

# Synchronization of different 3D chaotic systems by generalized active control

Mohammad Ali Khan

Department of Mathematics, Garhbeta Ramsundar Vidyabhaban,

Garhbeta, Paschim Medinipur, West Bengal, India

(Received May 19, 2012, accepted September 13, 2012)

**Abstract.** This paper designs a scheme for controlling a chaotic system to a period system using active control technique. We discuss this taking chaotic Genesio system as example. We have also discussed the synchronization scheme between two different coupled chaotic systems (Genesio and Nuclear spin generator(NSG)system) as well as two identical coupled chaotic systems (Four-scroll attractor) via active control. Numerical Simulation results are presented to show the effectiveness of the proposed scheme.

**Keywords:** Chaotic system, Chaos control, Chaos synchronization, Active control, Genesio system, NSG system, Four-scroll attractor.

## 1. Introduction

Chaos synchronization is an important topic in the non-linear science. In 1990, Pecorra and Carroll [1] introduced the idea of chaos synchronization. Two or more coupled chaotic systems are called synchronized if their behaviors are closely related. Chaos synchronization has received much attention due to its applications in many area such secure communication, information processing, biological systems and chemical reactions. Usually two dynamical systems are called synchronized if the distance between their corresponding states converges to zero as time goes to infinity. This type of synchronization is called identical synchronization. A generalization, of this concept for unidirectionally coupled dynamical systems was proposed by Rulkov et.al.(1995) [2] where two coupled systems are called synchronized if a static functional relationship exists between the states of the systems. This kind of synchronization is called generalized synchronization(GS). In 2009 synchronization in unidirectionally coupled Rossler system was proposed by Khan and Mandal [3] and also synchronization in bidirectionally coupled systems reported by Tarai et.al. [4,5]. In 2011 Khan et.al. [6] have discussed various type of control for controlling chaos in a unified chaotic system and recently in 2012 generalized anti-synchronization of different chaotic systems was proposed by Khan et.al. [7]. In 1996, Kocarev and Parlitz [8] formulated a condition for the occurrence of GS for the master and slave system. Using technique from active control theory, ER-Weibai and Karl.E.Lonngren(1997) [9] demonstrate that a coupled Lorenz systems can synchronize. Further ER-Weibai and Karl.E.Lonngren [10] have investigated sequential synchronization of two Lorenz system using active control. In 2001, synchronization of Rossler and Chen chaotical dynamical systems using active control was studied by Agiza and Yassen [11]. In 2002 Ming-Chuang Ho, Yao-Chen Hung [12] generalized the technique of active control theory and applied them to synchronize two different systems. Sinha et.al. [13] proposed a general approach in the design of active controllers for non-linear systems exhibiting chaos in 2003. Synchronization of two chaotic four-dimensional systems using active control technique was proposed by Youming and Wenxian in 2007 [14]. In 2009 Sudheer and Sabir [15] was investigated hybrid synchronization of hyperchaotic Lu system via active control. Recently in 2012 Shahzad [16] was investigated chaos synchronization of an ellipsoidal satellite via active control. Synchronization scheme for many coupled chaotic systems are not explored till now. Synchronization strategy for coupled chaotic Genesio system is very important from the theoretical point of view and this strategy is reported by us in this

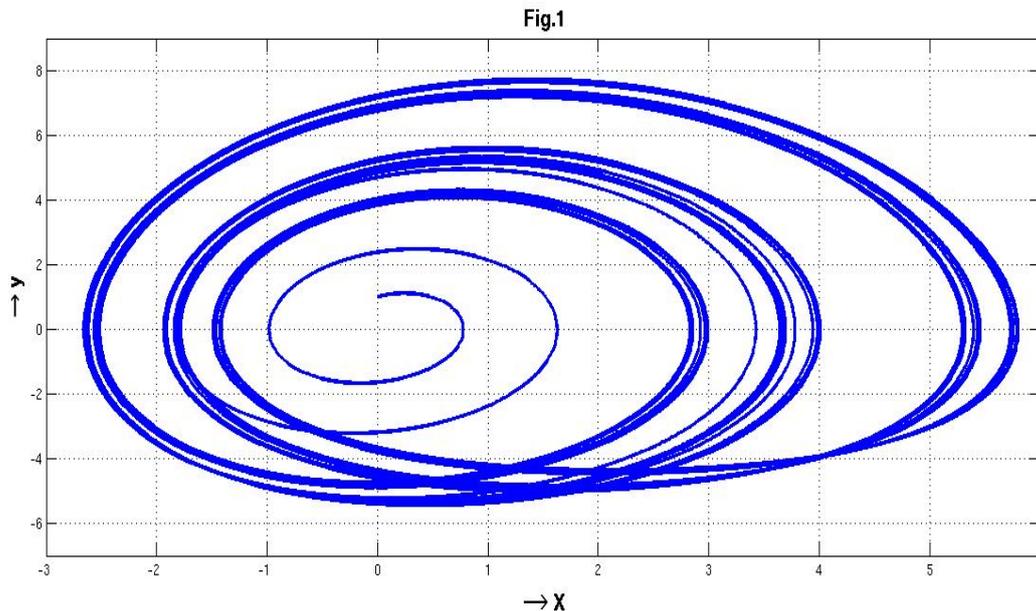
paper. The synchronization strategy between two identical coupled chaotic systems as well as two different chaotic systems are also very important from the application point of view. This motivates us to study the synchronization between Genesisio system and Nuclear spin generator systems and synchronization between Four-scroll attractor systems in this paper.

## 2. Genesisio system

The Genesisio system, proposed by Genesisio et.al [17], it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters. The dynamical equations are described by

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -cx - by - az + x^2 \end{aligned} \tag{1}$$

where  $x, y$  and  $z$  are state variables and  $a, b$  and  $c$  are positive constants, satisfying  $ab < c$ . The Genesisio system exhibits a chaotic attractor at the parameter values  $a=1.2, b=2.92$  and  $c=6$  shown in Fig.1.



### 2.1. Control of chaotic Genesisio system to a period system

In order to observe the control behavior we take three dimensional period systems as master system with three state variables denoted by the subscript 1 and the slave systems denoted by the subscript 2. The initial conditions for the master system are different from that of the slave system. The master and slave systems are defined as follows

Master system

$$\begin{aligned} \dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 \\ \dot{z}_1 &= -y_1 \end{aligned} \tag{2}$$

Slave system

$$\begin{aligned} \dot{x}_2 &= y_2 + u_a \\ \dot{y}_2 &= z_2 + u_b \\ \dot{z}_2 &= -cx_2 - by_2 - az_2 + x_2^2 + u_c \end{aligned} \tag{3}$$

where  $U = [u_a, u_b, u_c]^T$  is the controller function introduced in the slave system. Using active control techniques, we subtract equation (2) from equation (3) and get

$$\begin{aligned} \dot{x}_3 &= y_3 + u_a \\ \dot{y}_3 &= z_3 + u_b \\ \dot{z}_3 &= -cx_2 - by_2 - az_2 + x_2^2 + y_1 + u_c \end{aligned} \tag{4}$$

where  $x_3 = x_2 - x_1$ ;  $y_3 = y_2 - y_1$  and  $z_3 = z_2 - z_1$ . We define the active control functions  $u_a, u_b$  and  $u_c$  as

$$\begin{aligned} u_a &= V_a \\ u_b &= V_b \\ u_c &= cx_2 + by_2 + az_2 - x_2^2 - y_1 + V_c \end{aligned} \tag{5}$$

This leads to

$$\begin{aligned} \dot{x}_3 &= y_3 + V_a \\ \dot{y}_3 &= z_3 + V_b \\ \dot{z}_3 &= V_c \end{aligned} \tag{6}$$

Equation (6) describes the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input  $V_a, V_b$  and  $V_c$  as functions of  $x_3, y_3$  and  $z_3$ . Our aim is to stabilize the systems  $x_3, y_3$  and  $z_3$  at origin. This implies that two systems are synchronized with feedback control. There are many possible choices for the control  $V_a, V_b$  and  $V_c$ . We choose

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = A \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \tag{7}$$

where  $A$  is a  $3 \times 3$  constant matrix. For proper choice of elements of the matrix  $A$ , the feedback system must have all of the eigenvalues with negative real parts. In this case, the closed loop system will be stable. Let us choose a particular form of the matrix  $A$  that is given by

$$A = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \tag{8}$$

For this particular choice, the closed loop system has eigenvalues that are found to be -1,-1 and -1. This choice will lead to a stable systems and as we will observe in a numerical investigation, lead to the synchronization of Genesio system to be period system.

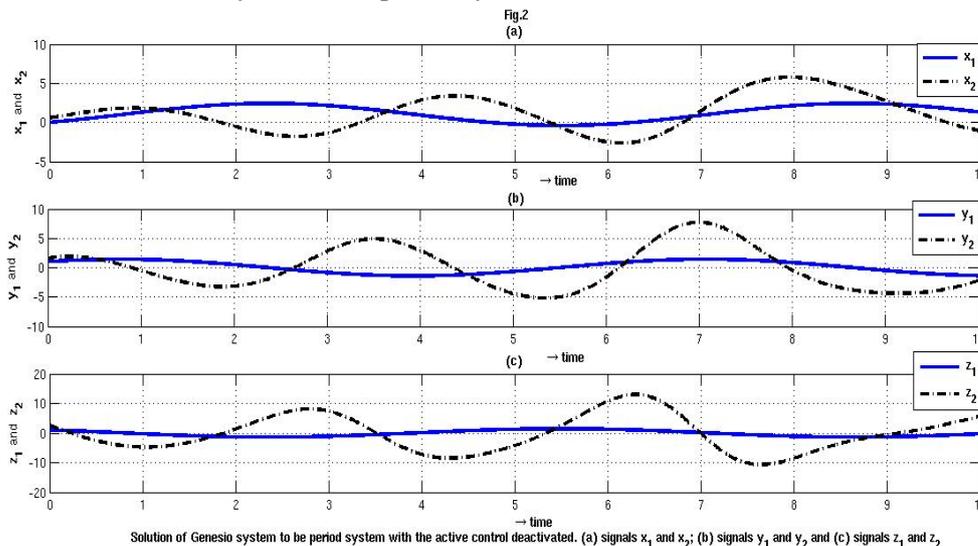


Fig.2. Solution of Genesio system to be period system with the active control deactivated. (a) signals  $x_1$  and  $x_2$ ; (b) signals  $y_1$  and  $y_2$  and (c) signals  $z_1$  and  $z_2$

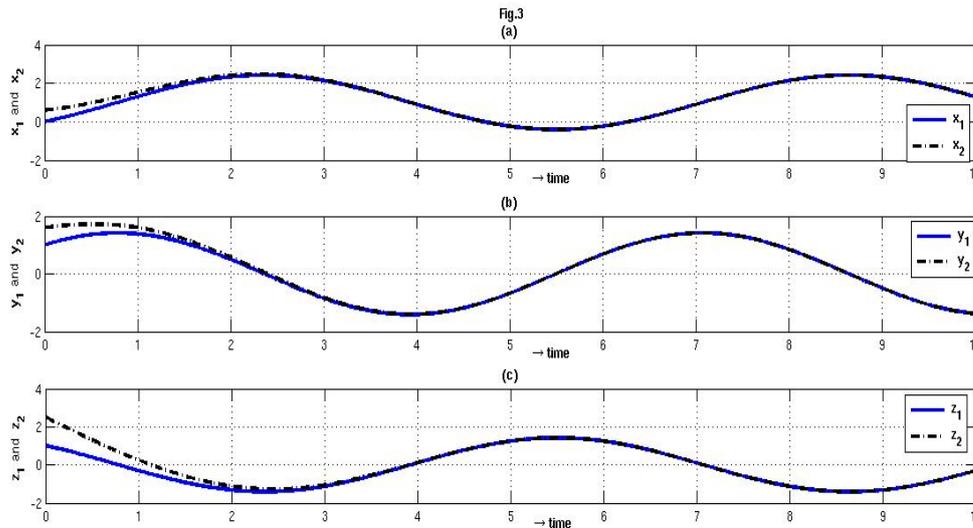


Fig.3. Solution of Genesisio system to be period system with the active control activated

### 2.2. Numerical Simulation

We select the parameters  $a=1.2$ ,  $b=2.92$  and  $c= 6$  to ensure the existence of the chaotic behavior(Fig.1). The initial values are taken as  $x_2(0) = 0.6$ ,  $y_2(0) = 1.6$  and  $z_2(0) = 2.5$ . Simulation results for uncoupled system are presented in Fig.2. and that of controlled system are shown in Fig.3. The synchronizing of Genesisio system to a period system can also be observed by monitoring the difference of the two signals  $x_3, y_3$  and  $z_3$ . For numerical simulation the initial values of difference of two signals are taken as  $x_3(0) = 4.0$ ,  $y_3(0) = -10.0$  and  $z_3(0) = 2.0$ . Fig.4. displays the difference signals with the active controller disconnected into the circuit. Fig.5. displays the same signals with the active controller connected into the circuit.

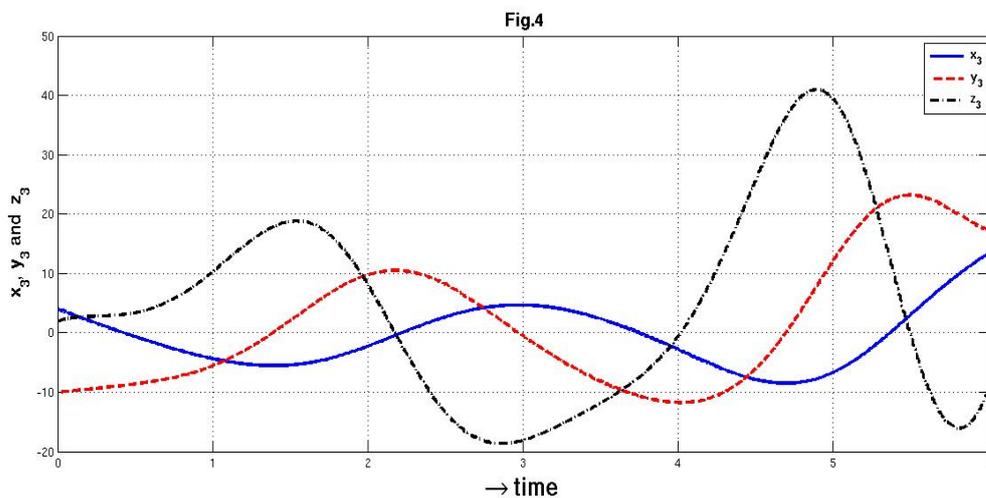


Fig.4. Display the difference signals when active controller deactivated

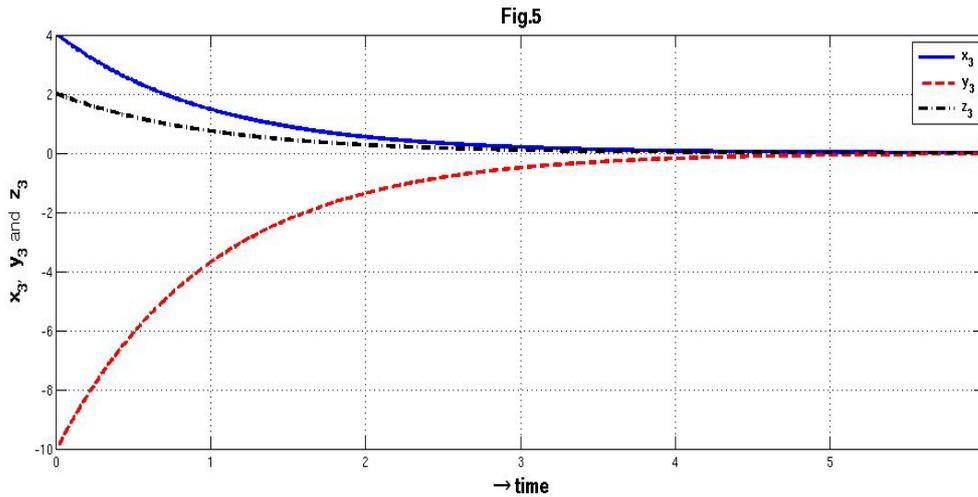


Fig.5. Display the difference signals when active controller activated.

### 3. Nuclear spin generator (NSG) system

The nuclear spin generator problem was studied by Sachdev and Sarthy [18] and Hegazi et.al. [19]. They showed that the system displays rich and typical bifurcation and chaotic phenomena for some values of the control parameters. The system consists a suitable simple of matter containing proper nuclei in a relatively strong magnetic field defining Z-direction, an exciting coil with axis in the X-direction, perpendicular to Z, a pick-up coil with axis in the X-direction, perpendicular to both X and Z and a high-gain amplifier feeding the voltage induced in the pick-up coil back to the exciting coil. We first reformulate its equation in the following form

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y(1 - \kappa z) \\ \dot{z} &= \beta(\alpha(1 - z)) - \kappa y^2 \end{aligned} \tag{9}$$

such that  $\alpha$ ,  $\beta$  and  $\kappa$  are parameters, where  $\beta\alpha \geq 0$  and  $\beta \geq 0$  are linear damping terms, the non-linearity parameters  $\beta\kappa$  is proportional to the amplifier gain in the voltage feedback. Physical consideration limits the parameters  $\alpha$  to the range  $0 < \alpha \leq 1$ .

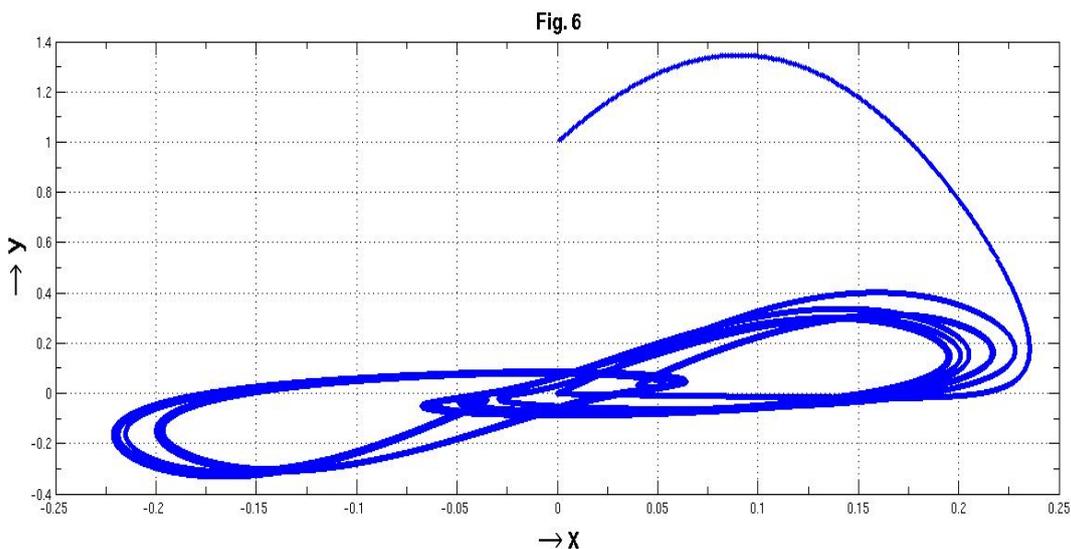


Fig.6. Chaotic attractor of NSG system at  $\beta = .75, \alpha = .15$  and  $\kappa = 2.0$

#### 3.1. Synchronization of two different chaotic systems

Now, we want to synchronize two different coupled chaotic systems. Take Genesis and Nuclear spin generating systems into consideration.

Master system(Genesio system)

$$\begin{aligned} \dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 \\ \dot{z}_1 &= -cx_1 - by_1 - az_1 + x_1^2 \end{aligned} \tag{10}$$

Slave system(NSG system)

$$\begin{aligned} \dot{x}_2 &= -\beta x_2 + y_2 + u_a \\ \dot{y}_2 &= -x_2 - \beta y_2(1 - \kappa z_2) + u_b \\ \dot{z}_2 &= \beta(\alpha(1 - z_2) - \kappa y_2^2) + u_c \end{aligned} \tag{11}$$

Subtract (10) from (11) we get

$$\begin{aligned} \dot{x}_3 &= -\beta x_2 + y_3 + u_a \\ \dot{y}_3 &= -x_2 - \beta y_2(1 - \kappa z_2) - z_1 + u_b \\ \dot{z}_3 &= \beta(\alpha(1 - z_2) - \kappa y_2^2) + cx_1 + by_1 + az_1 - x_1^2 + u_c \end{aligned} \tag{12}$$

We define the active control functions  $u_a, u_b$  and  $u_c$  as

$$\begin{aligned} u_a &= -\beta x_2 + V_a \\ u_b &= x_2 + \beta y_2(1 - \kappa z_2) + z_1 + V_b \\ u_c &= \beta(\alpha(z_2 - 1) + \kappa y_2^2) - cx_1 - by_1 - az_1 + x_1^2 + V_c \end{aligned} \tag{13}$$

This leads to

$$\begin{aligned} \dot{x}_3 &= y_3 + V_a \\ \dot{y}_3 &= V_b \\ \dot{z}_3 &= V_c \end{aligned} \tag{14}$$

Equation (14) describe the error dynamical system which is a linear system with a control input  $V_a, V_b$  and  $V_c$  as functions of  $x_3, y_3$  and  $z_3$ . Our goal is achieved when these feedbacks stabilize the system  $x_3, y_3$  and  $z_3$  at zero. There are many possible choices for the control  $V_a, V_b$  and  $V_c$ . We choose

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = A \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \tag{15}$$

Where  $A$  is a  $3 \times 3$  constant matrix. Let us choose a particular form of the matrix  $A$  that is given by

$$A = \begin{pmatrix} -\beta & 1 & 0 \\ -1 & -\beta & 0 \\ 0 & 0 & -\alpha\beta \end{pmatrix} \tag{16}$$

The equation (16) can be written as

$$\dot{X} = BX \tag{17}$$

where

$$B = \begin{pmatrix} -\beta & 1 & 0 \\ -1 & -\beta & 0 \\ 0 & 0 & -\alpha\beta \end{pmatrix} \tag{18}$$

Finally, we have to produce a matrix  $B$  such that three eigen values has negative real parts.

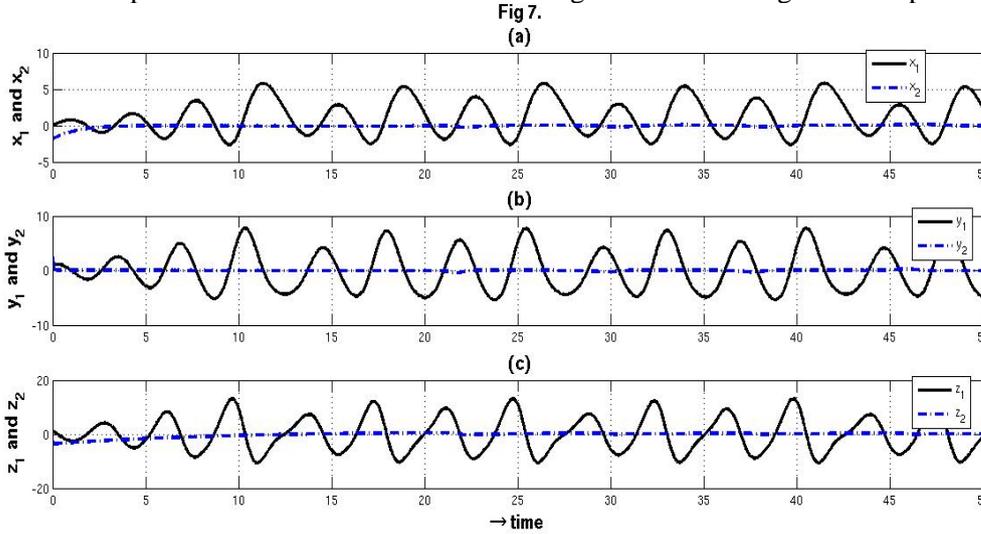


Fig.7. Solution of Genesio system and NSG systems with the active control deactivated.

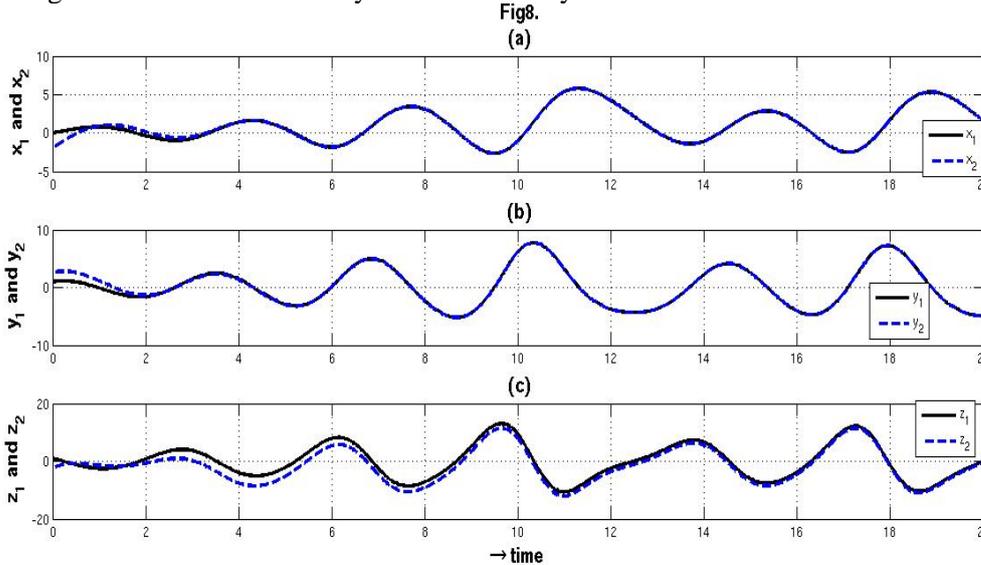


Fig.8. Solution of Genesio system and NSG systems with the active control activated.

### 3.2. Numerical Simulation

We select the parameters for NSG systems as  $\beta = 0.75, \alpha = 0.15$  and  $\kappa = 2$  to ensure the chaotic behavior shown in Fig.6. The initial values of Genesio system are taken as  $x_1(0) = 0.0, y_1(0) = 1.0, z_1(0) = 1.0$  and for Nuclear spin generator system  $x_2(0) = -2.0, y_2(0) = 2.6, z_2(0) = -2.5$ . Simulation results for uncontrolled system are presented in Fig.7. and that of controlled system are shown in Fig.8. Fig.9. displays the difference signals with the active controller into the circuit.

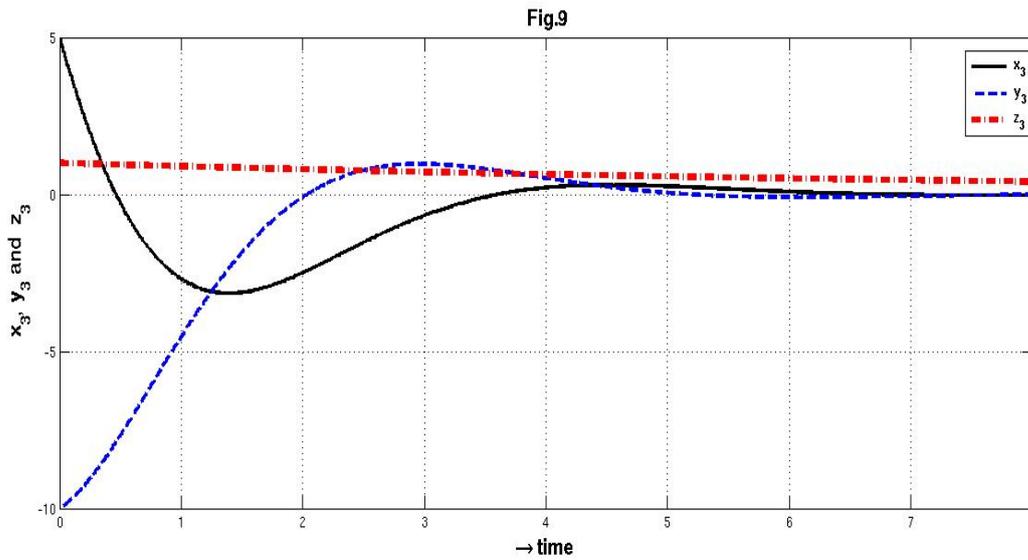


Fig.9. Display the difference signal of Genesio and NSG system when active control activated

#### 4. Four-scroll attractor

Consider the following Four-scroll attractor

$$\begin{aligned} \dot{x} &= ax - yz \\ \dot{y} &= -by + xz \\ \dot{z} &= -cz + xy \end{aligned} \tag{19}$$

where  $a, b$  and  $c$  are positive control parameters. This system has a chaotic attractor as shown in Fig.10. at the parameter values  $a = 0.4, b = 12$  and  $c = 5$ .

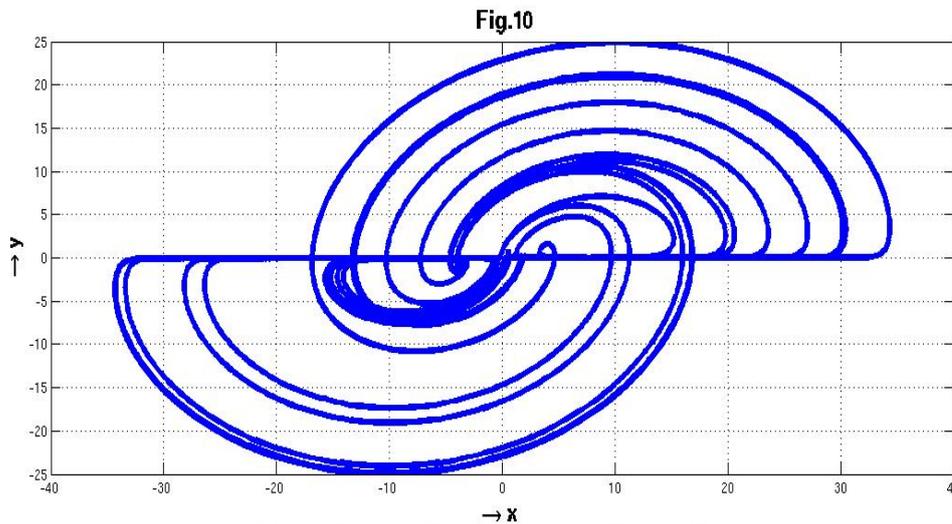


Fig.10. Four-scroll attractor at  $a=0.4, b=12$  and  $c=5$

##### 4.1. Synchronization of two identical chaotic systems

To observe the synchronization behavior for Four-scroll attractor, we have two Four-scroll attractor systems where the master system with three state variables denoted by the subscript 1 and slave system having identical equations denoted by the subscript 2.

The master and slave systems are defined as follows

$$\begin{aligned}\dot{x}_1 &= ax_1 - y_1 z_1 \\ \dot{y}_1 &= -by_1 + x_1 z_1 \\ \dot{z}_1 &= -cz_1 + x_1 y_1\end{aligned}\quad (20)$$

and

$$\begin{aligned}\dot{x}_2 &= ax_2 - y_2 z_2 + u_a \\ \dot{y}_2 &= -by_2 + x_2 z_2 + u_b \\ \dot{z}_2 &= -cz_2 + x_2 y_2 + u_c\end{aligned}\quad (21)$$

Subtracting (20) from (21) we get

$$\begin{aligned}\dot{x}_3 &= ax_3 + y_1 z_1 - y_2 z_2 + u_a \\ \dot{y}_3 &= -by_3 + x_2 z_2 - x_1 z_1 + u_b \\ \dot{z}_3 &= -cz_3 + x_2 y_2 - x_1 y_1 + u_c\end{aligned}\quad (22)$$

By suitable choice of  $u_a, u_b$  and  $u_c$  the system (22) becomes

$$\begin{aligned}\dot{x}_3 &= ax_3 + V_a \\ \dot{y}_3 &= -by_3 + V_b \\ \dot{z}_3 &= -cz_3 + V_c\end{aligned}$$

where

$$\begin{aligned}u_a &= -y_1 z_1 + y_2 z_2 + V_a \\ u_b &= x_1 z_1 - x_2 z_2 + V_b \\ u_c &= x_1 y_1 - x_2 y_2 + V_c\end{aligned}\quad (23)$$

There are many possible choices for controller  $V_a, V_b$  and  $V_c$ . We choose

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = A \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}\quad (24)$$

Where  $A$  is a  $3 \times 3$  constant matrix. Let us choose a particular form of the matrix  $A$  that is given by

$$A = \begin{pmatrix} -(a+1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\quad (25)$$

Therefore difference signal system becomes

$$\begin{aligned}\dot{x}_3 &= -x_3 \\ \dot{y}_3 &= -by_3 \\ \dot{z}_3 &= -cz_3\end{aligned}\quad (26)$$

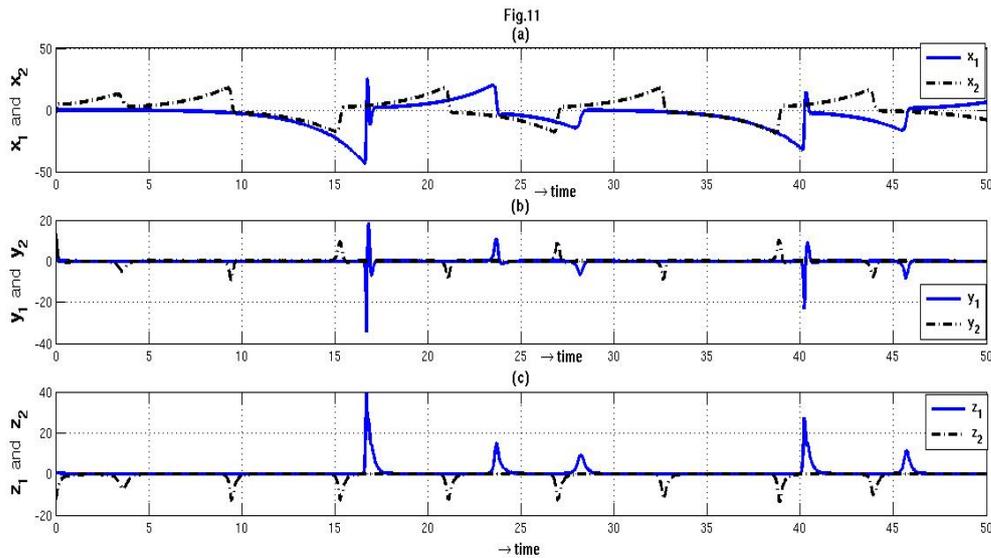


Fig.11. Display the solution of the coupled Four-scroll attractor system of equations with the active control deactivated.

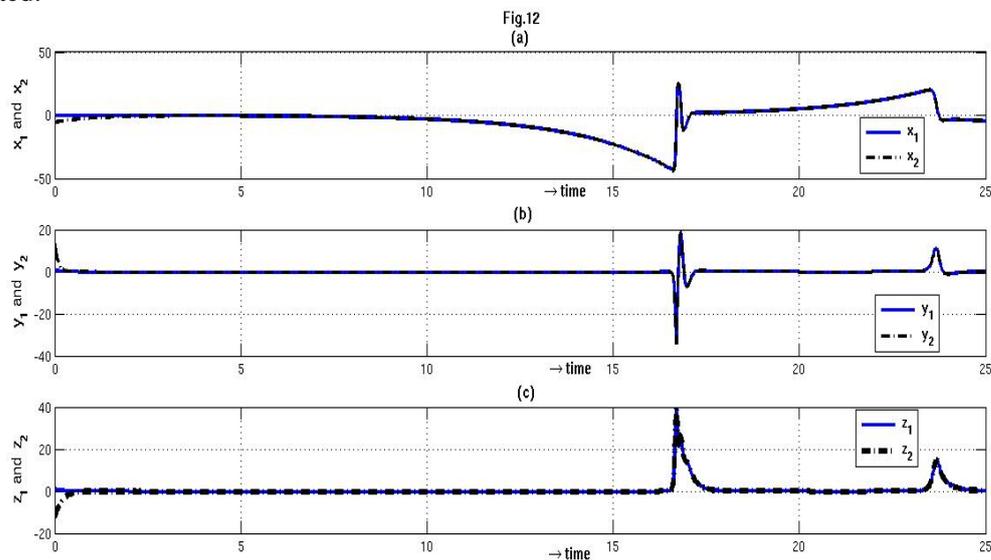


Fig.12. Display the solution of the coupled Four-scroll attractor system of equations with the active control activated.

### 4.2. Numerical Simulation

We select the parameters for Four-scroll attractor systems as  $a = 0.4, b = 12$  and  $c = 5$  to ensure the chaotic behavior shown in Fig.10. The initial values of coupled Four-scroll attractor system are taken as  $x_1(0) = 0.0, y_1(0) = 1.0, z_1(0) = 1.0$  and  $x_2(0) = -5.6, y_2(0) = 13.6, z_2(0) = -12.5$ . The results of the simulation of coupled Four-scroll attractor system with the active controller deactivated are shown in Fig.11. Fig.12. displays the same sequence of signals when active control activated and difference signals are shown in Fig.13. with the active controller connected into the circuit.

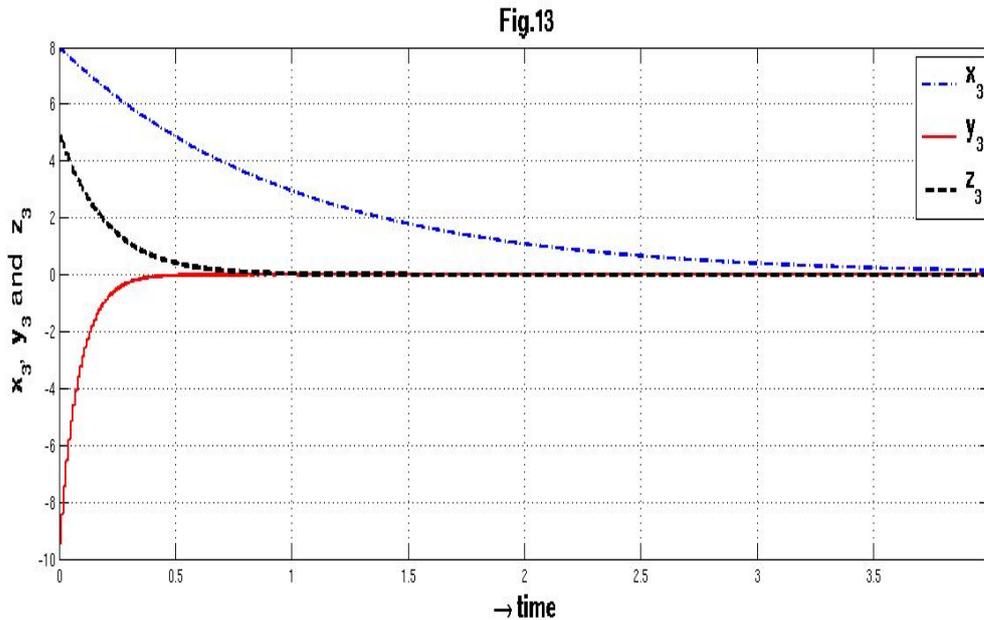


Fig.13. Display the difference signal of  $x_3$ ,  $y_3$  and  $z_3$  when active control activated.

## 5. Conclusion

We have discussed synchronization of Genesio system to a period system using active control technique. This strategy can be used to synchronize any chaotic system to a periodic system with desired period. This technique may be useful for controlling chaotic oscillations in electronic systems, biological cells etc. We have also discussed the synchronization between two different chaotic systems considering Genesio and NSG systems as well as two identical Four-scroll attractor systems via active control. We believe these synchronization scheme may be useful for sending secret message. We show that a chaotic system can be synchronized to a periodic system or any other system.

## Acknowledgement

I would like to express my sincere thanks to Dr. Swarup poria of the Dept. Of Applied Mathematics, Calcutta University for motivating me by valuable suggestions for preparation of this work.

## 6. References

- [1] Pecorra and Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (1990), pp.821.
- [2] Rulkov N.F, Suschik M.M and T. Simring L.S, *Generalized synchronization of chaos directionally coupled chaotic system*, Phys. Rev. E. 51 (1995), pp. 980.
- [3] M.A.Khan and A.K.Mandal, *Generalized chaos synchronization of coupled Rossler systems*, Bull. Cal. Math. Soc. 101 (2009), pp.197-204.
- [4] A.Tarai, S.Poria and P.Chatterjee, *Synchronization of bidirectionally coupled chaotic Chen's System with delay*, Chaos Solitons and Fractals, doi: 10.1016/j.chaos.2007.11.025.
- [5] A.Tarai, S.Poria and P.Chatterjee, *Synchronization of generalized linearly bidirectionally coupled unified chaotic system*, Chaos Solitons and Fractals. 40 (2009), pp.885-892.
- [6] M.A.Khan, A.K.Mondal and S.Poria, *Three control strategies for unified chaotic system*, Int. J. of Applied Mechanics and Engineering. 16 (2011), pp. 597-608.
- [7] M.A.Khan, S.N.Pal and S.Poria, *Generalized anti-synchronization of different chaotic systems*, Int. J. of Applied Mechanics and Engineering. 17 (2012), pp.83-89.

- [8] Kocarev L and Parlitz U, *Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems*, Phys. Rev. Lett. 76 (1996), pp.1816.
- [9] ER-Weibai and Karl E. Lonngren, *Synchronization of two Lorenz systems using active control*, Chaos, Solitons and Fractals. 8 (1997), pp.51-58.
- [10] ER-Weibai and Karl E. Lonngren, *Sequential synchronization of two Lorenz systems using active control*, Chaos, Solitons and Fractals. 11 (1997), pp.1041.
- [11] Agiza and Yassen, *Synchronization of Rossler and Chen chaotic dynamical systems using active control*, Physics Letters A. 278 (2001), pp.191
- [12] Ming-Chung Ho and YC Hung, *Synchronization of different systems using generalized active control*, Physics Letters A. 301 (2002), pp. 424-428.
- [13] Sinha SC, Henrichs JT and Ravindra BA, *A general approach in the design of active controllers for nonlinear systems exhibiting chaos*, Int. J. Bifurcat. Chaos 13 (2003) pp. 2343-51.
- [14] L.Youming, X. Wei and X. Wenxian, *Synchronization of two chaotic four-dimensional systems using active control*, Chaos, Solitons and Fractals. 32 (2007), pp.1823-1829.
- [15] K. Sebastian Sudheer and M. Sabir, *Hybrid synchronization of hyperchaotic Lu system*, Pramana-journal of physics. 73 (2009) pp.781-786.
- [16] Genesio R and Tesi A , *A harmonic balance methods for the analysis of chaotic dynamics In nonlinear systems*, Automatica, 28 (1992), pp.531-48.
- [17] Mohammad Shahzad, *Chaos synchronization of an ellipsoidal satellite via active control*, Progress in Applied Mathematics. 3 (2012), pp.16-23.
- [18] Sachdev and Sarthy, *Periodic and chaotic solutions for a non-linear system arising from a nuclear spin generator*, Chaos, solitons and Fractals. 4 (1994), pp.2015-41.
- [19] Hegazi, Agiza and Eledessoky, *Adaptive control and synchronization of a modified Chua's circuit system*, Chaos, solitons and Fractals 12. (2001), pp. 631-58.