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A Real Coded Genetic Algorithm to Entropy Bimatrix Game: Fuzzy Programming Technique

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Abstract. Here, we propose a mathematical model to analyze bimatrix game under entropy environment. In this new approach, the entropy function for each player are considered as objectives to the bimatrix game. This formulated model is named as Entropy Bimatrix Game Model which is a multi-objective non linear problem. To solve this kind of model, we have introduced a new solution technique, which determines the feasibility of fuzzy multiobjective non linear programming via a revised Genetic Algorithm (GA). By a real coded Genetic Algorithm, we obtained the bounds of objectives of said model and then applied fuzzy programming to determine the Nash equilibrium solution. To illustrate the methodology, numerical examples are included.

Keywords: Bimatrix Game, Nash Equilibrium, Entropy, Fuzzy Programming, GA.

1. Introduction

Every probability distribution has some "uncertainty" associated with it. The concept of "entropy" is introduced to provide a quantitative measure of uncertainty. Entropy models are emerging as valuable tools in the study of various social and engineering problems. The maximum entropy principle initiated by Jaynes'[8] is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available in the system. The principle has now been broadened and extended and has found wide applications in different fields of science and technology.

Two-person zero-sum game models are accurate when stakes are small monetary amounts. But in reality sense, when the stakes are more complicated, as often in economic situations, it is not generally true that the interests of the two players are exactly opposed. Such type of game models are non-cooperative game model. In other words, such situations give rise to two-person non-zero sum game, called bimatrix games. A bimatrix game can be considered as a natural extension of the matrix game, to cover situations in which the outcome of a decision process does not necessarily dictate the verdict that what one player gains the other one has to lose.

In bimatrix game, we see that family of probability distributions of strategies of every player are consistent with given information, we choose the distribution whose uncertainty or entropy is maximum. Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the players are involved in maximizing their entropies. Consequently, in the mathematical models of bimatrix game are incorporated an entropy function as one of their objectives. These models are known as entropy bimatrix game model.

In conventional mathematical programming, the coefficients or parameters of the bimatrix game models are assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may have some uncertainty in nature. Thus the decision-making methods under uncertainty are needed. The fuzzy programming has been proposed from this viewpoint. In fuzzy programming problems, the coefficients, constraints and the goals are viewed as fuzzy number or fuzzy set. In decision-making process, first Bellman and Zadeh [20] introduced fuzzy set theory. Tanaka applied the concepts of fuzzy sets to decision making problems by considering as fuzzy goals[18] and Zimmermann[21] showed that the classical algorithms could be used to solve multi-objective fuzzy linear programming problems.

In this paper, some references are presented including their work. Borm, Vermeulen and Voorneveld[2] analyzed the structure of the set of equilibria for the two-person multicriteria game. It turns out that the classical result for the set of equilibria for bimatrix games is valid for multicriteria games if one of players has two pure strategies. In another paper[19] they generalised some axioms of the Nash equilibria and it was shown that there exists no consistent refinement of Nash equilibria concept that satisfy individual rationality and non emptiness on a reasonably large class of games (Borm, Vermeulen and Voorneveld 2003). Nishizaki and Sakawa ([11],[12],[13]) proposed the resolution approach which can be regarded as a paradigm for bimatrix multi-objective non-cooperative game. Roy[17] presented the study of two different solution procedures for the two-person bimatrix game. The first solution procedure is applied to the game on getting the probability to achieve some specified goals along the player's strategy. The second specified goals along with the player's strategy by defining the fuzzy membership function to the pay-off matrix of the bimatrix game. Das and Roy ([5],[15]) have presented some two-persons zero sum game under entropy environment.

Several methodologies have been proposed to solve bimatrix game. Most of these methods are based on the concept of Pareto-optimal security strategies for linear models. However, no studies have been made on bimatrix entropy game. Genetic Algorithm(GA) and Fuzzy programming technique play an important role for determining the corresponding solution of the proposed model.

2. Mathematical Model of a Bimatrix Game

A bimatrix game can be considered as a natural extension of the matrix game. A two-person non zero-sum game can be expressed by a bimatrix game, comprised of two $m \times n$ dimensional matrices, namely A and B, where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

If player PI adopts the strategy "row" and player PII adopts the strategy "column" then denotes the expected payoff for player PI and denotes the expected payoff for player PII .

Definition 1:(Mixed Strategies)

The mixed strategies of the bimatrix game for player PI and PII are defined as follows:

$$Y = \{y \in R^m; \textstyle\sum_{i=1}^{m} y_i = 1; y_i \geq 0, i = 1,2,\ldots,m\} \quad (1)$$

$$Z = \{z \in R^n; \textstyle\sum_{j=1}^{n} z_j = 1; z_j \geq 0, j = 1,2,\ldots,n\} \quad (2)$$

Definition 2:(Nash equilibrium Solution)

For bimatrix game of two players, the Nash equlibrium solution (y^*, z^*) is found, if

$$y^{*t}Az^* \geq y^tAz^* \quad (3)$$

$$y^{*t}Bz^* \geq y^{*t}Bz \quad (4)$$

where $y \in Y$ and $z \in Z$; t denotes the transepose of a matrix.

Definition 3:(Expected Payoffs of Players)

If the mixed strategies are proposed by player PI and PII, then the expected payoff of player PI is y^tAz^*; the expected payoff of player PII is $y^{*t}Bz$. Therefore, the two person bimatrix game with mixed strategies can be formulated as follows:

$$\max_{y \in Y} y^tAz^* \quad (5)$$

and

$$\max_{z\in Z} y^{*t}Bz \tag{6}$$

Equations (5) and (6) can be regarded as the maximizing expected payoff of players PI and PII, respectively. The optimal strategy (y^*, z^*) can be achieved by simultaneously resolving equations (5) and (6).

2.1. Entropy Bimatrix Game Model

Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. The mathematical form of entropies are as follows:

$$H_1 = -\sum_{i=1}^{m} y_i ln(y_i) \tag{7}$$

$$H_2 = -\sum_{j=1}^{n} z_j ln(z_j) \tag{8}$$

Without any loss of generality, we combine the equations (5),(6),(7) and (8) and formulated a new mathematical model namely Entropy Bimatrix Game Model which is a multi-objective non-linear programming model. This model is defined as follows:

$$\begin{aligned} &\max_{y} y^t A z^* \\ &\max_{y} H_1 \\ &s.t.\, H_1 = -\sum_{i=1}^{m} y_i ln(y_i) \\ &\quad y \in Y \end{aligned} \tag{9}$$

and

$$\begin{aligned} &\max_{z} y^{*t} B z \\ &\max_{z} H_2 \\ &s.t.\, H_2 = -\sum_{j=1}^{n} z_j ln(z_j) \\ &\quad z \in Z \end{aligned} \tag{10}$$

The optimal strategy (y^*, z^*) can be achieved by simultaneously resolving the equations (9) and (10). The equlibrium solution of (3) and (4) can be obtained by solving the following equations:

$$\max_{y} v_1$$

$$\max_{z} v_2$$

$$\max_{y} H_1$$

$$\max_{z} H_2 (11)$$

$$s.t.\, v_1 = y^t A z^*$$

$$v_2 = y^{*t} B z$$

$$H_1 = -\sum_{i=1}^{m} y_i ln(y_i)$$

$$H_2 = -\sum_{j=1}^{n} z_j ln(z_j)$$

$$y \in Y$$

$$z \in Z$$

3. Solution Procedure

3.1. Basic Concepts of Fuzzy Set and Membership Function

Fuzzy sets first introduced by Zadeh[20] in 1965 as a mathematical way to representing impreciseness or vagueness in everyday life.

Fuzzy Set: A fuzzy set A in a discourse X is defined as the following set of pairs $A = (x, \mu_A): x \in X$, where $\mu_A: X \to [0,1]$ is a mapping, called membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set A. The larger $\mu_A(x)$ is the stronger grade of membership form in A.

Fuzzy Number: A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number A is a fuzzy set of real line R whose membership function $\mu_A(x)$ has following characteristic with $a < b$.

$$\mu_A(x) = \begin{cases} 0 & if \quad x \le a \\ \frac{x-a}{b-a} & if \ a \le x \le b \\ 1 & if \quad x \ge b \end{cases}$$

Fuzzy programming approach to conversion single objective non-linear problem from multi-objective non-linear problem(MONLP):

A MONLP or a Vector Maximization Problem(VMP) may be taken in the following form:

$$\max: f(x) = [f_1(x), \dots, f_k(x)]^T \quad (12)$$

$$s.t. \, x \in X = \{x \in R; g_j \le or = \ge b_j for j = 1, \dots, m; x \ge 0\} \quad (13)$$

Zimmermann showed that fuzzy programming technique could be used nicely to solve the multi-objective programming problem (Zimmermann 1978). To convert VMP [(12), (13)] as a single objective, following steps are used:

Step1: Solve the VMP [(12), (13)] as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step2: From the results of **Step1**, determine the corresponding values for every objective at each solution derived. Consider $x_1, \dots, x_k$ are the ideal solutions of the objectives $f_1(x), \dots, f_k(x)$ respectively. Then $U_r = \max(f_r(x_1), \dots, f_r(x_k))$ and $L_r = \min(f_r(x_1), \dots, f_r(x_k))$.

where L_r and U_r be the lower and upper bounds of the r-th objective function $f_r(x)$ for $r = 1, \dots, k$

Step3 : Using aspiration levels of each objective of VMP [(12), (13)], Find x so as to satisfy

$$f_r(x) \ge U_r, r = 1, \dots, k$$
$$x \in X$$

Here objective functions (12) are considered as fuzzy constraints. This type of fuzzy constraints can be quantified by eliciting a corresponding membership function.

$$\mu_r(f_r(x)) = \begin{cases} 0 & if \quad f_r(x) \le L_r \\ d_r(x) & if \quad L_r \le f_r(x) \le U_r \\ 1 & if \quad f_r(x) \ge U_r \end{cases} \quad (14)$$

where $d_r(x)$ is a strictly monotonic increasing function with respect to $f_r(x)$.

Having elicited the membership function [as in (14)] for $r = 1, \dots, k$), a general aggregation function $\mu_D = \mu_D(\mu_1(f_1(x)), \dots, \mu_k(f_k(x)))$ is introduced. So a fuzzy multi-objective decision making (MODM) can be defined as

$$\max \mu_D$$
$$\text{s.t. } x \in X$$

In previous section, we have seen that, (11) is a multi-objective non-linear programming (MONLP) problem. To get a satisfactory solution of the above model, we have introduced the fuzzy programming which is defined in the following subsection.

3.2 Fuzzy Programming:

In fuzzy programming, first we construct the membership function for each objective function of (11). Let $\mu_l(v_1), \mu_2(v_2)$ $\mu_3(H_1)$ and $\mu_4(H_2)$ be the membership functions for objectives respectively and they are defined as follows:

$$\mu_1(v_1) = \begin{cases} 0 & if \quad v_1 \le v_1^- \\ \frac{v_1 - v_1^-}{v_1^+ - v_1^-} & if \ v_1^- \le v_1 \le v_1^+, \\ 1 & if \quad v_1 \ge v_1^+ \end{cases} \quad (15)$$

$$\mu_2(v_2) = \begin{cases} 0 & if \quad v_2 \le v_2^- \\ \frac{v_2 - v_2^-}{v_2^+ - v_2^-} & if \ v_2^- \le v_2 \le v_2^+, \\ 1 & if \quad v_2 \ge v_2^+ \end{cases} \tag{16}$$

$$\mu_3(H_1) = \begin{cases} 0 & if \quad H_1 \le H_1^- \\ \frac{H_1 - H_1^-}{H_1^+ - H_1^-} & if \ H_1^- \le H_1 \le H_1^+ \\ 1 & if \quad H_1 \ge H_1^+ \end{cases} \tag{17}$$

and

$$\mu_3(H_2) = \begin{cases} 0 & if \quad H_2 \le H_2^- \\ \frac{H_2 - H_2^-}{H_2^+ - H_2^-} & if \ H_2^- \le H_2 \le H_2^+ \\ 1 & if \quad H_2 \ge H_2^+ \end{cases} \tag{18}$$

where v_1^+, v_1^- respectively, represent maximum and minimum values of v_1; v_2^+, v_2^- respectively, represent maximum and minimum values of v_2, and H_1^+, H_1^- respectively, represent maximum and minimum values of H_1,. H_2^+, H_2^- respectively, represent maximum and minimum values of H_2.

To conversion in a single objective non-linear model from multi-objective non-linear model, we have introduced the concept of fuzzy programming technique with the help of (15) to (18) and the (11) then we have formulated the following equations (19) as follows :

$$\begin{gathered} \max: \lambda \\ subject \ to \ \lambda \le \frac{v_1 - v_1^-}{v_1^+ - v_1^-}, \\ \lambda \le \frac{v_2 - v_2^-}{v_2^+ - v_2^-}, \\ \lambda \le \frac{H_1 - H_1^-}{H_1^+ - H_1^-} \\ \lambda \le \frac{H_2 - H_2^-}{H_2^+ - H_2^-} \\ v_1 = y^t A z^* \\ v_2 = y^{*t} B z \\ H_1 = -\sum_{i=1}^{m} y_i ln(y_i) \\ H_2 = -\sum_{j=1}^{n} z_j ln(z_j) \\ s.t. \sum_{i=1}^{m} y_i = 1; y_i \ge 0, i = 1,2, \dots, m \\ \sum_{j=1}^{n} z_j = 1; z_j \ge 0, j = 1,2, \dots, n \end{gathered} \tag{19}$$

To solve the equations (19), we can apply the GA which is defined in the next subsection. Now, we developed an algorithm for determining the v_l^+, v_l^-, and $H_l^+, H_l^-, l = 1,2$. The stepwise procedure of GA are shown as follows:

Step 1 : Initialize the parameters of GA of the proposed Entropy Bimatrix Game model.
Step 2: $t = 0$ (t represents the number of current generation.)
Step 3: Initialize $P(t)$ ($P(t)$ represents the population at the t-th generation.)
Step 4: Evaluate $P(t)$
Step 5: Find optimal result from $P(t)$.
Step 6: $t = t + 1$.
Step 7: If ($t >$ maximum generation number) go to Step 13.
Step 8: Alter $P(t)$ by mutation.
Step 9: Evaluate $P(t)$.
Step 10: Find optimal result from $P(t)$.
Step 11: Compared optimal results of $P(t)$ and $P(t-1)$ and store better one.
Step 12: Go to Step 6.
Step 13: Print optimal result.

Step 14: Stop.

To implement the above GA for the proposed model, the following basic components are considered:(i) parameters of GA, (ii) chromosome representation, (iii) initialization, (iv) evaluation function, (v) selection process, (vi) genetic operaters(crossover and mutation).

• **Parameters of GA :** GA depends on different parameters like population size(POPSIZE), probability of crossover(PCROS), probability of mutation(PMUTE) and maximum number of generation(MAXGEN). In our present study, we have taken the value of these parameters as follows:

POPSIZE= 25 PCROS= 0 PMUTE=0.6 MAXGEN= 80

• **Chromosome representation**

The chromosome is defined as $(y_1^a, y_2^a, y_3^a, \dots, y_m^a)$ where $y_i^a \in Y, i = 1,2,3,\dots,m$.

$(z_1^a, z_2^a, z_3^a, \dots, z_n^a)$ where $z_i^a \in Z, i = 1,2,3,\dots,m$.

• **Initialization**

In this study; $y_1^a, y_2^a, \dots, y_{m-1}^a$ are randomly given values such that chromosome must satisfy that $y_1^a + y_2^a + y_3^a + \cdots + y_m^a = 1$. This process is randomly generating each element in $(y_1^a, y_2^a, y_3^a, \dots, y_m^a)$ and $y_1^a + y_2^a + y_3^a + \cdots + y_m^a = 1$;

Again $z_1^a, z_2^a, \dots, z_{n-1}^a$ are randomly given values such that chromosome must satisfy that $z_1^a + z_2^a + z_3^a + \cdots + z_n^a = 1$. This process is randomly generating each element in $(z_1^a, z_2^a, z_3^a, \dots, z_n^a)$ and $z_1^a + z_2^a + z_3^a + \cdots + z_n^a = 1$; Moreover the number of chromosome is limited to 25 when each new run begins.

• **Evaluation function**

Once $(y_1^a, y_2^a, y_3^a, y_4^a)$ is determined, the corresponding v_1^a, v_2^a can be computed by (5), (6) and H_1^a, H_2^a can be computed by (7) and (8) respectively.

• **Optimum 1**

For 25 chromosomes, we get 25 set of values of v_1^a, v_2^a and H_1^a, H_2^a. Among these values of v_1^a we stored maximum and minimum values in v_1^{a+} and v_1^{a-}, respectively. Similarly, among these values of v_2^a we stored maximum and minimum values in v_2^{a+} and v_2^{a-}, respectively. In each iteration, these maximum and minimum values are globally stored in $VMAX1, VMIN1, VMAX2, VMIN2$, respectively. Similarly, among 25 values of H_1^a we stored maximum value in H_1^{a+} and minimum value in H_1^{a-} and they are also globally stored in another locations $HMAX1$ and $HMIN1$ respectively, in each iteration. And among 25 values of H_2^a, we stored maximum value in H_2^{a+} and minimum value in H_2^{a-} and they are also globally stored in another locations $HMAX2$ and $HMIN2$ respectively, in each iteration.

• **Selection**

Selection procedure is omitted because here objectives are more than one so we can not choose the weaker chromosome that serve worst value for all objectives.

• **Crossover**

Since it is not easy to design a crossover between chromosomes for satisfying that $y_1^a + y_2^a + y_3^a + \cdots + y_m^a = 1$, and $z_1^a + z_2^a + z_3^a + \cdots + z_n^a = 1$, therefore no crossover is applied in this study.

• **Mutation**

It is applied to single chromosome. It is designed as an order of elements in $(y_1^a, y_2^a, y_3^a, \dots, y_m^a)$ by randomly determined cut-point. Consider an example: if the original chromosome is $(y_1^a, y_2^a, y_3^a, \dots, y_m^a)$ and cut-point is randomly determined between the string: y_1^a and $y_2^a, y_3^a, \dots, y_m^a$, then moreover newly mutated chromosome $(y_1', y_2', y_3', \dots, y_m')$ is $(y_2^a, y_3^a, \dots, y_m^a, y_1^a)$.

Similarly, in $(z_1^a, z_2^a, z_3^a, \ldots, z_n^a)$ by randomly determined cut-point. Consider an example: if the original chromosome is $(z_1^a, z_2^a, z_3^a, \ldots, z_n^a)$ and cut-point is randomly determined between the string: z_1^a and $z_2^a, z_3^a, \ldots, z_n^a$, then moreover newly mutated chromosome $(z_1', z_2', z_3', \ldots, z_n')$ is $(z_3^a, z_2^a, \ldots, z_n^a, z_1^a)$.

In each iteration the (POPSIZE * PMUTE) number of chromosome are chosen for mutation.

- **Iteration**

The number of iteration is set to 80 runs, each of which begins with the different random seed.

- **Optimum 2**

After completing all the iterations, we determined v_1^+ as the maximum among all $VMAX1$ and v_1^- as the minimum among all $VMIN1$. Similarly, we determine v_2^+ as the maximum among all $VMAX2$ and v_2^- as the minimum among all $VMIN2$. Also, H_1^+ is the maximum among all $HMAX1$ and H_1^- is the minimum among all $HMIN1$, are determined and H_2^+ is the maximum among all $HMAX2$ and H_2^- is the minimum among all $HMIN2$, are determined.

4. Numerical Solution

Example 1: Consider the following bimatrix game

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 6 & 2 & 5 \\ 4 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 7 & 5 & 2 \\ 5 & 8 & 4 \\ 4 & 6 & 2 \end{bmatrix} \tag{20}$$

The following results are summarized in Table-1 which computed by Genetic Algorithm.

Table - 1

	maximum value	*minimum value*
v_1	$v_1^+ = 3.849926$	$v_1^- = 3.014539$
v_2	$v_2^+ = 4.665042$	$v_2^- = 2.161460$
H_1	$H_1^+ = 1.097173$	$H_1^- = 0.108341$
H_2	$H_2^+ = 1.097173$	$H_2^- = 0.108341$

With the help of above values from Table-1 and (19), we formulated (21) as follows:

$$\begin{gathered} \max: \lambda \\ \text{subject to} \\ \lambda \le \frac{v_1 - 3.014539}{3.849926 - 3.014539} \\ \lambda \le \frac{v_2 - 2.161460}{4.665042 - 2.161460} \\ \lambda \le \frac{H_1 - 0.108341}{1.097173 - 0.108341} \\ \lambda \le \frac{H_2 - 0.108341}{1.097173 - 0.108341} \\ v_1 = y^t A z^* \\ v_2 = y^{*t} B z \\ H_1 = -\sum_{i=1}^{m} y_i ln(y_i) \\ H_2 = -\sum_{j=1}^{n} z_j ln(z_j) \\ \sum_{i=1}^{3} y_i = 1; y_i \ge 0, i = 1,2,3 \\ \sum_{j=1}^{3} z_j = 1; z_j \ge 0, j = 1,2,3 \end{gathered} \tag{21}$$

The aspiration level λ^* with the objectives are determined from (21) by the help of Lingo package. The solutions are represented in the following Table-2.

Table - 2

aspiration level	$\lambda^* = 0.9881181$
expected payoffs	$v_1^* = 3.84$ $v_2^* = 4.66$
entropies	$H_1^* = 1.085424$ $H_2^* = 1.094622$
strategies	$y^* = (.2593744, .3590094, .3816162)$ z*=(0.3025743, 0.3234811, 0.3739446)

Thus when player PI is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PI plays his/her strategy $y^* = (.2593744, .3590094, .3816162)$ then he/she gets $v_1^* = 3.84$. Similarly, when player PII is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PII plays his/her strategy z*=(0.3025743, 0.3234811, 0.3739446) then he/she gets $v_2^* = 4.66$.

Example 2:

Consider the following bimatrix game

$$A = \begin{bmatrix} 3 & 9 & 7 \\ 5 & 4 & 6 \\ 3 & 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & 8 & 3 \\ 6 & 5 & 9 \\ 2 & 3 & 4 \end{bmatrix} \quad (22)$$

The following results are summarized in Table-3 which computed by Genetic Algorithm.

Table - 3

	maximum value	*minimum value*
v_1	$v_1^+ = 5.243864$	$v_1^- = 2.162876$
v_2	$v_2^+ = 6.011992$	$v_2^- = 2.731790$
H_1	$H_1^+ = 1.097173$	$H_1^- = 0.108341$
H_2	$H_2^+ = 1.097173$	$H_2^- = 0.108341$

With the help of above values from Table-3 and (19), we formulated (23) as follows:

max: λ

subject to

$$\begin{gathered} \lambda \leq \frac{v_1 - 2.162876}{5.243864 - 2.162876} \\ \lambda \leq \frac{v_2 - 2.731790}{6.011992 - 2.731790} \\ \lambda \leq \frac{H_1 - 0.108341}{1.097173 - 0.108341} \\ \lambda \leq \frac{H_2 - 0.108341}{1.097173 - 0.108341} \\ v_1 = y^t A z^* \\ v_2 = y^{*t} B z \\ H_1 = -\sum_{i=1}^{m} y_i ln(y_i) \\ H_2 = -\sum_{j=1}^{n} z_j ln(z_j) \\ \sum_{i=1}^{3} y_i = 1; y_i \geq 0, i = 1,2,3 \\ \sum_{j=1}^{3} z_j = 1; z_j \geq 0, j = 1,2,3 \end{gathered} \quad (23)$$

The aspiration level λ^* with the objectives are determined from (23) by the help of Lingo package. The efficient solutions are represented in the following Table-4.

Table - 4

aspiration level	$\lambda^* = 0.9401646$
expected payoffs	$v_1^* = 5.143019$ $v_2^* = 5.81572$
entropies	$H_1^* = 1.038006$ $H_2^* = 1.038006$
strategies	y*=(0.3126077,0.4847442,0.2026481)
	z*=(0.2443381,0.2533206,0.5023413)

Thus in entropy environment, when player PI is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PI plays his/her strategy y*=(0.3126077,0.4847442,0.2026481) then he/she gets $v_1^* = 5.143019$. Similarly, when player PII is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PII plays his/her strategy z*=(0.2443381,0.2533206,0.5023413) then he/she gets $v_2^* = 5.81572$.

4.1 Comparison

If we solve the Examples 1 and 2 separately by considering the equations (5) and (6) simultaneously with the help of our proposed GA and fuzzy programming technique, then the results are represented in Table-5 and Table-6 respectively as follows:

Table - 5

aspiration Level	$\lambda^* = 0.9881181$
expected payoffs	$v_1^* = 3.84$ $v_2^* = 4.66$
strategies	$y^* = (.3620,.2674,.3706)$
	$z^* = (.3308,.3278,.3414)$

Thus, when player PI is interested to maximize to expected payoff then it is seen that if PI plays his/her strategy $y^* = (.3620,.2674,.3706)$ then he/she gets $v_1^* = 3.84$. Similarly, when player PII is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PII plays his/her strategy $z^* = (.3308,.3278,.3414)$ then he/she gets $v_2^* = 4.66$.

Table - 6

aspiration level	$\lambda^* = 0.9996976$
expected payoffs	$v_1^* = 5.243$ $v_2^* = 6.011$
strategies	$y^* = (.6388,.2927,.0685)$
	$z^* = (.4734,.2338,.2928)$

Thus, when player PI is interested to maximize to expected payoff then it is seen that if PI plays his/her strategy $y^* = (.6388,.2927,.0685)$ then he/she gets $v_1^* = 5.243$. Similarly, when player PII is interested to maximize the measure of uncertainty together with expected payoff then it is seen that if PII plays his/her strategy $z^* = (.4734,.2338,.2928)$ then he/she gets $v_2^* = 6.011$.

3. Conclusion

This paper presents the study of bimatrix game and analyze the game under entropy environment. To obtain the solution, we apply fuzzy based genetic algorithm to Entropy Bimatrix Game Model. We have shown that all these strategies, together with their expected payoffs, can be obtained as the solution of a particular non-linear problem. It is seen that if players are interested to maximize the measure of uncertainty together with expected payoff then their expected payoff may be decreased. From numerical point of view, we suggested the players that if they have paid more attention to the opponents' strategies deviation then their expected payoffs may be decreased. Finally we concluded that the model incorporating entropy is highly significant related to the real world practical problem on bimatrix game.

4. References

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