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Fast Iterative Method-FIM: Application to the Convection-Diffusion Equations

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Abstract. In this paper we present a new algorithm for solving linear systems by iterative method. In this method the rate of convergence has been improved well. The results show that this new algorithm converges faster than AOR, SOR, SSOR, etc .and works with high accuracy. The Main difference between these methods with the others is that uses affine combinations and also less parameter. Besides, we have shown that a divergent process can be converges by using FIM method. Finally the method is tested by some numerical experiments.

Keywords: iterative method, spectral radius, AOR, SOR, SSOR, Convection-Diffusion equation

1. Introduction

The iterative methods for solving a linear system are well known and some of them like MAOR, AOR, SSOR and SOR, are very popular [1-4].In this article a new iterative method presents which uses less parameter and converges faster than the other methods. There are examples of divergent iterative methods, which converge when using this new algorithm. Let

$$Ax=b \tag{1.1}$$

Where $A\in R^{n\times n}$ & $b,x\in R^n$ and A is nonsingular. The basic iterative formula for (1.1) is

$$x^{(i+1)}=M^{-1}Nx^{(i)}+M^{-1}b \qquad i=0,1,\ldots \tag{1.2}$$

Where x^0 is an initial guess .if A is split in to A=M-N, where M is nonsingular, then the basic iterative method for solving (1.1) is (1.2) .this iterative process converges to the unique solution $X=A^{-1}b$ for any initial vector value $x^0\in R^n$ if and only if $\rho(\underbrace{M^{-1}N}_{B})<1$ where $B=M^{-1}N$ is called iteration matrix .Suppose diag(A)=I and A=I-L-U, Where L and U are strictly lower and strictly upper triangular part of A, respectively. Now for Jacobi iterative method M=I, N=L+U and for Gauss-Seidel M=I-L, N=U.

That is

$$\begin{aligned} &jacobi: && x^{(i+1)}=b+Lx^{(i)}+Ux^{(i)}\\ &gauss-seidel && x^{(i+1)}=b+Lx^{(i+1)}+Ux^{(i)} \end{aligned} \tag{1.3}$$

In the new method, which we call FIM, not only the new iteration values are used, the previous values also used for computing the next values. In other words we have

$$\begin{cases} Ax=b \\ x^{(i+1)}=b+L\{(1-k)x^{(i+1)}+kx^{(i)}\}+Ux^{(i)} & ;k\in R \end{cases} \tag{1.4}$$

Difinition1.1.[5] if $s=\{x^0,x^1,\ldots,x^n\}$, then the set of all affine combinations of s is defined as

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$$A(s)=\{x \mid x=\sum_{j=0}^{n}\lambda_j x^j \quad ;\sum_{j=0}^{n}\lambda_j=1 \quad ;\lambda_j \in R\} \tag{1.5}$$

2. Basic idea of FIM

In the Gauss – Seidel process, i.e. $x^{(i+1)}=b+Lx^{(i+1)}+Ux^{(i)}$ We would like to improve $Ux^{(i)}$ and substitute the better value instead. There are two suggestions for implementing the above goal:

i.statistical method

Using variance and standard deviation $Ux^{(i)}$ can be changed to $U(x^{(i)}+\delta(x))$ where, $\delta(x)$ is standard deviation. (Confidence interval, statistical tests)

ii.prediction method

2.1. Two steps predictions (the prediction method with two sub iteration)

This process like SSOR contains two half iterations in each step, but in our method first the better value of $Ux^{(i)}$ provides and then improved more, i.e.

$$\begin{aligned} x^{(i+\frac{1}{2})} &= b+Lx^{(i+\frac{1}{2})}+Ux^{(i)} \\ x^{(i+1)} &= b+Lx^{(i+1)}+Ux^{(i+\frac{1}{2})} \end{aligned} \tag{2.1}$$

2.2. Four steps predictions

In this process each iterations involves quarter iterations i. e,

$$\begin{cases} x^{(i+\frac{1}{4})} = b+Lx^{(i+\frac{1}{4})}+Ux^{(i)} \\ x^{(i+\frac{2}{4})} = b+Lx^{(i+\frac{2}{4})}+Ux^{(i+\frac{1}{4})} \\ x^{(i+\frac{3}{4})} = b+Lx^{(i+\frac{3}{4})}+Ux^{(i+\frac{2}{4})} \\ x^{(i+1)} = b+Lx^{(i+1)}+Ux^{(i+\frac{3}{4})} \end{cases} \tag{2.2}$$

Example 2.1 Let

$$A=\begin{bmatrix} 1 & -.7 & -.3 & 0 & -.2 & -.2 \\ 0 & 1 & 0 & -.4 & -.1 & 0 \\ -.1 & 0 & 1 & -.6 & -.1 & -.4 \\ -.3 & -.3 & -.2 & 1 & -.1 & -.2 \\ 0 & -.2 & 0 & -.1 & 1 & 0 \\ -.9 & 0 & 0 & 0 & -.2 & 1 \end{bmatrix}$$

The standard iterative methods and the FIM method are used for this matrix. The spectral radius in each case computed and compared with each other in Table 1. As the results show the spectral radius of FIM is smaller than the others which means faster convergence.

Table1 shows the results of example2.1

Iterative method	Spectral radius
Jacobi	0.9859
Gauss-Seidel	0.9760
Symmetric Gauss- Seidel	0.9632
Two steps FIM	0.9526
Four steps FIM	0.9127

To show a proof for the FIM method and to describe why it works well, we consider the following:

(i) It is easy to see that iteration matrix in Two steps predictions is

$$B_2 = M_2 N_2 = \left\{(I-L)^{-1}U\right\}^2$$

and for Four steps predictions $B_4 = M_4 N_4 = \left\{(I-L)^{-1}U\right\}^4$;generally we have

$$B_n = M_n N_n = \left\{(I-L)^{-1}U\right\}^n \qquad \forall n \geq 2$$

Now we obtain M2 ,N2 follows:

$$(I-L)^{-1}U(I-L)^{-1}U = M_2^{-1}N_2 \tag{2.3}$$

$$\Rightarrow (I-(I-L)^{-1}A)(I-(I-L)^{-1}A) = M_2^{-1}N_2$$

$$\Rightarrow I - M_2^{-1}N_2 = 2(I-L)^{-1}A - (I-L)^{-1}A(I-L)^{-1}A$$

$$\Rightarrow M_2^{-1} = (I-L)^{-1}\left\{2I - A(I-L)^{-1}\right\}$$

Since in Gauss-Seidel , MG.S=I-L

$$\Rightarrow M_2^{-1} = M_{G.S}^{-1}\left\{I + N_{G.S}M_{G.S}^{-1}\right\} \tag{2.4}$$

$$\Rightarrow M_2 = \left\{I + N_{G.S}M_{G.S}^{-1}\right\}^{-1} M_{G.S} \qquad \& N_2 = \left\{I + N_{G.S}M_{G.S}^{-1}\right\}^{-1} N_{G.S}M_{G.S}^{-1}N_{G.S} \tag{2.5}$$

Also we can to show that $\forall n \geq 2$

$$M_n^{-1} = M_{G.S}^{-1}\left\{I + (N_{G.S}M_{G.S}^{-1}) + \cdots + (N_{G.S}M_{G.S}^{-1})^{n-1}\right\} \tag{2.6}$$

(ii) Lemma2.1[6,Theorem3.36].let A=M1-N1=M2-N2 be two regular splittings of A(i.e. $M_i^{-1} \geq 0, N_i \geq 0$ i=1,2), where A-1≥0.If $M_1^{-1} \geq M_2^{-1}$,then $\rho(M_1^{-1}N_1) \leq \rho(M_2^{-1}N_2) < 1$

Now by considering (i),(ii)and referring to[7, Theorem 4.1]we have the following Comparison theorem

Theorem2.2.let $A^{-1} \geq 0$ and the Gauss-Seidel split is regular, then $\forall n \geq 2$

$$\rho(B_{n+1}) \leq \rho(B_n) \leq \cdots \leq \rho(B_2) < \rho(B_{G.S}) < \rho(B_{Jacobi}) < 1$$

Proof. By Stein-Rosenberg Theorem; see [6; Theorem3.8; p: 100] we have $\rho(B_{G.S}) < \rho(B_{Jacobi}) < 1$

To prove the other inequalities, we first show that $A=M_2-N_2$ is a regular splitting.by(2.4)we can conclude that $\forall n \geq 2$ the split is regular so with respect to (2.4)and(2.6)we have

$$M_{n+1}^{-1} \geq M_n^{-1} \geq \cdots \geq M_2^{-1}$$

And by (i)the theorem is true.■

Now we improve the convergence with affine combinations

2.3. Two steps modified FIM

We improve the iterations as follows.

$$x^{(i+\frac{1}{2})} = b + Lx^{(i+\frac{1}{2})} + Ux^{(i)} \tag{2.7}$$

$$x^{(i+1)} = b + Lx^{(i+1)} + U\{(1-k)x^{(i+\frac{1}{2})} + kx^{(i)}\} \quad , k \in R$$

2.4. Four steps modified FIM

In this case we have

$$\begin{cases} x^{(i+\frac{1}{4})} = b + Lx^{(i+\frac{1}{4})} + Ux^{(i)} \\ x^{(i+\frac{2}{4})} = b + Lx^{(i+\frac{2}{4})} + Ux^{(i+\frac{1}{4})} \\ x^{(i+\frac{3}{4})} = b + Lx^{(i+\frac{3}{4})} + Ux^{(i+\frac{2}{4})} \\ x^{(i+1)} = b + Lx^{(i+1)} + U\{(1-k)x^{(i+\frac{3}{4})} + kx^{(i+\frac{2}{4})}\} \quad ,k \in R \end{cases} \tag{2.8}$$

2.5. General FIM

The iterative matrices in two and four steps method respectively are:

$$B_2 = s\{(1-k)s + kI\} \;\&\; B_4 = s\{(1-k)s^3 + ks^2\} \tag{2.9}$$

Where $s = (I-L)^{-1}U$.

Therefore

$$B_n = s\{(1-k)s^{n-1} + ks^{n-2}\} \quad ,n \geq 2 \quad ,k \in R \tag{2.10}$$

$$\begin{cases} x^{(i+\frac{1}{n})} = b + Lx^{(i+\frac{1}{n})} + Ux^{(i)} \\ \quad \vdots \\ x^{(i+1)} = b + Lx^{(i+1)} + U\{(1-k)x^{(i+\frac{n-1}{n})} + kx^{(i+\frac{n-2}{n})}\} \quad ,k \in R \end{cases} \tag{2.11}$$

3. Some numerical tests and its theoretical analysis

Lemm3. 1[1]. Let A be a consistently ordered matrix, and assume that the eigenvalues μ of Jacobi iteration matrix (B = L + U) are real and $\rho_j = \rho(B) < 1$. Then the optimal relaxation parameter w in SOR is given by $w_{opt} = \dfrac{2}{1+\sqrt{1-\rho_j^2}}$

For this optimal value we have $\rho(B_{opt}) = w_{opt} - 1$.

Lemma 3. 2. [2] . Suppose the eigenvalues of iterative matrix of Jacobi method is $\mu_i, i = 1,2,\ldots,n$.

Table 2. Optimum values for the AOR method

case	identification	Overreactions factor w	Acceleration factor r	Spectral radius ρ
(i)	$\underline{\mu} = 0$	$\widetilde{w}_2$	$\widetilde{w}_2$	$\dfrac{1-(1-\overline{\mu}^2)^{\frac{1}{2}}}{1+(1-\overline{\mu}^2)^{\frac{1}{2}}}$
(iia)	$0 < \underline{\mu} < \overline{\mu}$ $(1-\overline{\mu}^2)^{1/2} < (1-\underline{\mu}^2)$	$\widetilde{w}_2$	$\widetilde{w}_2$	$\dfrac{1-(1-\overline{\mu}^2)^{\frac{1}{2}}}{1+(1-\overline{\mu}^2)^{\frac{1}{2}}}$
(iib)	$0 < \underline{\mu} < \overline{\mu}$ $(1-\overline{\mu}^2)^{1/2} > (1-\underline{\mu}^2)$	$\widetilde{w}_2$	$\dfrac{1-\underline{\mu}^2-(1-\overline{\mu}^2)^{\frac{1}{2}}}{(1-\underline{\mu}^2)(1+(1-\overline{\mu}^2)^{\frac{1}{2}})}$	$\dfrac{\underline{\mu}(\overline{\mu}^2-\underline{\mu}^2)^{\frac{1}{2}}}{(1-\underline{\mu}^2)(1+(1-\overline{\mu}^2)^{\frac{1}{2}})}$
(iii)	$0 < \underline{\mu} = \overline{\mu}$	$\widetilde{w}_2$	$1/(1-\overline{\mu}^2)^{1/2}$	0
		$\widetilde{w}_2$	$-1/(1-\overline{\mu}^2)^{1/2}$	0

They are all real and $|\mu_i| < 1$, then we define:

$$L_{r,w} = (I - wL)^{-1}[(1-r)I + (r-w)L + rU] \tag{3.1}$$

$$\lambda^2 - [2(1-r) + rw\mu^2]\lambda + (r-1)^2 + (w-r)r\mu^2 = 0 \tag{3.2}$$

$$0 \le \underline{\mu} \equiv \min_i |\mu_i| \le \max_i |\mu_i| \equiv \overline{\mu} < 1. \tag{3.3}$$

If λ_2, λ_1 are the roots of (3.2), $\mu \in [\underline{\mu}, \overline{\mu}]$, $w \in (-\infty, +\infty), r \in (-\infty, +\infty)$, $\tilde{w}_2 \equiv 2/(1+(1-\overline{\mu}^2)^{\frac{1}{2}})$, then the results of minimizing the spectral radius in [2]are in Table2 .

In the next discussion and examples, we use the parameters of this Table for comparisons of the results. Note that the second column of this Table is important and describes the case of method.

Example 3.1. Let

$$A_1 = \begin{bmatrix} 1 & 0 & 4.6043 & -13.8136 \\ 0 & 1 & -.224 & .88 \\ 0 & -.169 & 1 & .49 \\ 0 & .55 & .165 & 1 \end{bmatrix}$$

The eigenvalues by Jacobi method are 0,0.8390,-0.6929 and -0.1460.so we have case (i) from Table2 .there fore the optimum value of spectral radius in SOR and AOR is 0.2952.but in FIM method we have the following to results

Table3. shows the result of example3.1

n	**K**	ρ
2	0	.5070
	-1.68	.1625
3	-2.35	.0189
4	-2.46	.0017
5	-2.47	1.8946e-004

Example 3.2. Let

$$A_2 = \begin{bmatrix} 1 & -.1 & -.1 & .3 \\ .23 & 1 & .23 & .36 \\ .11 & .23 & 1 & .2 \\ .18 & 0.2 & .18 & 1 \end{bmatrix}$$

The eigenvalues by Jacobi method are 0.2764, 0.2311,-0.0265,-0.4810. Since $0 < (\underline{\mu} = .0265) < \overline{\mu} = .4810 \,\&\, (1-\overline{\mu}^2)^{1/2} = .8767 < (1-\underline{\mu}^2) = .9996$ .Then we have the case (iia) in Table2 .there for the optimum value for spectral radius of SOR and AOR is 0.0657. But in FIM method we have the following results

Table4 . Shows the results of example3.2

n	**k**	ρ
2	0	0.0131
	-.059	.0079
3	-0.1	7.1496e-004

Example 3.3. Let

$$A_3 = \begin{bmatrix} 1 & 0 & 0.2 & .2 \\ 0 & 1 & -7.1 & 11.3 \\ 3.2 & .2 & 1 & 0 \\ 2 & .2 & 0 & 1 \end{bmatrix}$$

The optimum value of spectral radius for SOR is 0.6667 and for AOR is 0.5652 but in FIM method we have the following results

Table5. Shows the resuls of example3.3

n	**K**	ρ
2	0	.9216
	- 15.8	.3165
3	-15.9	.2986
4	-16.03	.2822
5	-16.15	.2670

Example 3.4. Let

$$A_4 = \begin{bmatrix} 1 & 0 & .2 & .2 \\ 0 & 1 & -3.6 & 6 \\ 4.8 & .2 & 1 & 0 \\ 2.4 & .2 & 0 & 1 \end{bmatrix}$$

This is the case (iii) in Table2 and the optimum value of spectral radius for SOR is 0.6667 and for AOR is zero. But for FIM method for example with (n=2, k=-24) spectral radius is 3.8158e-007

Example 3.5. Let

$$A_5 = \begin{bmatrix} 1 & .2 & .3 & .4 \\ .2 & 1 & .5 & .3 \\ .6 & .3 & 1 & .5 \\ .12 & .23 & .34 & 1 \end{bmatrix}$$

Note that this is an arbitrary matrix and the Jacobi method for this matrix has real and also complex eigenvalues. The results are

Table6 shows the results of example 3.5

method	**w**	**r**	ρ
Jacobi	1	0	.9941
Gauss-Seidel	1	1	.2980
SOR	1.04	0	.2932
AOR	1.04	1.04	.2685

And for FIM we have

Table7. shows results for FIM in example3.5

n	k	ρ
2	0	.0888
	-.21	.0730
3	-.19	.0218
4	-.17	.0065
5	-.18	.0019

4. Divergent example

Application of stationary iterative methods such as Jacobi, Gauss-Seidel, SOR, SSOR, AOR, etc is easy but the main problem in these methods is that the process compare with the nonstationary iterative methods such as GMRES ,BICG,QMR,etc more beaks done .we can to see that FIM method work better than the other iterative methods when we have a divergent case in stationary iterative methods Let

$$A_6 = \begin{bmatrix} 1 & -.3129 & -.3605 & -.2015 \\ -1.81 & 1 & -.6074 & -.9325 \\ -.069 & -.1012 & 1 & -.6156 \\ -.1232 & -.029 & -.9233 & 1 \end{bmatrix}$$

In this example the methods like Jacobi, Gauss-Seidel, SOR, SSOR, AOR and etc; are all diverge. The spectral radius for all of them has the value grater than 1, e.g, for Jacobi and Gauss-Seidel we respectively have 1.1489 and 1.3408. but in FIM we have the following results.

Table8 .results for divergent example

n	**k**	ρ
2	0	1.7978
	3.3	.3256
3	3.87	.0395
4	3.9	.0280
5	3.93	.0044

Now we solve AX=b ,where $b = \begin{bmatrix} -1.5133 \\ -5.3622 \\ .2662 \\ 1.0489 \end{bmatrix}$ and $A = A_6$,(n=3,k=3.8) By FIM Method .the exact solution is x=[1,2,3,4].this example computed with MATLAB7 Codes on a personal computer Pentium 4-256 MHZ .

Table9. number of iteration FIM method for divergent example

Iterative	0	1	2	3	4
	0.0000	1.9840	1.0058	0.9981	1.0000
	0.0000	2.2314	1.9765	1.9986	2.0000
	0.0000	2.7015	2.9971	3.0002	3.0000
	0.0000	3.8524	3.9974	3.9999	4.0000

REMARK. In above examples, we obtain optimum parameter 'k' **,** just by computer computational. The main goal of this work is that we understand the fundamental different between our method and other methods in the optimal cases. but, the question that could be considered as a open problem is :how could we obtain the optimal parameter 'k'?

This problem is important . but by choice of usual value for 'k' ,the results are also amazing (see section5).We can to say ,when Jacobi iteration matrix is converge, the sign of 'k' is negative and when Jacobi iteration matrix is diverge, the k'sign is positive(see A_1,A_6) .

In the following section we can see that the application of our method in the model of convection-diffusion equation

5. Numerical experiments

In the following experiments ,without considering optimal 'k' and using an interval or usual value for

'k' we can see that FIM works very well.

Expriment5.1.(Application to the two-dimensional convection-diffusion equation)

We consider the two-dimensional convection-diffusion equation

$$-\underbrace{(u_{xx}+u_{yy})}_{\Delta u}+\delta\ u_x+\tau\ u_y=f(x,y)$$

On the unit square domain $\Omega=[0,1]\times[0,1]$, with constant coefficients δ,τ and subject to

Dirichlet boundary conditions. discretization by a five-point finite difference operator leads to a linear system AX=b where X now denotes a vector in a finite-dimensional space and $A\in R^{n^2\times n^2}$.

With discretization on a uniform n × n grid, using standard second-order differences for the Laplacian , and either centered or upwind differences for the first derivatives., the coefficient matrix has the form

$$A=tridiagonal[bI,tridiagonal[c,a,d],eI]$$

$$b=-(1+\partial);c=-(1+\gamma);a=4;d=-(1-\gamma);e=-(1-\partial)$$

Where $\partial=\frac{\tau h}{2};\gamma=\frac{\delta h}{2}$ are Reynolds numbers. Also, the equidistant step-size h = 1/n is used in the discretization and the natural lexicographic ordering is employed to the unknowns and the right-hand side satisfies $b_{i,j}=h^2f_{i,j}(x,y)$. For details, we refer to [8,9].

When $\delta=1,\tau=2$ we test this equation for some iterative methods , we denote spectral radius of iteration matrix, by ρ_{method} .

for n=16($[A]_{256\times256}$) and by computation of spectral radius of iteration matrix in Jacobi ,Gauss-Seidel, symmetric Gauss-Seidel, SOR ,SSOR and AOR ,we have;

$$\begin{cases}\rho_{Jacobi}=0.9818\\ \rho_{G.S}=0.9639\\ \rho_{symmetricG.S}=0.9307\\ \rho_{SOR}\xrightarrow{w=.9}=0.9704\\ \rho_{SSOR}\xrightarrow{w=.9}=0.9426\\ \rho_{AOR}\xrightarrow{w=.9,r=.8}=0.9729\end{cases}$$

In Fig1. We can see that the spectral radius of iteration matrix in FIM(five- steps)for different values of 'k'.

The observation can be further illustrated by the spectrum pictures plotted in following figure.

For n=24($[A]_{576\times576}$) in semi-logarithmic scale is given in the Fig.2, we can see that spectrum of the iteration matrices FIM (eight-step)and other methods. In this experiment parameters are given as the following;

W=.9,r=.8,k=-30

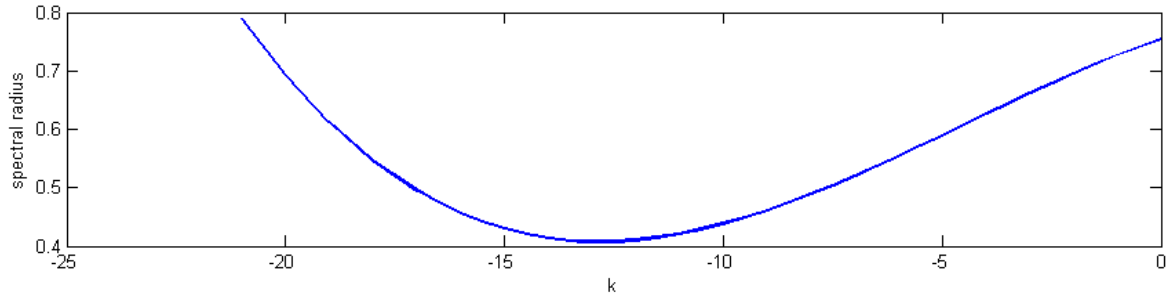

Fig. 1.spectral radius of FIM(five-steps) with some values of 'K'

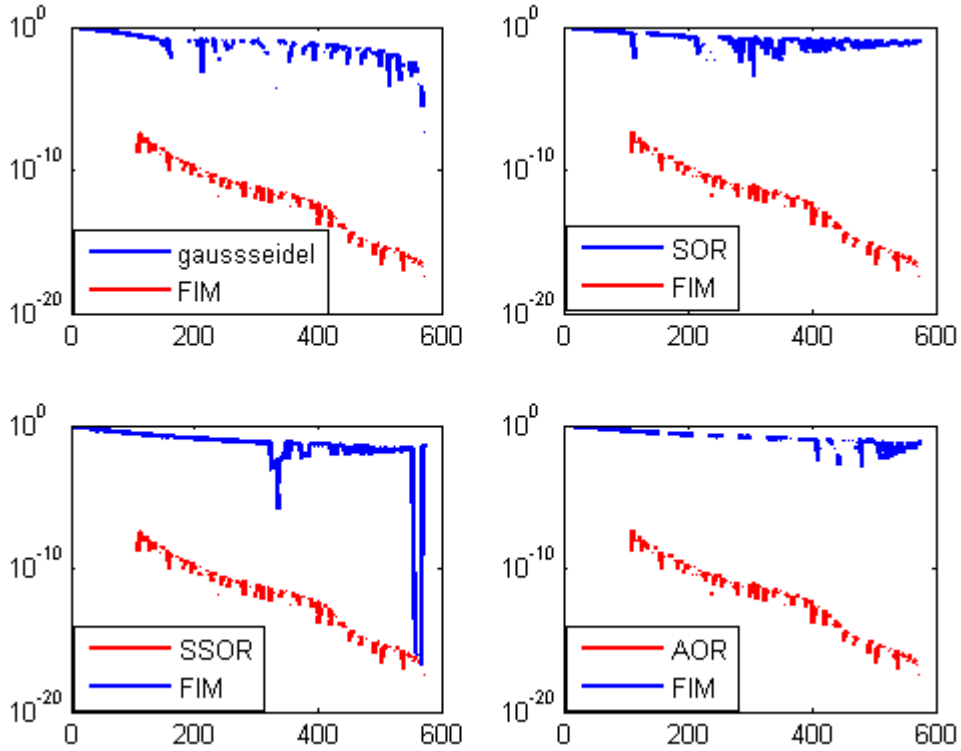

Fig. 2.spectra of the iteration matrix FIM and different iterative methods(for n=24)

Expriment2. (Application to the three-dimensional convection-diffusion equation)

Consider the three-dimensional convection-diffusion equation

$$-\underbrace{(u_{xx}+u_{yy}+u_{zz})}_{\Delta u}+2u_x+u_y+u_z=f(x,y,z)$$

On the unit cube domain $\Omega=[0,1]\times[0,1]\times[0,1]$, with Dirichlet boundary conditions.

When the seven-point finite difference discretization, for example, the centered differences to the diffusive terms, and the centered differences or the first order upwind approximations to the convective terms are applied to the above model convection-diffusion equation, we get the system of linear equations (1.1) with the coefficient matrix $A=T_x\otimes I\otimes I+I\otimes T_y\otimes I+I\otimes I\otimes T_z$

Where $A\in R^{n^3\times n^3}$ and the equidistant step-size h = 1/n is used in the discretization on all of the Three directions and the natural lexicographic ordering is employed to the unknowns.

In addition, $\otimes$ denotes the Kronecker product, and T_x , T_y, and T_z are tridiagonal Matrices given by

$$T_x = tridiagonal[-(\frac{2+2h}{12}),1,-(\frac{2-2h}{12})]$$

$$T_y = T_z = tridiagonal[-(\frac{2+h}{12}),0,-(\frac{2-h}{12})]$$

For details, we refer to [10-11].

For n=10($[A]_{1000\times1000}$) and by computation of spectral radius of iteration matrix in Jacobi ,Gauss-Seidel, symmetric Gauss-Seidel, SOR ,SSOR and AOR ,we have;

$$\begin{cases} \rho_{Jacobi} = 0.9571 \\ \rho_{G.S} = 0.9160 \\ \rho_{symmetricG.S} = 0.8453 \\ \rho_{SOR} \xrightarrow{w=.9} = 0.9312 \\ \rho_{SSOR} \xrightarrow{w=.9} = 0.8707 \\ \rho_{AOR} \xrightarrow{w=.9,r=.8} = 0.9369 \end{cases}$$

In Fig3. We can see that the spectral radius of iteration matrix in FIM(four-steps)for different values of 'k'.

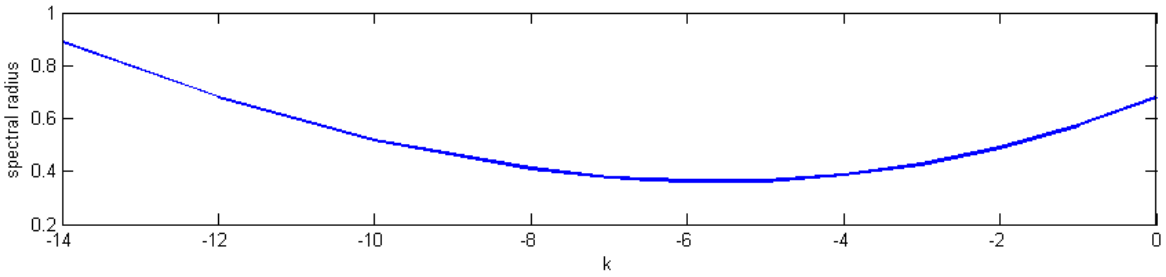

Fig. 3.spectral radius of FIM(four-steps) with some values of 'K'

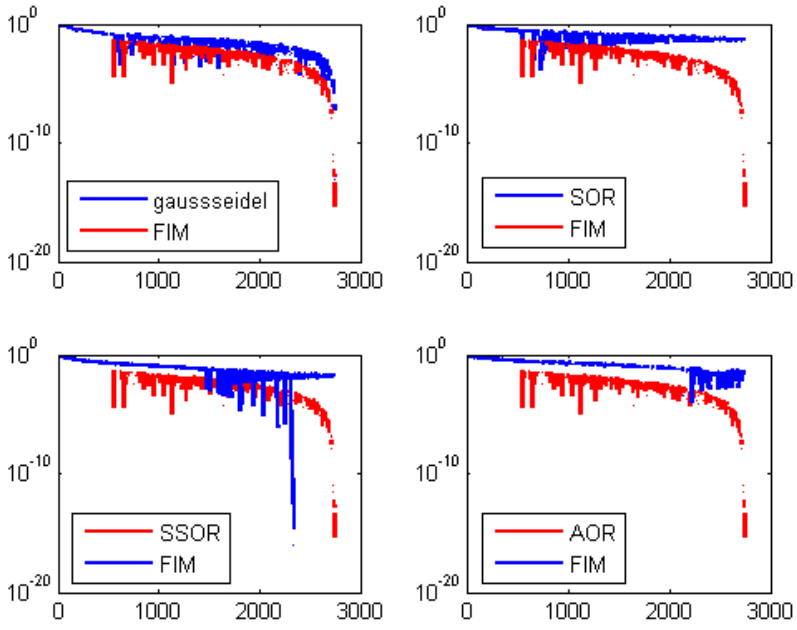

Fig. 4.spectra of the iteration matrix FIM and different iterative methods(for n=15)

For n=15($[A]_{3375\times3375}$) is given in the Fig.4, we can see that spectrum of the iteration matrices FIM (five-step)and other methods. In this experiment parameters are given as the following;
W=.9,r=.8,k=-10

6. Conclusion

The iterative methods like Jacobi, Gauss-seidel, SOR, etc are very popular for solving the linear systems and because the application of these methods is easy so they have been used very often by researchs.the only problem in these methods is that the process compare with the other methods more beaks done .in this article we have considered this point and developed the FIM method such that works fast with high accuracy and not accrued by breaking steps. The important points in this article are using affine combinations and less parameter. Finally we would like to suggest using FIM method instead of using common iterative methods also using this method as a base for preconditioning processes.

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