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Simulated Annealing Algorithm for Environmental/Economic Dispatch Problem

Aristidis Vlachos[1]

Department of Informatics, University of Piraeus, 80, Karaoli and Dimitriou Str.18534 PIRAEUS-GREECE

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Abstract. This paper presents a Simulated Annealing (SA) algorithm for solving the Environmental/Economic Dispatch Problem (EED). This problem is formulated as a multi-objective one with two competing functions, namely economic cost and emission functions, subject to different constraints. The inequality constraints considered are maximum and minimum limits of power generation while the equality constraint is demand-generation balance. The results are obtained for a simple power 3-generator system. The advantage of the SA algorithm is its robustness to find the global for our problem.

Keywords: Environmental/Economic Dispatch, Simulated Annealing Algorithm, Multi-objective optimization, Combinatorial optimization

1. Introduction

Major part of the power generation is due to fossil fuel power plants and their emission is a source of pollution for the environment [1].

Many countries around the globe have recently scheduled strategies for the reduction of the amount of the pollutants from fossil fuel power generation units.

Apart from particular pollutants, there are three different types of particular matter such as carbon di-oxide (CO_2), nitrogen oxides (NO_x) and sulphur oxides (SO_x) emitted from fossil fuel power plants. These pollutants have, mainly, ill effects to the human body.

This paper focuses on SO_2 and NO_x because their control is important at global level.

The Environmental/Economic Dispatch Problem (EED), is a multi-objective problem with conflicting objective because the minimum cost of power generation is conflicting with pollution minimization.

Many researches addressed the environmental and the economical objectives simultaneously by combining them linearly to form a single objective function [2]. King et al [3] suggested a Hopfield neural network for finding the optimal EED of thermal generation units. J. Nanda et al [4] solved th.e EED Problem using linear and non-linear goal programming techniques. Song et al [5] used a fuzzy logic controlled genetic algorithm for solving the EED Problem. Yalcinoz and Altum [6] solve the EED Problem via genetic algorithm with arithmetic crossover. Srinivasan and Tettamanzi [7] used a Heuristic-guided Evolutionary Approach to multi-objective Generation Scheduling suggesting a feasible solution.

The paper is organized as follows: Section 2 is an overview of the SA algorithm is presented. Section 3 formulates the SA algorithm for EED Problem. Section 5 presents the case study. Finally, Section 6 presents the conclusions.

2. Simulated Annealing Method

Simulated Annealing (SA) is a stochastic optimization technique which is based on the principles of statistical engineering. The search for a global minimum of a multidimensional cost function is a quite complex problem especially when a big number of local minimums correspond to the respective function. The main purpose of the optimization is to prevent hemming about to local minimums. The originality of the

[1] Aristidis Vlachos. Tel.: +30-693-255 3344.
E-mail address: avlachos@unipi.gr

SA method lies in the application of a mechanism that guarantees the avoidance of local minimums.

Following its introduction from Kirkpatrick et al [8], simulated annealing is mainly applied to large-scale combinatorial optimization problems.

2.1. The process of annealing in Thermodynamics

At high temperatures, the metal is in liquid phase. The molecules of liquidated metal, move freely with respect to each other. By gradually cooling (thermodynamic process of annealing) thermal mobility is lost. The atoms start to get arranged and finally form crystals, having the minimum energy which depends on the cooling rate. If the temperature is reduced at a very fast rate, the crystalline state transforms to an amorphous structure, a meta-stable state that corresponds to a local minimum of energy.

To conclude, the main point of the process is slow cooling, that leads to a crystallized solid state, which is a stable state, corresponding to a minimum energy. This is the technical definition of annealing and it is essential for insuring that a low energy state will be achieved.

There are similarities between the thermodynamic simulation annealing process and a combinatorial optimization problem.

Table I. Correspondence between thermodynamic simulation and combinatorial optimization

Thermodynamic Simulation	**Combinatorial Optimization**
System States	Feasible Solutions
Energy	Cost
Change of State	Neighboring Solutions
Temperature	Control Parameter
Frozen State	Heurestic Solution

The annealing process of metal influences SA algorithm.

If the system is at a thermal balance for given temperature T, then the probability $P_T(s)$ that it has a configuration s depends on the energy of the corresponding configuration $E(s)$, and is subject to the Boltzmann distribution:

$$P_T(s) = \frac{e^{-E(s)/\kappa T}}{\sum_W e^{-E(w)/\kappa T}} \quad (2)$$

Where κ is the Boltzmann constant and the sum Σ_W includes all possible states W.

Metropolis et al [9] were the first to suggest a method for calculating a distribution of a system of elementary particles (molecules) at the thermal balance state.

Let's suppose that the system has a configuration g, which corresponds to energy $E(g)$. When one of the molecules of the system is displaced from its starting position, a new state σ occurs which corresponds to energy $E(\sigma)$. The new configuration is compared with the old one. If $E(\sigma) \leq E(g)$, then the new state is accepted. If $E(\sigma) > E(g)$, then the new state is accepted with probability :

$$e^{-(E(\sigma)-E(g))/\kappa T}$$

Where κ is the Boltzmann constant.

The basic structure of the algorithm is presented at the following flow diagram (Figure 1) :

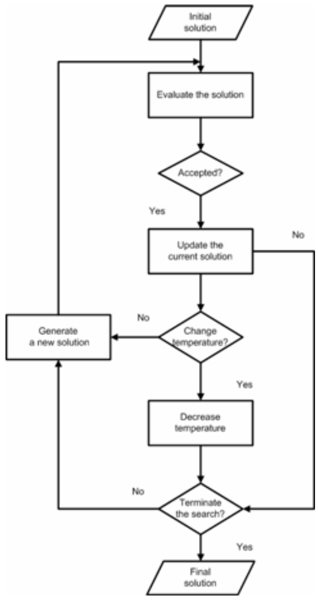

Figure 1. The basic structure of the simulated annealing algorithm

2.2. SA algorithm Control Parameters

Of great importance for the successful application of the SA algorithm is the annealing schedule, which refers to four control parameters that directly influence its convergence (to an optimized solution) and consequently its efficiency [10]. The parameters are the following:

- Starting Temperature
- Final Temperature
- Temperature Decrement
- Iterations at each Temperature

2.2.1 Starting Temperature

The starting temperature must be set to a big enough value, in order to make possible a big probability of acceptance non optimized solutions during the first stages of the algorithm's application.

However, if the value of the starting temperature gets too big, SA algorithm becomes non-effective because of its slow convergence and in general, the optimization process degenerates to a random walk. On the contrary, if the starting temperature is low then there is a big probability to hem about at one of the local minimums.

There is no particular method for finding the proper starting temperature that deals with the entire range of problems. Various techniques for finding the proper starting temperature have been developed.

Dowsland [11], suggests to quickly raise the temperature of the system initially up to the point where a certain percentage of the worst solutions is acceptable and after that point, a gradual decrement of temperature.

2.2.2 Final Temperature

During the application of the SA algorithm it is common to let the temperature fall to zero degrees. However, if the decrement of the temperature becomes exponential, SA algorithm can be executed for much longer. Finally, the stopping criteria can either be a suitable low temperature or the point when the system is "frozen" at current temperature.

2.2.3 Temperature Decrement

Since the starting and final temperatures have been defined, it is necessary to find the way of transition from the starting to the final temperature. The way of the temperature decrement is very important for the success of the algorithm.

Aart et al [12] suggested the following way to decrement the temperature:

$$T(t) = d / \log(t) \quad (2)$$

Where d is a positive constant.

An alternative, is the geometric relation:

$$T(t) = a \cdot t \quad (3)$$

Where parameter a is a constant near 1. In effect, its typical values range between 0.8 and 0.99.

2.2.4 Iterations at each Temperature

For increased efficiency of the algorithm, the number of iterations is very important. Using a certain number of iterations for each temperature is the proper solution.

Lundy [13] suggests the realization of only one iteration for each temperature, while the temperature decrement should take place at a really slow pace that can be expressed as:

$$T(t) = t / (1 + \beta t) \quad (4)$$

where β takes a very low value.

3. Problem Formulation

In this section, we shall formulate the Environmental/Economic Dispatch Problem (EED). It is an optimization problem that requires simultaneously the minimization of both fuel cost and emissions objective functions, which are conflicting ones.

The problem is formulated as follows.

3.1. Objective functions

Minimization of fuel cost : The problem of an Economic Load Dispatch (ELD) is to find the optimal combination of power generation, which minimizes the total fuel cost, under some constraints [14].

The ELD Problem can be, mathematically, formulated as the following optimization problem:

$$\text{Minimizes } F_{\cos t} = \sum_{i=1}^{n} (a_i + b_i P_{G_i} + c_i P_{G_i}^2) \quad (5)$$

Where:

- F_{cost} : the total fuel cost (\$/hr)
- a_i, b_i, c_i : the fuel cost coefficients of generator i
- P_{G_i} : the power generated by generator i (MW), and
- n : the number of generators

The emission dispatch problem, including the SO_2 and NO_x emission objectives, can be modeled using second order polynomial functions [15]:

$$\text{Minimizes } E_{SO_2} = \sum_{i=1}^{n} (a_{is} P_{G_i}^2 + b_{is} P_{G_i} + c_{is}) \quad (6)$$

$$\text{Minimizes } E_{NO_x} = \sum_{i=1}^{n} (a_{iN} P_{G_i}^2 + b_{iN} P_{G_i} + c_{iN}) \quad (7)$$

Units of E_{SO_2} and E_{NO_x} are ton/h

3.2. Problem Constraints

3.2.1 Power balance constraints

The total power generation must cover the total demand P_D (MW) and the real power loss, P_L (MW) in transmission lines. Hence,

$$\sum_{i=1}^{n} P_{G_i} + P_D - P_L = 0 \tag{8}$$

The transmission losses are given by:

$$P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{G_i} B_{ij} P_{G_j} \tag{9}$$

Where

B_{ij} : transmission losses coefficient

3.2.2 Generation capacity constraints

The power output of each generator should lie between its minimum and maximum limits:

$$P_{G_i \min} \le P_{G_i} \le P_{G_i \max} \tag{10}$$

Where

$P_{Gi}min$: minimum power generated and

$P_{Gi}max$: maximum power generated

Finally, the EED problem can be transformed to a minimization problem of three objective functions, always taking into mind that the corresponding circumstances must apply.

4. SA algorithms implementation of EED Problem

The SA algorithm for dispatch problem is stepped as follows:

Step 1. Initialization of the values temperature, T, parameter α and iterations number criterion. Find, randomly, an initial feasible solution, which is assigned as the current solution S_i and perform (?) ELD in order to calculate the total cost, F_{cost}, with the preconditions (8) and (10) fulfilled.

Step 2. Set the iteration counter to μ=1

Step 3. Find a neighboring solution S_j through a random perturbation of the counter one and calculate the new total cost, F_{cost}.

Step 4. If the new solution is better, we accept it, if it is worse, we calculate the deviation of cost $\Delta S=S_j-S_i$ and generate a random number uniformly distributed over $\Omega\in(0,1)$.

If

$$e^{-\Delta S/t} \ge \Omega \in (0,1) \tag{11}$$

accept the new solution S_j to replace S_i.

Step 5. If the stopping criterion is not satisfied, reduce temperature using parameter α:

$$T(t) = a \cdot t$$

And go to Step 2.

5. Case Study

The SA algorithm was applied to a 3-generator test system [14]. The system demand is 850 MW. Table II shows the data for the three generators.

Table II: Generators Data of 3-generator test system

Unit i	a_i	b_i	c_i	$P_{Gi}min$	$P_{Gi}max$
1	561.0	7.92	0.001562	150.0	600.0
2	310.0	7.85	0.00194	100.0	400.0
3	78.0	7.97	0.00482	50.0	200.0

The system transmission losses are given:

$$P_L = 0.00003 P_{G_1}^2 + 0.00009 P_{G_2}^2 + 0.00001 P_{G_3}^2 \quad \text{(MW)}$$

SO_2 and NO_x emission coefficients are taken from [16] and are presented at Tables III and IV respectively.

Table III : SO_2 Emission coefficients

Unit i	a_{is}	b_{is}	c_{is}
1	1.6103e-6	0.00816466	0.5783298
2	2.1999e-6	0.00891174	0.3515338
3	5.4658e-6	0.00903782	0.0884504

Table IV : NO_x Emission coefficients

Unit i	a_{iN}	b_{iN}	c_{iN}
1	1.4721848e-7	-9.4868099e-5	0.04373254
2	3.0207577e-7	-9.7252878e-5	0.055821713
3	1.9338531e-6	-3.5373734e-4	0.027731524

The results of the algorithms application for 10 iterations are presented at Table V.

Table V : Results of SA application.

Number of execution	Fuel Cost	$S0_2$	$N0_x$
1	8.346.175	9.007	.102
2	8.381.244	9.137	.098
3	8.398.395	8.983	.1
4	8.357.504	8.972	.097
5	8.355.257	**8.966**	.098
6	8.406.170	8.969	.106
7	**8.344.962**	9.103	.102
8	8.353.855	8.975	**.096**
9	8.371.037	8.968	.108
10	8.389.078	8.987	.1

Best results are indicated by bold lettering.

At Figures 2, 3 and 4 are presented the graphs of the objective functions : Fuel cost, SO_2 waste emission and NO_x waste emission respectively.

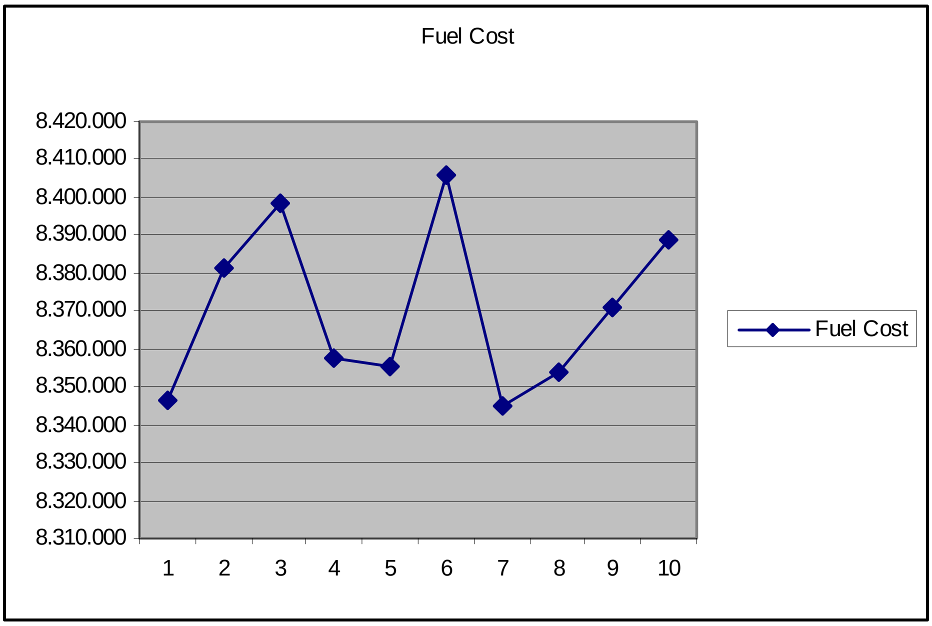

Figure 2. Graph for the objective function of Fuel Cost

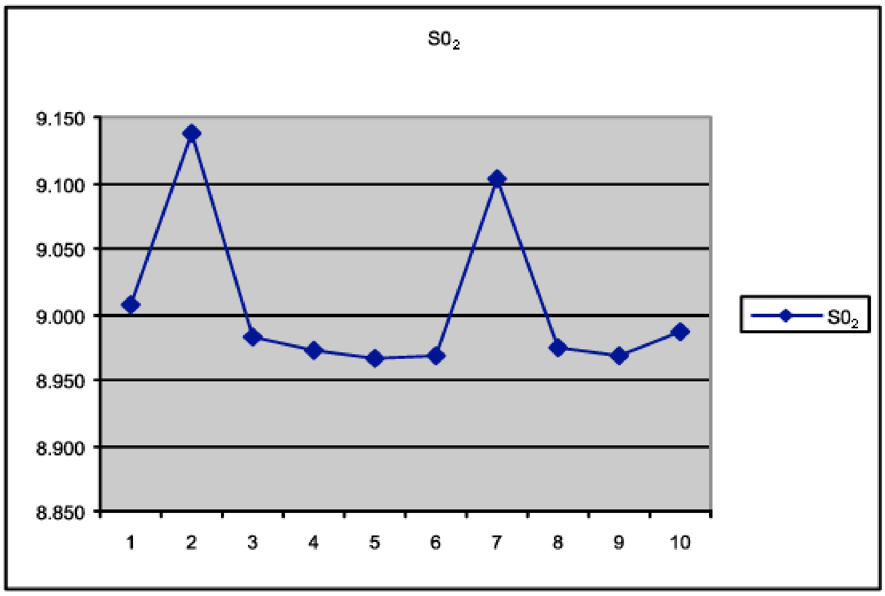

Figure 3. Graph for the objective function of SO_2 emissions

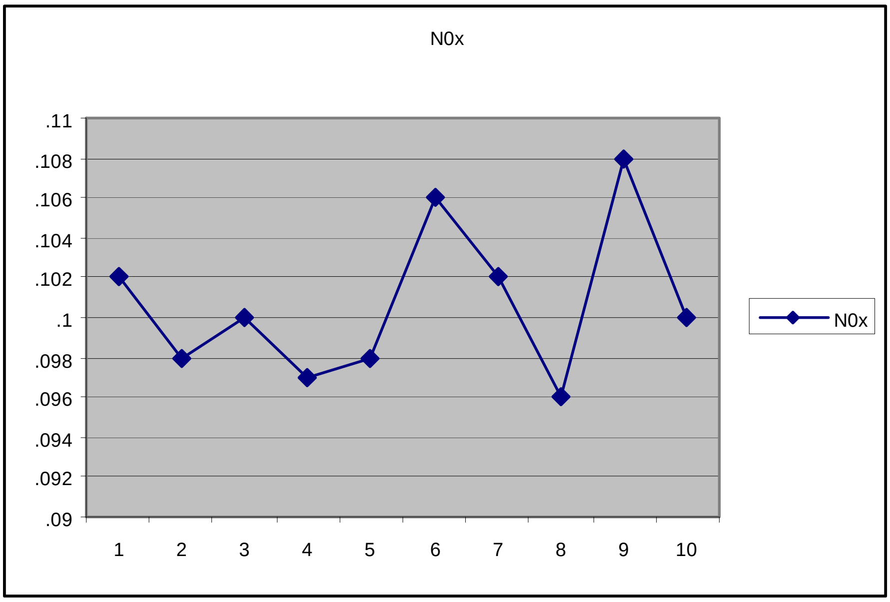

Figure 4. Graph for the objective function of NO_x emissions

The application was developed using Borland Delphi 7 with graphical MDI environment. All of the simulations took place at a Core Duo Processor running at 1,8 GHz with 2GB of RAM memory. The starting date was selected to be 100, parameter α equal to 0.9 and 10 iterations were completed.

6. Final Solution - convergence solution - compromise solution

From the above results we have now 3 optimum solutions for the 3 objective functions that define

- Best Fuel Cost
- Best SO_2 Emissions
- Best NO_x Emissions

There are many theories for finding the best compromise solutions like the fuzzy logic theory. One of them is the simple average. In order to apply the simple average to our results we must take the best results we have come up with. Then we must calculate the average of the two power generators. The third power generator value will derive after we solve a second-degree equation. Then we will simply replace the P_{G1},

P_{G2} and P_{G3} to our initial equations and we will have the final compromise solution.

Table VI : Best Solutions so far

	Best Fuel Cost		Best SO_2 Emission		Best NO_x emission
PG1	435.237	PG1	552.187	PG1	473.696
PG2	300.088	PG2	219.466	PG2	285.204
PG3	130.507	PG3	92.864	PG3	106.514
Losses	15.832	Losses	14.517	Losses	15.414
Fuel Cost	**8.344.593**	Fuel Cost	8.396.533	Fuel Cost	8.351.418
SO_2 Em.	9.022	SO_2 Em.	**8.966**	SO_2 Em.	8.992
NO_xEm.	0.099	NO_x Em.	0.097	NO_x Em.	**0.096**

Initially we find the average of P_{G1} and P_{G2}.

P_{G1} = (435.237 + 552.187 + 473.696) / 3 = 487.040

P_{G2} = (300.088 + 219.466 + 285.204) / 3 = 268.253

In order to find P_{G3} we must solve the following equation

$$\mathbf{\Sigma P_i - P_D - P_L = 0}$$

Which will end up to a second-degree polynomial with $\mathbf{P_{G3}}$ as the unknown value. We solve and we come up with $\mathbf{P_{G3}}$ = 109.745

Since we have the $\mathbf{P_{G1}}$, $\mathbf{P_{G2}}$, $\mathbf{P_{G3}}$ values we put them to the initial equations. The following table shows the best compromise solution.

Table VII : Best compromise solution

$\mathbf{P_{G1}}$	487.040
$\mathbf{P_{G2}}$	268.253
$\mathbf{P_{G3}}$	109.745
Losses	15.038
Fuel Cost	8.354.983
SO_2 Em.	8.983
NO_x Em.	0.096

7. Conclusions

Simulated annealing (SA) algorithm is a probabilistic meta-heuristic method for global optimization problems. The name derives from annealing in metallurgy, a process involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. In this paper SA algorithm solves the Environmental/Economic Dispatch problem, which is to minimize three objective functions : Fuel Cost, SO_2 Emission and NO_x Emission.

The proposing algorithm was tested on a 3-generator system with demand of 850 MW and gave good results. Furthermore, we have considered the transmission losses of the system. Finally, for finding a final solution, we used the simple method of averaging.

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