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Solution of Large Scale Economic Load Dispatch Problem using Quadratic Programming and GAMS: A Comparative Analysis

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Abstract. This paper presents a comparative analysis of efficient and reliable modern programming approach using quadratic programming (QP) and general algebraic modeling system (GAMS) to solve economic load dispatch (ELD) problem. The proposed methodology easily takes care of different equality and inequality constraints of the power dispatch problem to find optimal solution. To validate the effectiveness of algorithm simulations have been performed over four different cases i.e. Crete Island system of 18 thermal generating units, twenty generating units system with losses, practical Taiwan Power Company (TPC) data which consists of 40 generating unit system and a very large system consisting of 110 generating unit. Results obtained with the proposed method have been compared with other existing relevant approaches available in literatures. Experimental results support the claim of proficiency of the method over other existing techniques in terms of robustness and most importantly its optimal search behavior.

Keywords: Economic load dispatch, quadratic programming, general algebraic modelling system, quadratic cost function, operating limit constraints..

1. Introduction

The idea behind economic dispatch problem in a power system is to determine the optimal combination of power output for all generating units which will minimize the total fuel cost while satisfying load and operational constraints. The economic dispatch problem is very complex to solve because of its massive dimension, a non-linear objective function and large number of constraints. Various investigations on the ELD have been undertaken till date. Suitable improvements in the unit output scheduling can contribute to significant cost savings [1, 2]. Also information about forming market clearing prices is provided by it.

To improve the quality of solution, lot of researches have been done and various methods have been evolved so far in the field of economic load dispatch.

Several classical optimization techniques, such as the lambda iteration approach, the gradient method, the linear programming method and Newton's method were used to solve the ELD problem [3]. Lambda iteration method is the most common, which has been applied to solve ELD problems. But for effective implementation of this method, the formulation must be continuous. Though fast and reliable, the main drawback of the linear programming methods is that they are associated with the piecewise linear cost approximation [4].

In order to get the qualitative solution for solving the ELD problems, Artificial Neural Network (ANN) techniques such as Hopfield Neural Network (HNN) [4] have been used. The objective function of the Economic Dispatch problem is transformed into a Hopfield energy function and numerical iterations are applied to minimize the energy function. The Hopfield model has been employed to solve the ED problems for units having continuous or piecewise quadratic fuel cost functions and for units having prohibited zone constraints. In the conventional Hopfield Neural Network, the input-output relationship for its neurons can be described by sigmoid function. Due to the use of the sigmoid function to solve the ED problems, the Hopfield model takes more iteration to provide the solution and often suffers from large computational time.

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Recently, different heuristic approaches have been proved to be effective with promising performance. These include evolutionary programming (EP) [6], genetic algorithm (GA) [7], differential evolution (DE) [8], particle swarm optimization (PSO) [9] etc. Improved fast Evolutionary programming algorithm has been successfully applied for solving the ELD problem [1, 5]. Other algorithms like Biogeography-Based optimization(BBO) [10], Chaotic particle swarm optimization (CPSO) [11], new particle swarm with local random search (NPSO-LRS) [12], Self-Organizing Hierarchical PSO [13], Bacterial foraging optimization [14], improved coordination aggregated based PSO [15], quantum-inspired PSO [16], improved PSO [17], HHS algorithm [18] and HIGA [19] are some of the those which have been successfully applied to solve the ELD problem.

In this paper a comparative analysis of Quadratic programming (QP) and General Algebraic Modeling System (GAMS) approach has been proposed to solve economic load dispatch problems.

Quadratic programming is an effective tool to find global minima for optimization problem having Quadratic objective function and linear constraints. The objective function for the entire test considered here is Quadratic in nature for all cases but the constraints are not linear. Constraints are liberalized by transformation of variable technique and the Quadratic programming is applied recursively till the convergence is achieved [20].

General Algebraic Modeling System (GAMS) [21] is a high-level model development environment that supports the analysis and solution of linear, non linear and mixed integer optimization problems. GAMS is especially useful for handling large dimension and complex problem easily and accurately.

In this paper the effectiveness of the proposed algorithm is demonstrated using four standard test problems (i) Crete Island system of 18 thermal generating units, (ii) 20 generating units system with losses, (iii) practical large scale Taiwan Power Company (TPC) system consisting of 40 generating units and (iv) a very large system consisting of 110 generating units.

The paper is organized as follows: Section 2 provides a brief description and mathematical formulation of ELD problems. The concept of QP and GAMS is discussed in Section 3 and Section 4 respectively. The performance of both proposed approaches and the simulation studies are discussed in Section 5. Finally, Section 6 presents the conclusions.

2. Problem formulation

In a power system, the unit commitment problem has various sub-problems varying from linear programming problems to complex non-linear problems. The concerned ELD problem is one of the different non-linear programming sub-problems of unit commitment. The ELD problem is about minimizing the fuel cost of generating units for a specific period of operation so as to accomplish optimal generation dispatch among operating units and in return satisfying the system load demand considering power system operational constraints.

The objective function corresponding to the production cost can be approximated to be a quadratic function of the active power outputs from the generating units. Symbolically, it is represented as

$$\text{Minimize} \quad F_t^{\cos t} = \sum_{i=1}^{N_G} f_i(P_i) \tag{1}$$

Where

$$f_i(P_i) = a_i P_i^2 + b_i P_i + c_i, \quad i = 1,2,3,\dots,N_G \tag{2}$$

is the expression for cost function corresponding to i^{th} generating unit and a_i, b_i and c_i are its cost coefficients. P_i is the real power output (MW) of i^{th} generator corresponding to time period t. N_G is the number of online generating units to be dispatched.
The constraints are:

1) Power Balance Constraints:

This constraint is based on the principle of equilibrium between total system generation $(\sum_{i=1}^{N_G} P_i)$ and total system loads (P_D) and losses (P_L). That is,

$$\sum_{i=1}^{N_G} P_i = P_D + P_L \tag{3}$$

Where the transmission loss P_L is expressed using B- coefficients [3], given by

$$P_L = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} P_i B_{ij} P_j + \sum_{i=1}^{N_G} B_{0i} P_i + B_{00} \tag{4}$$

The following conditions for optimality can be obtained after applying lagrangian multiplier and K.T. condition

$$2\, a_i P_i + b_i = \lambda \left(1 - 2 \sum_{j=1}^{n} B_{ij}\right) \quad , i = 1, 2, 3, 4.....n \tag{5}$$

2) The Generator Constraints:

The power generated by each generator should be within its lower limit P_i^{min} and upper limit P_i^{max} so that

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{6}$$

3. Quadratic Programming Algorithm

Quadratic Programming is an effective optimization method to find the global solution if the objective function is quadratic and the constraints are linear. It can be applied to optimization problems having non-quadratic objective and nonlinear constraints by approximating the objective to quadratic function and the constraints as linear. For all the problems the objective is quadratic but the constraints are also quadratic so the constraints are to be made linear [20].

The non linear equations and inequalities are solved by the following steps.

Step 1: To initialize the procedure allocate lower limit of each plant as generation, evaluate the transmission loss and incremental loss coefficients and update the demand.

$$P_i = P_i^{min} \text{ , } x_i = 1 - \sum_{j=1}^{n} B_{ij} P_i, \qquad \text{and} \qquad PD^{new} = PD + P_L^{old} \tag{7}$$

Step 2: Substitute the incremental cost coefficients and solve the set of linear equations to determine the incremental fuel cost λ as.

$$\lambda = \frac{\sum_{i}^{n} 0.5 \times \frac{b_i}{a_i}}{P_D^{new} + \sum_{i}^{n} 0.5 \times \frac{b_i}{a_i}} \tag{8}$$

Step 3: Determine the power allocation of each plant

$$P_i^{new} = \frac{\lambda - \left(\frac{b_i}{ai}\right)}{2 \times \left(\frac{a_i}{x_i}\right)} \tag{9}$$

If plant violates its limits it should be fixed to that limit and only the remaining plants only should be considered for the next iteration.

Step 4: Check for convergence

$$\left| \sum_{i}^{n} P_i - PD^{new} - P_L \right| \leq \in \tag{10}$$

$\in$ is the tolerance value, for power balance violation.

Step 5: Carry out the steps 2-4 till convergence is achieved.

For all the above four steps the objective is quadratic but the constraints are also quadratic so the constraints are to be made linear.

$$\text{Minimize} \quad XH\,X^T + f^T X$$
$$\text{Subjected to} \quad KX \leq R \quad , X^{min} \leq X \leq X^{max}$$
$$X = [x_1, x_2, x_3, \ldots\ldots, x_n]^n$$
$$f = [f_1, f_2, f_3, \ldots\ldots, f_n]^n$$
$$R = [R_1, R_2, R_3, \ldots\ldots, R_m]^T$$

H is a Hessian matrix of size, $n \times n$ and *A* is a $m \times n$ matrix representing inequalities.
For the economic dispatch with losses the quadratic programming algorithm can be effectively implemented by defining the matrices *H, f, K* and *R.*

$$H = diag\left(\left[\frac{a_1}{x_1}, \frac{a_2}{x_2}, \ldots\ldots \frac{a_n}{x_n}\right]\right)$$

$$f = \left|\frac{b_1}{x_1}, \frac{b_2}{x_2}, \ldots\ldots \frac{b_n}{x_n}\right|$$

K = [1, 1,… …1] 1×n matrix, and $\boldsymbol{R = PD} + P_L^{old}$

4. General Algebraic Modeling System (GAMS)

The General Algebraic Modeling System (GAMS) is a high-level model specially designed for modeling linear, nonlinear and mixed integer optimization problems. GAMS can easily handle large and complex problems. It is especially useful for handling large complex problems, which may require much revision to establish an accurate model. Conversion of linear to nonlinear optimization is also very simple. Models can be developed, solved and documented simultaneously, maintaining the same GAMS model file. The basic structure of a mathematical model coded in GAMS has the components: sets, data, variable, equation, model and output [22] and the solution procedure are shown below.

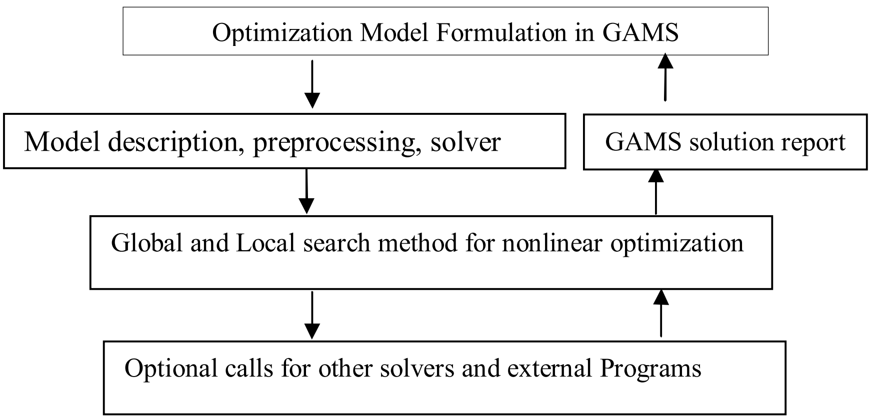

Fig. 1: GAMS modeling and solution procedure

STEPS FOR PROBLEM FORMULATION WITH GAMS

GAMS formulation follows the basic format as given below:

1. SETS
   Declaration
   Assignment of members
2. Data (PARAMETERS, TABLES, SCALARS)
   Declaration
   Assignment of values
3. VARIABLES
   Declaration
   Assignment of type
   Assignment of bounds and/or initial values (optional)
4. EQUATIONS
   Declaration
   Definition
5. MODEL and SOLVE statements
6. DISPLAY statements (optional)

5. Result and Discussion

The Quadratic programming and GAMS have been applied on four different standard systems. Test case I consists of *Crete Island system* of 18 thermal generating units, Test case II consists of 20 generating units system with losses, Test case III consists of practical *Taiwan Power Company* (TPC) 40 generators system and Test case IV consists of a large scale system consisting of 110 generating units .The programs were written in MATLAB 7.8 for quadratic programming and implementation on GAMS with system configuration Core 2 Duo processor and 3GB RAM.

5.1. Test case 1

Crete Island system of 18 thermal generating units having quadratic (Convex) cost function: The parameters of all thermal units are taken from [23], and the maximum power demand of the system set at MD = 433.22 MW. The results are compared with λ-iteration and Binary GA [23], RGA [23] and ABC [24] for this system. The summarized results of test case 1 for different demands without loss for the QP and GAMS algorithm are listed in Table: 1 and the comparative results are provided in Table 2 which shows that QP and GAMS both provides superior result then earlier reported results; But GAMS provides much better result then QP.

5.2. Test case 2

The system consists of twenty generating units having quadratic cost function with generating and transmission loss coefficient and power demand is set at 2500 MW. The parameters of all thermal units and loss coefficient are taken from [25].The results are compared with λ-iteration and Hopfield Model [25], BBO [10] and SA [26] methods for this system. The results obtained by quadratic programming approach and GAMS are listed in Table: 3. It can be clearly seen from Table: 3 the proposed GAMS provides better results as compared to other reported evolutionary algorithm techniques like λ-iteration, Hopfield Model, BBO and SA.

5.3. Test case 3

A 40 unit practical ED system of Taiwan Power Company (TPC) is employed as in this example uses quadratic (convex) unit cost functions. The input data of the entire system are given in [27]. In this case there are two different load demands 9000 MW and 10500 MW without transmission losses are considered. The results are compared with VSDE [27] and SA [26] methods for this system. The results obtained by quadratic programming approach and GAMS are listed in Table: 4.

TABLE 1: RESULT OF 18 UNIT SYSTEM (MD=433.33 MW)

Unit No.	QP				GAMS			
	0.95*MD	**0.9*MD**	**0.80*MD**	**0.70*MD**	**0.95*MD**	**0.9*MD**	**0.80*MD**	**0.70*MD**
P_{g1}	15	15	15	15	15	15	15	15
P_{g2}	45	45	45	44.6317	45	45	45	44.6318
P_{g3}	25	25	25	25	25	25	25	25
P_{g4}	25	25	25	25	25	25	25	25
P_{g5}	25	25	25	25	25	25	25	25
P_{g6}	13.7063	8.2134	3	3	13.7063	8.2134	3	3
P_{g7}	13.7063	8.2134	3	3	13.7063	8.2134	3	3
P_{g8}	12.28	12.28	12.28	12.28	12.28	12.28	12.28	12.28
P_{g9}	12.28	12.28	12.28	12.28	12.28	12.28	12.28	12.28
P_{g10}	12.28	12.28	12.28	12.28	12.28	12.28	12.28	12.28
P_{g11}	12.28	12.28	12.28	12.28	12.28	12.28	12.28	12.28
P_{g12}	24	24	20.7264	14.8823	24	24	20.7264	14.8823
P_{g13}	6.4132	3.1491	3	3	6.4132	3.1491	3	3
P_{g14}	36.2	36.2	30.8651	21.1329	36.2	36.2	30.8651	21.1329
P_{g15}	45	42.4842	32.3651	23.2393	45	42.4842	32.3651	23.2393
P_{g16}	37	37	33.2497	24.1239	37	37	33.2497	24.1239
P_{g17}	45	43.3688	33.2497	24.1239	45	43.3688	33.2497	24.1239
P_{g18}	6.4132	3.1491	3	3	6.4132	3.1491	3	3
Total power O/P (MW)	**411.559**	**389.898**	**346.576**	**303.254**	**411.559**	**389.898**	**346.576**	**303.254**
Power Demand (MW)	**411.559**	**389.898**	**346.576**	**303.254**	**411.559**	**389.898**	**346.576**	**303.254**
Power Mismatch	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Total Generation cast ($/hr)	**29622**	**27544.5**	**23746.04**	**20276.966**	**29621.817**	**27544.476**	**23746.036**	**20276.9**

TABLE 2: COMPARISION OF RESULT OF 18 UNIT SYSTEM

Demand	λ-iteration cost($/h)[23]	Binary GA cost($/h)[23]	Real-coded GA cost($/h) [23]	ABC cost($/h)[24]	**QP cost($/h)**	**GAMS cost($/h)**
0.95*MD	29731.05	29733.42	29731.05	29730.8	**29622**	**29621.817**
0.90*MD	27652.47	27681.05	27655.53	27653.3	**27544.5**	**27544.476**
0.80*MD	23861.58	23980.24	23861.58	23859.4	**23746.0363**	**23746.036**
0.70*MD	20393.43	20444.68	20396.39	20391.6	**20276.966**	**20276.9**

5.4. Test case 4

A large scale system consisting of 110 generating units system is employed in this example uses quadratic (convex) unit cost functions without losses .The input data of the entire system is taken from [28]. To investigate the robustness of the large system, here there are three different low, medium and high power demand of 10000 MW, 15000 MW and 20000 MW are considered. The results are compared with Analytical approach [29], SA [30], SAB [30], SAF [30] and RQEF [31] methods for this system. The results obtained by quadratic programming approach and GAMS are listed in Table: 5 and the comparative results are provided in Table: 6 which shows that QP and GAMS both provides better results as compared to other reported evolutionary algorithm techniques like Analytical approach , SA ,SAB, SAF and RQEA But GAMS provides much better result then QP.

TABLE 3: COMPARISION OF RESULT OF 20 UNIT SYSTEM (PD=2500 MW)

Unit No.	λ-iteration Method[25]	Hopfield Model[25]	BBO [10]	SA [26]	**QP**	**GAMS**
P_{g1}	521.785	512.7804	513.0892	512.7892	600	512.782
P_{g2}	169.133	169.1035	173.3533	169.0758	200	169.102
P_{g3}	126.8898	126.8897	126.9231	126.8792	50	126.891
P_{g4}	102.8657	102.8656	103.3292	102.8603	56.92	102.867
P_{g5}	113.6836	113.6836	113.7741	113.6844	94.28	113.683
P_{g6}	73.571	73.5709	73.06694	73.5866	33.72	73.572
P_{g7}	115.2878	115.2876	114.9843	115.2922	125	115.29
P_{g8}	116.3994	116.3994	116.4238	116.4042	60.24	116.4
P_{g9}	100.4062	100.4063	100.6948	100.4054	103.28	100.405
P_{g10}	106.0267	106.0267	99.99979	106.0245	79.49	106.027
P_{g11}	150.2394	150.2395	148.977	150.2226	221.14	150.239
P_{g12}	292.7648	292.7647	294.0207	292.7769	347.05	292.766
P_{g13}	119.1154	199.1155	119.5754	119.1098	127.38	119.114
P_{g14}	30.834	30.8342	30.54786	30.8353	60.29	30.832
P_{g15}	115.8057	115.8056	116.4546	115.7987	116.7	115.805
P_{g16}	36.2545	36.2545	36.22787	36.2566	36.25	36.254
P_{g17}	66.859	66.859	66.85943	66.8741	30	66.859
P_{g18}	87.972	87.972	88.54701	87.9766	58.21	87.971
P_{g19}	100.8033	100.8033	10.9802	100.8082	85.52	100.803
P_{g20}	54.305	54.305	54.2725	54.305	30	54.305
Power loss (MW)	91.967	91.9669	92.1011	91.9662	**15.48**	**91.967**
Total Power O/P (MW)	2591.967	2591.9669	2592.1011	2591.9662	**2515.48**	**2591.967**
Power Demand (MW)	2500	2500	2500	2500	**2500**	**2500**
Power Mismatch	0	0	0	0	**0**	**0**
Total Generation Cost ($/h)	62456.6391	62456.6341	62456.7926	62456.63309	**62456.633**	**62456.633**

TABLE 4: BEST POWER OUTPUT FOR FORTY GENERATING UNITS SYSTEM

Unit No.	VSHDE[27]	SA[26]	**QP**	**GAMS**	VSHDE[27]	SA[26]	**QP**	**GAMS**
P_{g1}	80	80	80	80	79.63	79.999	80	80
P_{g2}	120	120	120	120	119.99	120	120	120
P_{g3}	190	190	190	190	189.98	190	190	190
P_{g4}	42	42	42	42	36.27	41.0057	40.87	40.874
P_{g5}	42	41.5093	42	42	42	38.1259	37.76	37.765
P_{g6}	140	140	140	140	140	140	140	140
P_{g7}	300	300	300	300	300	300	300	300
P_{g8}	300	300	300	300	299.98	300	300	300
P_{g9}	300	300	300	300	300	300	300	300
P_{g10}	300	275.349	276.81	276.81	131.97	130.0164	130	130
P_{g11}	295.21	318.414	317.61	317.612	94.03	96.594	94	94
P_{g12}	324.1	316.2014	304.15	304.165	94	95.1518	95.3	95.304
P_{g13}	424.75	436.6008	446.13	446.128	174.03	166.0629	195	195
P_{g14}	500	500	493.1	493.098	327.7	335.1617	335.06	335.063
P_{g15}	500	500	500	500	339.51	338.7512	333.55	333.549
P_{g16}	500	500	500	500	339.49	341.5566	333.55	333.549
P_{g17}	500	492.8414	500	500	350.34	345.8268	333.55	333.549
P_{g18}	500	500	500	500	500	500	500	500
P_{g19}	500	500	500	500	500	500	500	500
P_{g20}	550	500	550	550	550	550	550	550
P_{g21}	550	550	550	550	550	550	550	550
P_{g22}	550	550	550	550	550	550	550	550
P_{g23}	550	550	550	550	550	550	550	550
P_{g24}	550	550	550	550	550	550	550	550
P_{g25}	550	550	550	550	550	550	550	550
P_{g26}	550	550	550	550	550	550	550	550
P_{g27}	550	550	550	550	549.99	550	550	550
P_{g28}	10	12.7003	12.4	12. 396	10	10.3708	10.12	10.12
P_{g29}	10.94	10.996	12.4	12. 396	10	10.0818	10.12	10.116
P_{g30}	10	12.3847	12.4	12. 396	10	10.2946	10.12	10.116
P_{g31}	20	20.0004	20	20	20.01	20.0001	20	20
P_{g32}	20	20.0004	20	20	20.01	20.0001	20	20
P_{g33}	20	20.0002	20	20	20	20	20	20
P_{g34}	20	20.0001	20	20	20	20.0001	20	20
P_{g35}	18	18.0003	18	18	18.01	18.0001	18	18
P_{g36}	18	18.0003	18	18	18	18	18	18
P_{g37}	20	20.0007	20	20	20	20.0002	20	20
P_{g38}	25	25.0005	25	25	25.06	25	25	25
P_{g39}	25	25.0001	25	25	25	25.0001	25	25
P_{g40}	25	25.0001	25	25	25	25.0001	25	25
Total Power O/P (MW)	10500	10500	**10500**	**10500**	9000	9000	**9000**	**9000**
Power Demand (MW)	10500	10500	**10500**	**10500**	9000	9000	**9000**	**9000**
Power Mismatch	0	0	**0**	**0**	0	0	**0**	**0**
Total Generation Cost($/Hr)	143943.9	143930.409	**143926.32**	**143926.32**	121253.01	121245.164	**121247.1**	**121247.1**

TABLE 5: BEST POWER OUTPUT FOR 110 GENERATING UNITS SYSTEM

Unit no.	QP	GAMS	QP	GAMS	QP	GAMS	Unit no.	QP	GAMS	QP	GAMS	QP	GAMS
	Low load		Medium load		High load			Low load		Medium load		High load	
P_{g1}	2.4	2.4	2.4	2.4	12	12	P_{g56}	25.2	25.2	25.2	25.2	96	96
P_{g2}	2.4	2.4	2.4	2.4	12	12	P_{g57}	25.2	25.2	25.2	25.2	96	96
P_{g3}	2.4	2.4	2.4	2.4	12	12	P_{g58}	35	35	35	35	100	100
P_{g4}	2.4	2.4	2.4	2.4	12	12	P_{g59}	35	35	35	35	100	100
P_{g5}	2.4	2.4	2.4	2.4	12	12	P_{g60}	45	45	45	45	120	120
P_{g6}	4	4	4	4	20	20	P_{g61}	45	45	45	45	120	120
P_{g7}	4	4	4	4	20	20	P_{g62}	45	45	45	45	120	120
P_{g8}	4	4	4	4	20	20	P_{g63}	54.3	54.3	185	185	185	185
P_{g9}	4	4	4	4	20	20	P_{g64}	54.3	54.3	185	185	185	185
P_{g10}	15.2	15.2	63.731	63.731	76	76	P_{g65}	54.3	54.3	185	185	185	185
P_{g11}	15.2	15.2	61.4981	61.498	76	76	P_{g66}	54.3	54.3	185	185	185	185
P_{g12}	15.2	15.2	58.7179	58.718	76	76	P_{g67}	70	70	70	70	197	197
P_{g13}	15.2	15.2	56.0035	56.004	76	76	P_{g68}	70	70	70	70	197	197
P_{g14}	25	25	25	25	100	100	P_{g69}	70	70	70	70	197	197
P_{g15}	25	25	25	25	100	100	P_{g70}	150	150	360	360	360	360
P_{g16}	25	25	25	25	100	100	P_{g71}	400	400	400	400	400	400
P_{g17}	122.6104	122.61	155	155	155	155	P_{g72}	400	400	400	400	400	400
P_{g18}	117.7676	117.768	155	155	155	155	P_{g73}	60	60	103.8579	103.858	300	300
P_{g19}	113.0225	113.022	155	155	155	155	P_{g74}	50	50	190.4242	190.424	250	250
P_{g20}	108.573	108.573	155	155	155	155	P_{g75}	30	30	90	90	90	90
P_{g21}	68.9	68.9	68.9	68.9	197	197	P_{g76}	50	50	50	50	50	50
P_{g22}	68.9	68.9	68.9	68.9	197	197	P_{g77}	160	160	160	160	450	450
P_{g23}	68.9	68.9	68.9	68.9	197	197	P_{g78}	150	150	293.1882	293.188	600	600
P_{g24}	320.0052	320.005	350	350	350	350	P_{g79}	50	50	173.4258	173.426	200	200
P_{g25}	400	400	400	400	400	400	P_{g80}	20	20	96.8026	96.803	120	120
P_{g26}	400	400	400	400	400	400	P_{g81}	10	10	10	10	55	55
P_{g27}	140	140	500	500	500	500	P_{g82}	12	12	12	12	40	40
P_{g28}	140	140	500	500	500	500	P_{g83}	20	20	20	20	80	80
P_{g29}	50	50	200	200	200	200	P_{g84}	50	50	200	200	200	200
P_{g30}	25	25	100	100	100	100	P_{g85}	80	80	325	325	325	325
P_{g31}	10	10	10	10	50	50	P_{g86}	275.6619	275.662	440	440	440	440
P_{g32}	5	5	20	20	20	20	P_{g87}	10	10	11.5871	11.587	35	35
P_{g33}	20	20	80	80	80	80	P_{g88}	20	20	22.5251	22.525	55	55
P_{g34}	75	75	250	250	250	250	P_{g89}	20	20	80.6862	80.686	100	100
P_{g35}	200.2652	200.265	360	360	360	360	P_{g90}	40	40	87.6575	87.658	220	220
P_{g36}	223.4902	223.49	400	400	400	400	P_{g91}	30	30	56.7171	56.717	140	140
P_{g37}	10	10	40	40	40	40	P_{g92}	40	40	98.6821	98.682	100	100

P_{g38}	20	20	70	70	70	70	P_{g93}	440	440	440	440	440	440
P_{g39}	25	25	100	100	100	100	P_{g94}	383.8196	383.82	500	500	500	500
P_{g40}	20	20	120	120	120	120	P_{g95}	600	600	600	600	600	600
P_{g41}	40	40	156.1309	156.131	180	180	P_{g96}	300.1309	300.131	470.6926	470.693	700	700
P_{g42}	50	50	220	220	220	220	P_{g97}	3.6	3.6	3.6	3.6	15	15
P_{g43}	440	440	440	440	440	440	P_{g98}	3.6	3.6	3.6	3.6	15	15
P_{g44}	560	560	560	560	560	560	P_{g399}	4.4	4.4	4.4	4.4	22	22
P_{g45}	660	660	660	660	660	660	P_{g100}	4.4	4.4	4.4	4.4	22	22
P_{g46}	419.5534	419.553	615.5721	615.572	700	700	P_{g101}	10	10	10	10	60	60
P_{g47}	5.4	5.4	5.4	5.4	32	32	P_{g102}	10	10	10	10	80	80
P_{g48}	5.4	5.4	5.4	5.4	32	32	P_{g103}	20	20	20	20	100	100
P_{g49}	8.4	8.4	8.4	8.4	52	52	P_{g104}	20	20	20	20	120	120
P_{g50}	8.4	8.4	8.4	8.4	52	52	P_{g105}	40	40	40	40	150	150
P_{g51}	8.4	8.4	8.4	8.4	52	52	P_{g106}	40	40	40	40	171.1582	171.158
P_{g52}	12	12	12	12	60	60	P_{g107}	50	50	50	50	136.7398	136.74
P_{g53}	12	12	12	12	60	60	P_{g108}	30	30	30	30	150	150
P_{g54}	12	12	12	12	60	60	P_{g109}	40	40	40	40	310.1019	310.102
P_{g55}	12	12	12	12	60	60	P_{g110}	20	20	20	20	200	200
							Total Power Output (MW)	10000	10000	15000	15000	20000	20000
							Power Demand(MW)	10000	10000	15000	15000	20000	20000
							Power mismatch	0.00001	0	0.00001	0	0	0
							Total Generation Cost ($/h)	131935.5	131935.4998	197968.23	197968.226	313179.65	313179.6488

TABLE6: COMPARISION OF RESULTS FOR 110 UNITS SYSTEM (Cost ($/h))

Loading condition	Analytical [29]	SA[30]	SAB [30]	SAF [30]	RQEA [31]	QP	GAMS
Low							
Best	131941.8838	145550.4412	140385.7586	141107.8541	131941.8851	**131935.5**	**131935.4998**
average	----	146757.706	141213.4207	141215.1159	131942.0439		
worst	----	147476.4295	141900.2431	141398.0923	131942.4931		
Medium							
best	197988.1775	216100.5475	206921.9057	207380.5164	197988.1393	**197968.23**	**197968.226**
average	----	216365.7269	207764.7398	207813.3717	197988.1835		
worst	----	216823.5408	208197.0059	208012.6248	197988.2006		
High							
best	313211.5688	314647.0416	313279.8825	314532.8747	313211.5688	**313179.65**	**313179.6488**
average	----	315695.1453	314271.7484	314635.3244	313211.5983		
worst	----	317385.2167	314723.8825	314783.5061	313211.8189		

6. Conclusion

In this paper, Quadratic Programming (QP) and General Algebraic Modeling System (GAMS) for optimization have been used for solving four practical power dispatch problems. Case I consisting *Crete Island system* 18 generating units with quadratic cost characteristics without transmission loss, which is investigated by change in percentage of maximum demand and comparison is made with λ-iteration, Binary GA, RGA and ABC.

Based on the simulated results, performance comparison among four above listed different methods, we can say that QP and GAMS provides superior result than previously reported methods.

In Case II the system consists of twenty generating units having quadratic cost function with loss coefficient and obtained result is compared with λ-iteration, Hopfield Model, RGA and ABC algorithms.

In Case III a 40 generating units data of practical ELD system of Taiwan Power Company (TPC) is employed as example uses quadratic (convex) unit cost functions which is investigated on two different load demand and comparison is made with VSDE and SA reported in literature. And finally in Case IV a large scale system consists of 110 unit generating units is employed uses quadratic (convex) unit cost functions without losses to investigate the robustness of algorithm. This investigated on three different high, medium and low power demands. And compared to those obtained with, RGA and SGA and Hybrid GA reported in literature. The comparison shows that GAMS performs better then above mentioned methods. The GAMS algorithm has superior features, including quality of solution and good computational efficiency.
Therefore, this results shows that GAMS is a promising technique for solving complicated problems in power system.

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