattern, because customers' purchasing behavior may be affected by factors such as selling price, inventory level, seasonality, and so on.

A large number of academic papers (for a review, see [11]) have been published describing numerous variations of the basic EOQ model. The body of the research assumes that the parameters involved in the EOQ model, such as the demand and the purchasing cost, are crisp values or random variables. However, in reality, the demand and the cost of the items often change slightly from one cycle to another.

For example, inventory carrying cost may be different in rainy season compared to summer or winter seasons (costs of taking proper action to prevent deteriorations of items in different seasons and also the labour charges in different seasons are different). Ordering cost, being dependent on the transportation facilities may also vary from season to season. Changes in the price of fuels, mailing charges, telephonic charges may also make the ordering cost fluctuating. Unit purchase cost is highly dependent on the costs of raw materials and labour charges, which may fluctuate over time.

To solve the problem with such imprecise numbers, stochastic, fuzzy and fuzzy-stochastic approaches [5, 6, 9, 10] may be used. In stochastic approach, the parameters are assumed to be random variables with known probability distribution. In fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, if the membership function of the fuzzy variable is complex, for example when a trapezoidal fuzzy number and a Gaussian fuzzy number coexist in a model, it is hard to obtain the membership function of the total cost. Therefore, these membership functions play a significant role in the method. However, in practice one may not be able to get exact membership function for fuzzy values and probability distribution for stochastic variable. Since precise

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information is required, the lack of accuracy will affect the quality of the solution obtained. For these reasons, we have represented the imprecise number by interval numbers [3, 13].

Thus, the interval number theory, rather than the traditional probability theory and fuzzy set theory, is well suited to the inventory problem. According to the decision maker's point of view under changeable conditions, we may replace the real numbers by the interval valued numbers to formulate the problems more appropriately.

We organize the paper as follows : In section 2, we give some basic definitions, notations and comparison on interval numbers. In section 3, we give the model formulation and the solution procedure.

2. Interval number

An interval number proposed by Moore [13], is considered as an extension of a real number and as a real subset of the real line $\Re$.

Definition 1. An interval number $\tilde{A}$ is a closed interval defined by $\tilde{A} = [a_L, a_R] = \{x \in \Re : a_L \le x \le a_R;\ \Re \text{ be the set of all real numbers}\}$. The numbers a_L, a_R are called respectively the lower and upper limits of the interval $\tilde{A}$. An interval number A alternatively represented in mean-width or center-radius form as

$$\tilde{A} = \langle m(\tilde{A}), w(\tilde{A})\rangle = \{x \in \Re : m(\tilde{A}) - w(\tilde{A}) \le x \le m(\tilde{A}) + w(\tilde{A})\}, \tag{1}$$

where $m(\tilde{A}) = \frac{1}{2}(a_L + a_R)$ and $w(\tilde{A}) = \frac{1}{2}(a_R - a_L)$ are the mid-point and half-width of the interval $\tilde{A}$. Actually, each real number can be regarded as an interval, such as, for all $x \in \Re$, x can be written as an interval $[x, x]$, which has zero length.

The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

2.1. Basic interval arithmetic

Let $\tilde{A} = [a_L, a_R] = \langle m_1, w_1\rangle$ and $\tilde{B} = [b_L, b_R] = \langle m_2, w_2\rangle \in I(\Re)$ be two interval numbers, then

$$\tilde{A} + \tilde{B} = [a_L + b_L, a_R + b_R];\ \ \tilde{A} + \tilde{B} = \langle m_1 + m_2, w_1 + w_2\rangle.$$

The multiplication of an interval by a real number $c \ne 0$ is defined as

$$c\tilde{A} = [ca_L, ca_R];\quad if\ \ c \ge 0\ and\, c\tilde{A} = [ca_R, ca_L];\quad if\ \ c < 0.$$

$$c\tilde{A} = c\langle m_1, w_1\rangle = \langle cm_1, |c|\, w_1\rangle.$$

The difference of these two interval numbers is

$$\tilde{A} - \tilde{B} = [a_L - b_R, a_R - b_L].$$

The product of these two distinct interval numbers is given by

$$\tilde{A}\tilde{B} = \left[\min\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}, \max\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}\right].$$

The division of these two interval numbers with $0 \notin \overset{\frown}{B}$ is given by

$$\tilde{A} \setminus \tilde{B} = \left[\min\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\}, \max\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\}\right].$$

2.2. Comparison between interval numbers

Let $\tilde{A} = [a_L, a_R] = \langle m_1, w_1\rangle, \tilde{B} = [b_L, b_R] = \langle m_2, w_2\rangle$ be two interval numbers within $I(\Re)$. These two intervals may be one of the following types:

1. Two intervals are completely disjoint (non-overlapping).
2. Two intervals are nested, (fully overlapping).
3. Intervals are partially overlapping.

A brief comparison on different interval orders is given in [1, 12].

Case 1 (Disjoint subintervals): Moore [13] defined transitive order relations over intervals as :

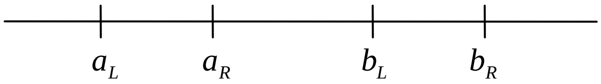

Figure 1: Disjoint subintervals

$\tilde{A}$ is strictly less than $\tilde{B}$ if and only if $a_R < b_L$ and this is denoted by $\tilde{A} < \tilde{B}$. This relation is an extension of `<' on the real line. This relation seems to be strict order relation that $\tilde{A}$ is smaller than $\tilde{B}$.

Case 2 (Nested subintervals) : Let $\tilde{A} = [a_L, a_R], \tilde{B} = [b_L, b_R] \in I(\Re)$ be such that $a_L \le b_L < b_R \le a_R$.

Figure 2: Nested subintervals

Then $\tilde{B}$ is contained in $\tilde{A}$ and it is denoted by $\tilde{B} \subseteq \tilde{A}$ which is the extension of the concept of the set inclusion [13]. The extension of the set inclusion here only describes the condition that, $\tilde{B}$ is nested in $\tilde{A}$ but it can not order $\tilde{A}$ and $\tilde{B}$ in terms of value.

Let $\tilde{A}$ and $\tilde{B}$ be two cost intervals and minimum cost interval is to be chosen. If the decision maker (DM) is optimistic then he/she will prefer the interval with maximum width along with the risk of more uncertainty giving less importance. Again, if the DM is pessimistic then he/she will pay more attention on more uncertainty i.e., on the right end points of the intervals and will choose the interval with minimum width. The case will be reverse when $\tilde{A}$ and $\tilde{B}$ represent profit intervals. In this case, we define the ranking order of $\tilde{A}$ and $\tilde{B}$ as

$$\tilde{A} \vee \tilde{B} = \begin{cases} \tilde{A}, & \textit{if the player is optimistic} \\ \tilde{B}, & \textit{if the player is pessimistic.} \end{cases}$$

The notation $\tilde{A} \vee \tilde{B}$ represents the maximum among the interval numbers $\tilde{A}$ and $\tilde{B}$. Similarly

$$\tilde{A} \wedge \tilde{B} = \begin{cases} \tilde{B}, & \textit{if the player is optimistic} \\ \tilde{A}, & \textit{if the player is pessimistic.} \end{cases}$$

The notation $\tilde{A} \wedge \tilde{B}$ represents the minimum among the interval numbers $\tilde{A}$ and $\tilde{B}$.

Case 3 (Partially overlapping subintervals) : The above mentioned order relations introduced by Moore [13] can not explain ranking between two overlapping closed intervals.

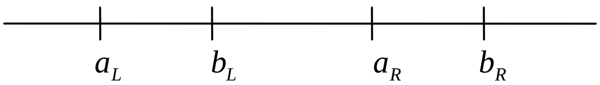

Figure 3: Partially overlapping subintervals

We define an acceptability index to compare and order any two interval numbers on the real line in terms of value as in [1, 12], which are used throughout the paper.

Definition 2 For $m_1 \le m_2$ and $w_1 + w_2 \ne 0$, the value judgement index or acceptability index (AI) of the premise $\tilde{A} \prec \tilde{B}$ is defined by

$$AI(\tilde{A} \prec \tilde{B}) = \frac{m_2 - m_1}{w_1 + w_2},$$

which is the value judgement by which $\tilde{A}$ is inferior to $\tilde{B}$ ($\tilde{B}$ is superior to $\tilde{A}$) in terms of value. Here `inferior to', `superior to', are analogous to `less than', `greater than', respectively.

Here `$\prec$' be an extended order relation between the intervals $\tilde{A}$ and $\tilde{B}$ on the real line $\Re$. For any sort of value judgement the AI index consistently satisfies the decision maker i.e., if $\tilde{A}, \tilde{B} \in I(\Re)$, then either $AI(\tilde{A} \prec \tilde{B}) > 0$ or $AI(\tilde{B} \prec \tilde{A}) > 0$ or $AI(\tilde{A} \prec \tilde{B}) = AI(\tilde{B} \prec \tilde{A}) = 0$ holds. Thus, on the basis of comparative position of mean and width of intervals $\tilde{A}, \tilde{B}$ the values of $AI(\tilde{A} \prec \tilde{B})$ of the premise $\tilde{A} \prec \tilde{B}$ are given by

(i) $AI(\tilde{A} \prec \tilde{B}) \geq 1$ when $m_1 < m_2$ and $a_R \leq b_L$, which refer to Case 1;

(ii) $0 < AI(\tilde{A} \prec \tilde{B}) < 1$ when $m_1 < m_2$ and $a_R > b_L$;

(iii) $AI(\tilde{A} \prec \tilde{B}) = 0$ when $m_1 = m_2$.

Using the ***AI*** index, we have presented the ordering for closed intervals $\tilde{A}, \tilde{B} \in I(\Re)$ reflecting decision maker's preference as

(i) When $AI(\tilde{A} \prec \tilde{B}) \geq 1$ we have $m_1 < m_2$ and $a_R \leq b_L$. In this case, $\tilde{A}$ is preferred over $\tilde{B}$ (i.e., $\tilde{A}$ is less than $\tilde{B}$) with acceptability index greater than or equal to 1 and so the decision maker (DM) accepts it with absolute satisfaction.

(ii) When $0 < AI(\tilde{A} \prec \tilde{B}) < 1$ we have $m_1 < m_2$ and $a_R > b_L$, then $\tilde{A}$ is preferred over $\tilde{B}$ with different grades of satisfaction lying between 0 and 1, excluding 0 and 1.

(iii) If $AI(\tilde{A} \prec \tilde{B}) = 0$ then obviously $m_1 = m_2$. Now, if $w_1 = w_2$ then there is no question of comparison as $\tilde{A}$ is identical with $\tilde{B}$. But, if $w_1 \neq w_2$, then the intervals $\tilde{A}$ and $\tilde{B}$ are non-inferior to each other, i.e., acceptability index becomes insignificant. In this case, DM has to negotiate with the widths of $\tilde{A}$ and $\tilde{B}$. Let $\tilde{A}$ and $\tilde{B}$ be two cost intervals and minimum cost interval is to be chosen. If the DM is optimistic then he/she will prefer the interval with maximum width along with the risk of more uncertainty giving less importance. Again, if the DM is pessimistic then he/she will pay more attention on more uncertainty i.e., on the right end points of the intervals and will choose the interval with minimum width. The case will be reverse when $\tilde{A}$ and $\tilde{B}$ represent profit intervals.

Ex 1. Let $\tilde{A} = [20,30] = \langle 25,5 \rangle$ and $\tilde{B} = [34,38] = \langle 36,2 \rangle$ be two intervals. Then $AI(\tilde{A} \prec \tilde{B}) = \frac{36-25}{5+2} = 1.57 > 1$. Hence the DM accept the decision that `$\tilde{A}$ is less than $\tilde{B}$' with full satisfaction.

Ex 2. Let $\tilde{A} = [20,26] = \langle 23,3 \rangle$ and $\tilde{B} = [24,28] = \langle 26,2 \rangle$ then $AI(\tilde{A} \prec \tilde{B}) = \frac{26-23}{3+2} = 0.6 < 1$. Here `$\tilde{A}$ is less than $\tilde{B}$' with grade of satisfaction 0.6.

Ex 3. Let $\tilde{A} = [8,16] = \langle 12,4 \rangle$ and $\tilde{B} = [6,18] = \langle 12,6 \rangle$ be two intervals. Here $m_1 = m_2 = 12$ and $w_1 < w_2$ and so $AI(\tilde{A} \prec_P \tilde{B}) = 0$. Hence both the intervals are non-inferior to each other. In this case, from the optimistic point of view the DM will prefer the interval $\tilde{B}$ instead of $\tilde{A}$. Because, if $\tilde{A}$ and $\tilde{B}$ are both the profit intervals then DM will pay more attention on the highest possible profit of 18 unit ignoring the risk of minimum profit of 6 unit. In the same manner if $\tilde{A}$ and $\tilde{B}$ are cost intervals then the DM will pay his attention on the minimum cost of 6 units i.e., the left end points of both the cost interval $\tilde{A}$ and $\tilde{B}$ and select $\tilde{B}$ instead of $\tilde{A}$ as $6 < 8$. Again, when both the intervals are profit intervals then the DM with a pessimistic outlook will prefer the profit interval $\tilde{A}$ because his attention will be drawn to the fact that the minimum profit of 8 unit will never be decreased, whereas his choice of $\tilde{A}$ might cost him the loss of 2 unit profit and this apprehension will determine from selecting the interval $\tilde{B}$. Similar is the explanation when $\tilde{A}$ and $\tilde{B}$ are cost intervals.

The above observations can be put into a compact form as follows

$$\tilde{A} \vee \tilde{B} = \begin{cases} \tilde{B}, & if \;\; AI(\tilde{A} \prec \tilde{B}) > 0 \\ \tilde{A}, & if \;\; AI(\tilde{A} \prec_P \tilde{B}) = 0 \;\; and \;\; w_1 < w_2 \;\; and \;\; DM \;\; is \;\; pessimistic \\ \tilde{B}, & if \;\; AI(\tilde{A} \prec_P \tilde{B}) = 0 \;\; and \;\; w_1 < w_2 \;\; and \;\; DM \;\; is \;\; optimistic. \end{cases}$$

Similarly, if $m_1 \geq m_2$ and $w_1 \geq w_2$, then there also exist a strict preference relation between $\tilde{A}$ and $\tilde{B}$. Thus similar observations can be put into a compact form as

$$\tilde{A} \wedge \tilde{B} = \begin{cases} \tilde{B}, & if \;\; AI(\tilde{B} \prec \tilde{A}) > 0 \\ \tilde{A}, & if \;\; AI(\tilde{B} \prec_P \tilde{A}) = 0 \;\; and \;\; w_1 > w_2 \;\; and \;\; DM \;\; is \;\; pessimistic \\ \tilde{B}, & if \;\; AI(\tilde{B} \prec_P \tilde{A}) = 0 \;\; and \;\; w_1 > w_2 \;\; and \;\; DM \;\; is \;\; optimistic. \end{cases}$$

An unified algorithm involving the dominance of interval numbers: Two interval numbers $\tilde{A} = \langle m_1, w_1 \rangle$ and $\tilde{B} = \langle m_2, w_2 \rangle$ are said to be *non-dominating* if (i) $m_1 = m_2$ and (ii) $w_1 \neq w_2$. The following function computes the minimum between two interval numbers.

```
    Function min(Ã, B̃)
if Ã = B̃ then minimum = Ã;
else
  if Ã = ⟨m1, w1⟩ and B̃ = ⟨m2, w2⟩ are not
   non-dominating then
    if ((Ã ≺ B̃) or (Ã ≺P B̃)) then
      minimum = Ã;
    else
      minimum = B̃;
    endif;
  else
    if (w1 < w2) then
      if the decision maker is optimistic minimum = B̃;
          if the decision maker is pessimistic minimum = Ã;
          endif;
      endif;
   endif;
return(minimum);
End Function.
```

Similarly, in the following we have given another function *max* which determines the maximum between two interval numbers.

```
  Function max(Ã, B̃)
if Ã = B̃ then maximum = Ã;
else
  if Ã = ⟨m1, w1⟩ and B̃ = ⟨m2, w2⟩ are not
   non-dominating then
    if ((Ã ≺ B̃) or (Ã ≺P B̃)) then
      maximum = B̃;
    else
      maximum = Ã;
    endif;
  else
    if (w1 > w2) then
      if the decision maker is optimistic maximum = Ã;
      if the decision maker is pessimistic maximum = B̃;
    endif;
  endif;
```

```
endif;
return(maximum);
 End Function.
```

3. Model formulation and analysis

The purpose of the EOQ model is to find the optimal order quantity of inventory items at each time such that the sum of the order cost and the carrying cost, i.e., total cost is minimal. In the classical EOQ model without shortage, an instantaneous replenishment is assumed to take place when the inventory level drops to zero, and the stock items are exhausted with a fixed demand rate. Moreover, the setup cost in each replenishment are assumed to be deterministic. But in real situations, the setup cost is usually affected by various uncontrollable factors and often show some fluctuation. Similarly, also for demand. In most cases, these are described by "lies between α and β". It is more reasonable, therefore, to characterize these as interval numbers.

Notations : For the sake of clarity, the following notations are used throughout the paper.

T length of one cycle;

$\tilde{D} = [d_l, d_r]$ demand rate;

$Q =$ order quantity per cycle;

$\tilde{C}(T) =$ total cost in the plan period;

$\tilde{C}_1 = [C_{1L}, C_{1R}]$ the inventory carrying cost per unit item per unit time;

$\tilde{C}_3 = [C_{3L}, C_{3R}]$ the ordering or setup cost/ unit item.

Assumptions : We have the following assumptions:

1. No shortages are allowed.
2. Lead time is zero.
3. The inventory planning horizon is infinite and the inventory system involves only one item and one stocking point.
4. Only a single order will be placed at the beginning of each and the entire lot is delivered in one batch.
5. The quantities $\tilde{C}_1$, $\tilde{C}_3$ and $\tilde{D}$ are assumed to be interval number, belongs to $I(\Re)$

A typical behavior of the EOQ lot size model with uniform demand and without shortage is depicted in Fig 4.

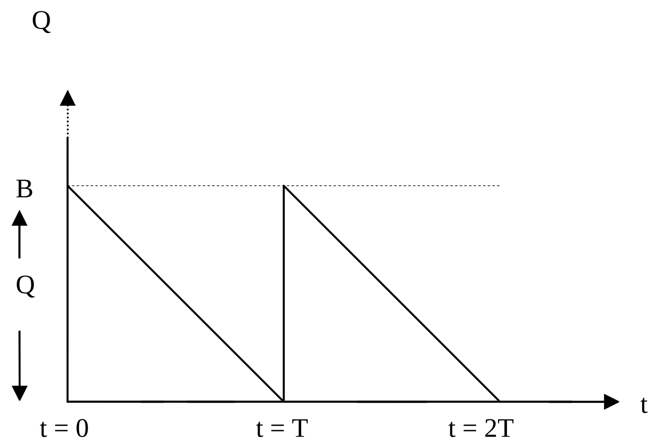

Figure 4: EOQ model without shortage

Let us assume that an enterprize purchases an amount of Q units of item at time $t = 0$. This amount will be depleted to meet up the customers demand. Ultimately, the stock level reaches to zero at time $t = T$. The total demand D in plan period $[0, T]$ can be expressed as,

$$Q = DT.$$

The inventory carrying cost for the entire cycle T is given by

$$C_1 \times \textit{area of } \Delta AOB = C_1 \times \left(\frac{1}{2} \times Q \times T \right) = \frac{1}{2}.C_1.Q.T$$

and the ordering cost for that cycle T is C_3. Hence the total cost in the plan period $[0,T]$ can be expressed as

$$X = C_3 + \frac{1}{2}.C_1.Q.T.$$

Therefore total average cost $C(Q)$ is given by $C(Q) = \frac{X}{T}$, i.e.,

$$C(Q) = \frac{C_3.D}{Q} + \frac{1}{2}C_1.Q \quad or,\ C(T) = \frac{C_3}{T} + \frac{1}{2}.C_1.Q = \frac{C_3}{T} + \frac{1}{2}.C_1.D.T. \tag{2}$$

By using calculus, we optimize $C(Q)$ and we get optimum values of Q, T and C as

$$Q^* = \sqrt{\frac{2.C_3.D}{C_1}}, \quad T^* = \sqrt{\frac{2.C_3}{C_1.D}}, \quad C^* = \sqrt{2.C_1.C_3.D}.$$

Usually, in mathematical programming we deal with the real numbers which are assumed to be fixed in value. In usual models- Carrying cost (C_1), set up cost (C_3), demand (D) are always fixed in value. But in real life, business cannot be properly formulated in this way due to uncertainty. Because the demand of customers can never be fixed, similarly the other costs also never be fixed in value. In such cases demand and other costs are assumed to be interval valued. But in interval oriented system we cannot use the calculus method for optimization.

3.1. Interval valued EOQ model

Let us assume interval valued demand by $\tilde{D} = [d_L, d_R]$, carrying cost by $\tilde{C}_1 = [c_{1L}, c_{1R}]$ and set up cost by $\tilde{C}_3 = [c_{3L}, c_{3R}]$, where first term within the bracket denote lower limit and 2nd term within the bracket denote the upper limit of the variable. Replacing D by $[d_L, d_R]$, C_1 by $[c_{1L}, c_{1R}]$ in equation (2) we have,

$$\tilde{C}(T) = [c_{3L}, c_{3R}].\frac{1}{T} + \frac{1}{2}.[c_{1L}, c_{1R}].[d_L, d_R].T \tag{3}$$

Addition and other composition rules (seen in the section 2.1 in this paper) on interval numbers are used in this equation. But in interval oriented system we cannot use the calculus method for optimization of $\tilde{C}(T)$. If we take $T = [T_L, T_R]$, then the expression (3) becomes,

$$\tilde{C}(T) = [c_{3L}, c_{3R}].\left[\frac{1}{T_R}, \frac{1}{T_L}\right] + \frac{1}{2}.[c_{1L}, c_{1R}].[d_L, d_R].[T_L, T_R]. \tag{4}$$

Since the value of $\tilde{C}(T)$ is interval valued, we cannot use the calculus method for optimization. In the next section, we have presented a new method dependent on interval computing technique to solve the unconstrained optimization problems. By using multi-section method, we are to find $T^* = [T_L^*, T_L^*]$, for which $\tilde{C}(T)$ have the optimal (minimum) value.

Multi-section method and solution procedure of the system:

Here, we use the multi-section algorithm as described by Mahato and Bhunia [14]. The idea of multi-section comes out from the concept of multiple bisection, where more than one bisection are done at a single iteration cycle. The basis of this method is the comparison of intervals according to the DM's point of view.

Let us consider a bound unconstrained optimization (maximization or minimization) problem with fixed coefficients as follows:

$$z = f(x), \quad l \le x \le u,$$

where $x = (x_1, x_2, \ldots, x_n), l = (l_1, l_2, \ldots, l_n), u = (u_1, u_2, \ldots, u_n)$, n represents the number of decision variables, the j^{th} decision variable x_j; $(j = 1,2,\ldots,n)$ lies in the prescribed interval $[l_j, u_j]$. Hence, the search space of the above problem is as follows:

$$D = x \in \Re^n : l_j \le x_j \le u_j, j = 1,2,\ldots,n.$$

Suppose, a firm divides the sales season into λ periods. Now our object is to split the accepted region(reduced)region (for the first time, it is the given search space or assumed if the search space is not given)into finite number of distinct equal subregions $R_1, R_2, \ldots, R_\lambda$ to select the subregion containing the best function value.

Let $f(R_i) = [\underline{f_i}, \overline{f_i}]; i = 1,2,\ldots,\lambda$ be the interval valued objective function $f(x)$ in the i^{th} subregion R_i, where $\underline{f_i}, \overline{f_i}$ denote the upper and lower bounds of $f(x)$ in R_i, computed by the application of finite interval arithmetic. Now, comparing all the interval-valued values of objective function, $f(x)$ in $R_i (i = 1,2,\ldots,\lambda)$ with the help of interval order relations mentioned in earlier section, the subregion containing best objective function value is accepted. Again, this accepted subregion is divided into other smaller distinct subregions $R'_i (i = 1,2,\ldots,\lambda)$ by the aforesaid process and applying the same acceptance criteria, we get the reduced subregion. This process is terminated after reaching the desired degree of accuracy and finally, we get the best value of the objective function and the corresponding values of the decision variables in the form of closed intervals with negligible width.

Algorithm :

To solve the problem (4), the optimal solution or an approximation of it has been obtained by applying the following algorithm.

Step 1**:** *Initialize* n, *(here* $n = 1$ *for* T *)* λ, l_j *(lower bounds) and* u_j *(upper bounds), where* $j = 1,2,\ldots,n.$

Step 2 **:** *Divide the accepted region* X *into* λ *equal subregions* R_i, *where* $i = 1,2,\ldots,\lambda$ *such that* $\bigcup_{i=1}^{\lambda} R_i = X$.

Step 3**:** *Using interval arithmetic find the interval value* $F(R_i) = [\underline{f_i}, \overline{f_i}]$ *of the objective function in the subregions* R_i *for* $i = 1,2,\ldots,\lambda.$

Step 4 **:** *Applying pessimistic order relations (defined in the section 2.2) between any two interval numbers, choose the subregion* R^{opt} *among* $R_i (i = 1,2,\ldots,\lambda)$ *which has better objective function value by comparing the interval values* $F(R_i), i = 1,2,3,\ldots,\lambda$ *to each other.*

Step 5**:** *Compute the widths* $w_j = (u_j - l_j), j = 1,2,3,\ldots,n.$

Step 6 **:** *If* $w_j < \varepsilon$, *a pre-assigned very small positive number for* $j = 1,2,3,\ldots,n$, *go to next step; otherwise go to step* 3.

Step 7 **:** *Print the values of the variables and of the objective function in the form of closed intervals with negligible width.*

Step 8**:** *Stop.*

3.2. Numerical example

Numerical example has been carried out to test the performance of the proposed approach described in this paper. To illustrate the developed model, an example with the following data has been considered. Consider a interval valued EOQ inventory system without shortage in which the carrying cost

$(\hat{C}_1) = [2.64, 2.70]$ and the ordering or setup cost $(\tilde{C}_3) = [1500, 2000]$, the demand quantity $\tilde{D} = [2000, 3000]$. The approach for computing the best found value in each subregion of the given search region of the test problem has been coded in $C++$ programming language. The solution is $T^* = 0.7510$, optimal cost $\tilde{C}^* = [3979.9770, 5704.6621]$ and the optimum $\tilde{Q}^* = [1501.9850, 2252.9775]$.

Based on the numerical example considered above, we now study sensitivity of $\tilde{T}^*, \tilde{C}^*$ and $\tilde{Q}^*$ to changes in the values of the system parameters C_1, C_3 and D. The sensitivity analysis is performed by changing each of the parameters by $+50\%, +25\%, -25\%$ and -50%; taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown the following table 1.

Table1: Effect of changes in the various parameters of the inventory model

Mid value of the parameter	*% change*	*%Change in* T^*	$m(\tilde{C}^*)$	$m(\tilde{Q}^*)$
$m(\tilde{C}_1)$	$+50$	-18.64	$+22.41$	-18.64
	$+25$	-10.65	$+11.77$	-10.65
	-25	$+15.98$	-13.35	$+15.98$
	-50	$+33.29$	-29.24	$+33.29$
$m(\tilde{C}_3)$	$+50$	$+26.30$	$+22.67$	$+26.30$
	$+25$	$+14.07$	$+11.91$	$+14.07$
	-25	-15.98	-13.46	-15.98
	-50	-35.21	-29.25	-35.21
$m(\tilde{D})$	$+50$	-21.30	$+22.34$	$+18.04$
	$+25$	-12.38	$+11.71$	$+9.51$
	-25	$+21.05$	-13.11	-9.21
	-50	$+33.29$	-29.24	-33.35

From the table 1, it is seen that

1. $\tilde{C}^*$ is fairly sensitive while T^* and $\tilde{Q}^*$ are less sensitive to changes in the value of the carrying cost C_1.

2. Each of T^*, C^* and Q^* are moderately sensitive to changes in the value of the setup cost C_3.

3. T^* is less sensitive while $\tilde{C}^*$ and $\tilde{Q}^*$ are fairly sensitive to changes in the value of the demand rate $\tilde{D}$.

4. Conclusion

In this paper, we have presented an inventory model without shortage, where carrying cost, the ordering or setup cost and demand are assumed as interval numbers instead of crisp or probabilistic in nature. We have considered the nature of these quantities as interval numbers to make the inventory model more realistic. At first, we have formulated a solution procedure to optimize a general function with coefficients as interval valued numbers using interval arithmetic. Using multi-section technique, we have derived the solution of the model. The algorithm has been tested using numerical example. Lastly, to study the effect of the determined quantities on changes of different parameters, a sensitivity analysis is also presented.

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