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Conceptual Category of Knowledage and Its Correlation-based Metrizable Space

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Abstract. The phenomenon was understood as the basic element to constitute knowledge in this study, where the granular structure was taken as the basic structure of phenomenon formalized representation. Based on the isomorphism between formal concept analysis and rough set theory, the granular structure model was established in conceptual category of phenomenon, and the correlation among concept categories was given as well as correlation distance, proving that the concept element space induced by correlation distance Θ was a metrizable space. Based on the conclusions in this study, the knowledge representation model can be established more completely than conceptual model in semantic description, providing a basis for phenomenon formalization research by using the topologic method.

Keywords: Knowledge Representation; Phenomenon; Formal Concept Analysis; Rough Set; Metric Space

1. Introduction

In knowledge engineering, semantic Web, knowledge discovery, information retrieval and other fields, it is required to conduct representation and description on knowledge as well as establish the formalized conceptual model. However, the description on knowledge is derived from that on “phenomenon” in natural, social and artificial systems. The human cognitive process on objective world was generalized as granulation, organization, and causal reasoning by Zadeh [1]. This process can also be mapped as the phenomenon structure in objective world. The phenomenon was taken as a research object in this study, and its structure contains the granular structure of categories, their correlation and relevance logic. Among them, the categories of phenomenon include the conceptual category of its semantics and the time - state category as the description on its movement and evolution basis. The correlation is able to link loose individual phenomena, providing a basis for the formation of knowledge with a core and boundaries. While, specific constraints are given to the correlation among phenomena by relevance logic. This study was carried out on granular structure of conceptual category in phenomenon as well as its correlation.

According to good definition on “concept” and its representative form consistent with granular structure in knowledge discovery and other fields proposed by formal concept analysis (FCA) theory [2,3] as well as the process on incomplete and insufficient knowledge in rough set (RS) theory [4,5,9], the descriptions on conceptual category of phenomenon were carried out by combining both theories and then incorporated into granular structure. Meanwhile, the correlation and correlation distance were also studied. Based on correlation distance, the definition of concept element space was given and proven as a metrizable space.

2. Phenomenon and the formalized representation of its categories

2.1. Granular structure

Human brain is a hierarchical structure, where the human knowledge structure is determined as a hierarchy by the formation and learning process of knowledge. Then, the phenomenon in objective world is taken as the mapping of human knowledge structure also with the same hierarchy. Such structure is considered as “granular structure” in view of granular computing theory [6,7], where the three basic attributes are satisfied[8], including (1) context attribute indicating the existence of a grain in special environment; (2) internal attribute reflecting elemental interaction in a grain; (3) external attribute reflecting the interaction between a grain and others. Based on the above attributes, the basic form of granular structure is given as below.

Definition 2.1 (granular structure): A granular structure is described with a triple (E_G, I_G, R), where E_G and I_G are the sets called as granular extension and granular connotation respectively. Binary relation $R \subseteq E_G \times I_G$ is called as context relation.

In general, granular structure can be established in a special domain based on specific context. A certain constraint is defined in this domain, where granular connotation I_G is expressed by constraint rules, reflecting the general characteristics of all elements in the domain; while granular extension E_G is the element set in this domain satisfying the constraint, i.e., the set of elements covered in granular structure. However, context relation R represents the external attribute of interaction among granular structures. The formalized representation on conceptual category of phenomenon is carried out through granular structure as below.

2.2. Conceptual category

First, considering that the conceptual categories of most phenomena are ambiguous with indistinct boundaries, the knowledge domain is required with classification capacity for a rough identification and organization on scattered grains. Thereby, the indiscernibility relation of RS theory was used to process incomplete and inadequate information, and then the knowledge domain can be divided into different equivalence classes, i.e., the phenomena are classified according to different attributes or characteristics. Meanwhile, the granular structure forms of conceptual category in phenomena are given by combining the scale theory of FCA. The isomorphism between both theories is first given by the following propositions.

Proposition 2.1. Any knowledge base (U, A) is given, where U is the domain and $A := \{B_m \mid m \in M\}$. $\mathrm{S}(U, A) := ((U, A, W, I), (S_B \mid B \in A))$ is known, and S_B is the rated scale with its derived context (U, N, J). γ is set to represent the mapping of object concept in the context (U, N, J), and thereby, if $(u,v) \in IND(P)$ ($(u,v) \in U^2$, $P \subseteq A$), then $\gamma(u) = \gamma(v)$.

Proof. $[v]_P = \{v \in U \mid u\,J(m,n) \Leftrightarrow v\,J(m,n), P \subseteq A\}$ is set as well as $v \in [u]_P$, $[u]_P = [v]_P$ ($u, v \in U$) has been proven previously.

In fact, if $(u,v) \in IND(P)$, then $I(u,m) = I(v,m) = w$ for any $m \in P$, where $w \in W_m$ and $wI_m n$ ($n \in M_m$). According to $u\,J(m,n) \Leftrightarrow v\,J(m,n)$, $v \in [v]_P$ is known, i.e., $[u]_P \subseteq [v]_P$. In addition, if $v \in [v]_P$, $w_1, w_2 \in m(U)$ must exist, known by $u\,J(m,n) \Leftrightarrow v\,J(m,n)$, as well as $I(u,m) = w_1$, $w_1 I_m n \Leftrightarrow I(v,m) = w_2$ and $w_2 I_m n$ .Thereby, $w_1 = n \Leftrightarrow w_2 = n$, $w_1 = w_2 = n$, and $I(u,m) = I(v,m)$. According to indiscernibility relation of rough set theory, $\mathrm{c}(u,v) \in IND(P)$ exists as well as $v \in [u]_P$, i.e., $[v]_P \subseteq [u]_P$, then $[u]_P = [v]_P$. Thus, $[u]_B = [v]_B$ exists for $\forall B \in P$.

Thereby, J can be redefined, and $[u]_B = [v]_B \Leftrightarrow B(u)\,J\,[v]_B \Leftrightarrow uJ(B, [v]_B)$ exists, i.e., the derived context (U, N, J) of $\mathrm{S}(U, A)$ can be expressed as $(U, \{(B, [u]_B) \mid B \in A, [u]_B \in U/B\}, J)$. $(u,v) \in IND(B) \Leftrightarrow uJ(B, [v]_B) \Leftrightarrow \gamma(u) = \gamma(v)$ can be obtained simultaneously. □

It is known by Proposition 2.1 that if a knowledge base (U, A) is given, then many-valued context can be generated under the effect of scale operator, and the derived context of many-valued context is $(U, \{(B, [u]_B) \mid B \in A, [u]_B \in U/B\}, J)$, i.e., the knowledge base (U, A) can be expressed with the derived context of many-valued context of formal concept. Thereby, the consistence between basic concepts of both theories is characterized in the form as a basis for establishing granular structure. Based on this proposition, the formalized description of conceptual category is given and called as concept element.

Definition 2.2 (concept element): A knowledge base (U, A) is given as well as a many-valued context (G, M, W, I), of which, $M \subseteq A, G \subseteq U/M, W := \{[u]_B \mid u \in G, B \in M\}$ and $I \subseteq G \times M \times W$. The concept element is the derived context of rated scale as $\left(G, \left\{\left(B, [u]_B\right) \mid B \in M, [u]_B \in U/B\right\}, J\right)$. $uJ\left(B, [v]_B\right) \Leftrightarrow [u]_B = [v]_B$ is satisfied, where $\forall u, v \in U$ and $u \neq v$.

It is abbreviated as $\chi := (G, N, J)$, where $N := \left\{\left(B, [u]_B\right) \mid B \in M, [u]_B \in U/B\right\}$.

It is known from granular structure of concept element that the constraints among all elements with similarity in the domain U can be reflected in granular connotation $\left\{\left(B, [u]_B\right) \mid B \in M, [u]_B \in U/B\right\}$ through indiscernibility relation. While granular extension G is a set of elements satisfying this indiscernibility relation, and specific characterization of constraints are given by relation J. Then, conceptual category is a granular structure satisfying granular attributes.

2.3. Correlation among phenomena

The relation among phenomena was studied in this section, i.e., the external attributes of phenomenon under granular structure. The conceptual category of phenomenon is described with roughness, i.e., ambiguous and indistinct boundaries exist. Same or similar parts may be included in conceptual categories of two different phenomena, i.e., different phenomena are correlated due to their conceptual similarity, and then a new phenomenon with more abundant knowledge connotation is constituted elemented.

Definition 2.4 (correlation among concept elements): In the knowledge base (U, A), any two many-valued contexts, i.e., (G, M, W, I) and (G', M', W', I'), are given. Then, the two concept elements are shown as $\chi := (G, N, J)$ and $\chi' := (G', N', J')$ respectively, and $\exists C \subseteq M$ and $\exists C' \subseteq M'$ exist. If the following conditions are satisfied:

1. $N \cap N' \neq \varnothing$
2. $\bigcup_{Y \in X/C'} \left\{u \in G \mid [u]_C \subseteq Y\right\} \cap \bigcup_{Y' \in X/C} \left\{v \in G' \mid [v]_{C'} \subseteq Y'\right\} \neq \varnothing$

Then, the two concept elements are correlated. C （C'） is called as the correlative knowledge set of concept element χ （χ'）, where the elements are called as correlative knowledge. And the nonempty intersection in condition 2 is denoted as $\Lambda(\chi, \chi')$.

The correlation among concept elements is reflected in its granular extension and connotation: first, the interaction degree among the subsets constituted by elements is reflected in the division of different knowledge contexts in two sub-domains, i.e., the interaction degree among granular extensions. Second, this intensity depends on element quantity in the domain U covered in lower approximation of G' （G） determined by B - knowledge (B' - knowledge), reflecting the correlation degree among granular connotations of concept elements.

It is reflected in Condition 2 of Definition 2.4 that the correlation between two concept elements in their respective sub-domains depends on that among elements in the domain under correlative knowledge context. In order to lose no generality, the correlation between the elements of two sub-domains is denoted as $\Re_U$. When $\forall u \in G$ and $\exists v \in G'$, $u \Re_U v$ is satisfied, where the correlation $\Re_U$ between the elements of G and G' is expressed by a many-valued mapping relation η as $\eta : G \to 2^{G'}$, i.e., $\eta(u) = \{v \in G' \mid u \Re_U v\}$. $\eta(u)$ is a set of all elements correlated to u in G', and the contrary is also true. It is reflected in mapping η that intersection (inclusion) exists between the subsets of elements in G and that of G' correlated to element knowledge of G under the premise of knowledge correlation between two concept elements.

Definition 2.5: In the set knowledge base (U, A), any two concept elements, i.e., $\chi := (G, N, J)$ and $\chi' := (G', N', J')$, are given. If correlative knowledge sets of C and C' exist in the concept elements of χ and χ' respectively, then $B \in C$. If the following is satisfied:

$$\bigcup_{Y \in X/(C'-\{B\})} \{u \in G \mid [u]_C \subseteq Y\} = \bigcup_{Y \in X/C'} \{u \in G \mid [u]_C \subseteq Y\}$$

Then, B is called as M'-reducible knowledge in M, otherwise, B is M'-irreducible knowledge in M. Similarly, M-reducible knowledge in M' can be defined.

Definition 2.6 (knowledge core): In the knowledge base (U, A), any two many-valued contexts, i.e., (G, M, W, I) and (G', M', W', I'), are given. Then, the two concept elements are shown as $\chi := \left(G, \left\{\left(B, [u]_B\right) \mid B \in M, [u]_B \in W\right\}, J\right)$ and $\chi' := \left(G', \left\{\left(B', [v]_{B'}\right) \mid B' \in M', [v]_{B'} \in W'\right\}, J'\right)$ respectively. $B \in M$ ($B' \in M'$), where B (B') is not only reducible knowledge but also correlative knowledge, called as core knowledge. The set constituted by intra-core knowledge of correlative concept elements is called as knowledge core, denoted as $core(A)$.

Knowledge core is also considered as the identity of boundaries and characteristics with more advanced knowledge grains formed by phenomena with correlations.

3. Concept element space and its topology under correlation

In order to measure the correlation among concept elements, it can be considered to give the similar definition of distance in Euclidean Space. In granular structure of concept element, the granular connotation can be taken as "knowledge dimension" of granular extension, and the knowledge core $core(A)$ as the coordinates of reference systems. Thereby, the distance among elements in granular extension can be defined based on this knowledge dimension, and then that between concept elements can be defined.

Definition 3.1 (correlation):

Two concept elements of $\chi := (G, N, J)$ and $\chi' := (G', N', J')$ are correlated, and then the correlation degree between the elements u, v ($u, v \in \Lambda(\chi, \chi')$) in two concept elements is defined as follows:

$$\rho = \left| card\left(\left\{[u] \in N \mid \eta(u) \subseteq G'\right\}\right) - card\left(\left\{[v] \in N' \mid \eta(v) \subseteq G\right\}\right)\right| / card\left(core(A)\right).$$

Where, $[u]$ and $[v]$ indicate the equivalence class on knowledge core $core(A)$ of $u \in G$ and $v \in G'$ respectively. $\rho(u, v)$ is called as the local distance between u and tv. The distance between G and G' as correlation-based granular extensions is defined as $\rho(G, G') = \inf\{\rho(u, v) \mid u \in G, v \in G'\}$.

It is known from the above definition that the measurement of correlation degree between concept elements is required for no prior knowledge of any assumptions, but depends on the given knowledge base. Meanwhile, the contrast relation between knowledge similarity among phenomena and knowledge content is also given by correlation degree.

Proposition 3.1. According to the definition of local distance, the significant properties are shown in the two correlative concept elements, i.e., $\chi := (G, N, J)$ and $\chi' := (G', N', J')$, shown as below.

(1) $0 < \rho(G, G') < 1$;

(2) $\rho(u, v) \geq \rho(u, G')$, $u \in G$、$v \in G'$

(3) When $\rho < 1$, if

$$\left\{[u] \in N \mid \eta(u) \subseteq G'\right\} \cap \left\{[v] \in N' \mid \eta(v) \subseteq G\right\} = \left\{[v] \in N' \mid \eta(v) \subseteq G\right\},$$

Then, the concept element χ' is covered in χ or as the sub concept element of χ.

Definition 3.2 (concept element space): As a set of concept elements with a given local distance ρ, X is called as a concept element space, denoted as a two-tuple (X, ρ).

The correlation degree herein is the distance paradigm based on rough set, used for calculating the similar degree (distance) between elements in granular extensions of two concept elements. The concept element space is expected to correspond to metric space, so as to introduce the topology for studying more abundant properties of phenomenon. The following definition of correlation is another express of the distance between concept elements based on local distance, used for inducing a metric space.

Definition 3.3: In the knowledge base (U, A), the two concept elements, i.e., $\chi := (G, N, J)$ and $\chi' := (G', N', J')$, are given. For $\forall \delta \geq 0$, as a set of G' with the local distance from G less than δ, $\mathbf{Z}(G,\delta) := \{G' \mid \rho(G,G') < \delta\}$ is called as G-ball with G as the center and δ as the radius. In the domain U, the union of all G-balls is shown as $G_\delta := \bigcup_{G\in U} \mathbf{Z}(G,\delta)$. When $\delta = 0$, $G_\delta = G$.

Definition 3.4 (correlation distance): Two concept elements, i.e., $\chi := (G, N, J)$ and $\chi' := (G', N', J')$, are correlated, and then, the correlation distance is Θ. The correlation distance between χ and χ' is defined as $\Theta(\chi, \chi') := \inf\{\delta \geq 0 \mid G \subset G'_\delta \wedge G' \subset G_\delta\}$.

The concept element space can be redefined by the extension of local distance ρ to correlation distance Θ.

Definition 3.5 (concept element space): As a family of concept elements with given correlation distance Θ, X is called as a concept element space, denoted as a two-tuple (X, Θ).

Proposition 3.2: The two concept elements, i.e., $\chi := (G, N, J)$ and $\chi' := (G', N', J')$, are assumed correlated. If $\mathrm{A} := \{\delta \geq 0 \mid G \subset G'_\delta \wedge G' \subset G_\delta\}$ is set, then $\Theta(\chi, \chi') \in \mathrm{A}$.

Proof. $\Theta(\chi, \chi') = \inf A$ is known from the definition of correlation distance, and $\Theta(\chi, \chi')$ is abbreviated as Θ. The conclusion of previous proofs is required to prove $G \subset G'_\Theta$ and $G' \subset G_\Theta$ simultaneously.

In fact, for $\forall \varepsilon > 0$, $\delta \in A$ exists with $0 \leq \delta \leq \Theta + \varepsilon$. It is known from the definition of A that $G \subset G'_\delta$ and $G' \subset G_\delta$. Meanwhile, due to $\delta \leq \Theta + \varepsilon$, $G_\delta \subset G_{\Theta+\varepsilon}$ and $G'_\delta \subset G'_{\Theta+\varepsilon}$ are easy to know. Then,

$$\forall \varepsilon > 0,\ G \subset G'_{\Theta+\varepsilon} \text{ and } G' \subset G_{\Theta+\varepsilon} \tag{1}$$

$G \not\subset G'_\Theta$ is assumed, i.e., $g \in G$ always with $g \notin G'_\Theta$. Due to the correlation between χ and χ', if $\rho(g, G'_\Theta) = m > 0$ is set according to Proposition 3.1 as well as $\rho(g, g') \geq m > m/2$, and then, $g \notin (G'_\Theta)_{m/2}$ can be obtained. It can be known from the Minkowski sum that $(G'_\Theta)_{m/2} = G'_\Theta + \mathbf{Z}(v, m/2) = G' + \mathbf{Z}(v,\Theta) + \mathbf{Z}(v, m/2) = G' + \mathbf{Z}(v, \Theta + m/2) = G'_{\Theta+m/2}$, （$v \in G'$）. Then, when $\varepsilon = m/2$, $G \subset G'_{\Theta+\varepsilon}$ will not exist, i.e., conflict with (1). Thus, $G \subset G'_\Theta$. $G' \subset G_\Theta$ is proven by the same methods, i.e., because $G \subset G'_\Theta$ and $G' \subset G_\Theta$, $\Theta(\chi, \chi') \in \mathrm{A}$. □

In order to prove the concept element space as a topological space induced by a metric, it is first required to prove the correlation distance Θ is a metric, i.e., the following definition is satisfied:

Definition 3.6 [11]: A metric in the set X is a function as $d: X \times X \to R$, leading to the existence of following properties:

(1) For $\forall x, y \in X$, $d(x, y) \geq 0$. If and only if $x = y$, $d(x, y) = 0$.

(2) For $\forall x, y \in X$, $d(x, y) = d(y, x)$.

(3) For $\forall x, y, z \in X$, $d(x, y) + d(y, z) \geq d(x, z)$.

Theorem 3.1. If any two concept elements in concept element space (X,Θ) are correlated, then the correlation distance in concept element space (X,Θ) is a metric.

Proof. It order to prove Θ as a metric, it is required to prove three conditions in Definition 3.6 are satisfied. The two given concept elements, i.e., $\chi := (G,N,J)$ and $\chi' := (G',N',J')$, are correlated, and $\mathrm{A} := \{\delta \geq 0 \mid G \subset G'_\delta \wedge G' \subset G_\delta\}$ is set.

First, $\Theta(\chi,\chi') \geq 0$ exists obviously. Second, according to Definition 3.3, $G = G_0$ exists as well as $G' = G'_0$. When $G = G'$, $G = G' \subset G'_0$ exists as well as $G' = G \subset G_0$, i.e., $0 \in A$, and $\inf A = \Theta(\chi,\chi') = 0$. In addition, when $\Theta(\chi,\chi') = 0$, i.e., $\inf A = 0$, $0 \in A$ is known according to Proposition 3.2, and then, $G \subset G'_0 = G'$ exists as well as $G' \subset G_0 = G$, i.e., $G = G'$. Thus, Condition (1) exists.

Condition (2) exists obviously;

The following is to prove the correlation distance satisfying Condition (3). It is assumed that the concept element $\chi'' := (G'',N'',J'')$ is correlated to χ and χ' respectively. Then, if

$\Theta(\chi,\chi') = 0$ or $\Theta(\chi',\chi'') = 0$, $G = G'$ or $G' = G''$ exists, i.e., $\Theta(\chi,\chi'') = \Theta(\chi,\chi') + \Theta(\chi',\chi'')$.

However, if $\Theta(\chi,\chi') \neq 0$ and $\Theta(\chi',\chi'') \neq 0$, then $\mathrm{A}_1 := \{\delta \geq 0 \mid G \subset G'_\delta \wedge G' \subset G_\delta\}$ and $\mathrm{A}_2 := \{\delta \geq 0 \mid G' \subset G''_\delta \wedge G'' \subset G'_\delta\}$. Thus, $\Theta(\chi,\chi') = \inf A_1$ and $\Theta(\chi',\chi'') = \inf A_2$. It is known according to Proposition 3.2 that $\Theta(\chi,\chi') \in A_1$ and $\Theta(\chi',\chi'') \in A_2$. Thus, $G' \subset G_{\Theta(\chi,\chi')}$ and $G'' \subset G'_{\Theta(\chi',\chi'')}$ can be obtained according to the definitions of A_1 and A_2, i.e., $G'_{\Theta(\chi',\chi'')} \subset \left(G_{\Theta(\chi,\chi')}\right)_{\Theta(\chi',\chi'')}$. Then, the following can be obtained through the proof process of Proposition 3.2,

$$G'' \subset G'_{\Theta(\chi',\chi'')} \subset \left(G_{\Theta(\chi,\chi')}\right)_{\Theta(\chi',\chi'')} = G_{\Theta(\chi,\chi')+\Theta(\chi',\chi'')} \quad (1)$$

The following can be obtained with the same method,

$$G \subset \left(G''_{\Theta(\chi,\chi')}\right)_{\Theta(\chi',\chi'')} = G''_{\Theta(\chi,\chi')+\Theta(\chi',\chi'')} \quad (2)$$

According to (1) and (2), $\Theta(\chi,\chi') + \Theta(\chi',\chi'') \in \{\delta \geq 0 \mid G \subset G''_\delta \wedge G'' \subset G_\delta\}$ can be obtained, i.e., $\Theta(\chi,\chi'') := \inf\{\delta \geq 0 \mid G \subset G''_\delta \wedge G'' \subset G_\delta\} \leq \Theta(\chi,\chi') + \Theta(\chi',\chi'')$. Then, Condition (3) exists. □

Definition 3.7: In concept element space (X,Θ), the concept elements of $\chi := (G,N,J)$ and $\chi' := (G',N',J')$ are given. For $\forall \delta \geq 0$, as a set of χ' with correlation distance from χ less than δ, $\mathbf{Z}(\chi,\delta) := \{\chi' \mid \Theta(\chi,\chi') < \delta\}$ is called as χ-ball with χ as the center and δ as the radius.

Theorem 3.2: If the concept element space (X,Θ) is given, then (X,Θ) is a metrizable space induced by Θ.

Proof. First, for $\forall \delta > 0$, $\chi \in \mathbf{Z}(\chi,\delta)$ exists obviously. Second, χ' is set as the concept element correlated to χ, and $\chi' \in \mathbf{Z}(\chi,\delta_1)$, then $\mathbf{Z}(\chi',\delta_2)$ exists as the χ-ball with χ' as the center and δ_2 as the radius. χ'' is set as the concept element correlated to χ' and χ as well as $\chi'' \in \mathbf{Z}(\chi',\delta_2)$. If $\delta_2 = \delta_1 - \Theta(\chi,\chi')$ is set, then $\Theta(\chi,\chi'') \leq \Theta(\chi,\chi') + \Theta(\chi',\chi'') = \delta_1 - (\delta_2 - \Theta(\chi',\chi''))$ exists according to Theorem 3.1. Because $\chi'' \in \mathbf{Z}(\chi',\delta_2)$, i.e., $\delta_2 - \Theta(\chi',\chi'') > 0$, $\Theta(\chi,\chi'') < \delta_1$, and $\chi'' \in \mathbf{Z}(\chi,\delta_1)$ can be obtained. Then, the following exists.

$$\mathbf{Z}(\chi',\delta_2)\subset\mathbf{Z}(\chi,\delta_1) \tag{3}$$

$\mathbf{Z}_1$ and $\mathbf{Z}_2$ are set as the two χ -balls. If $\chi'\in\mathbf{Z}_1\cap\mathbf{Z}_2$, then $\mathbf{Z}(\chi',\varepsilon_1)\subset\mathbf{Z}_1$ and $\mathbf{Z}(\chi',\varepsilon_2)\subset\mathbf{Z}_2$ always exist according to (3). If $\varepsilon=\min(\varepsilon_1,\varepsilon_2)$ is set, then $\mathbf{Z}(\chi',\varepsilon)\subset\mathbf{Z}_1\cap\mathbf{Z}_2$. From the above conclusions, it can be obtained that the family consisting of $\mathbf{Z}(\chi,\delta)$ is the topological base of a certain metric induced by metric Θ in X. □

4. Conclusions

The formalized representation was carried out on the conceptual category of phenomenon with granular structure as the basic framework in this study, i.e., by adding the indiscernibility relation under multi-value context of FCA, the conceptual category was generated with the capacity of processing incomplete and inadequate information, i.e., the capacity of classifying phenomena based on different attributes or characteristics. Meanwhile, the correlation between phenomena was understood as that between conceptual categories (concept elements) in this study, where the correlation conditions and correlation distance as a metric of correlation degree were defined. In the meantime, the concept element space induced by correlation distance was proven as a metric space. The work in this study laid the foundation for studying phenomenon properties in the structure of topological space.

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