Journal of Information and Computing Science
J. Info. Comput. Sci., Vol. 18, No. 2, pp. 141-160

DOI: 10.4208/JICS-2023-010
December 2023

School Bus Routing Considering Operation and Travel Costs

Kailei Li[1], Han Bai[1,*], Xiang Yan[1], Manxi Zhu[1] and Xiuguang Wang[1]

[1] *School of Transportation and Logistics Engineering, Shandong Jiaotong University, Jinan 250357, China*

Abstract. In order to solve the problems of high operating cost and poor service quality of school bus due to the scattered distribution of bus stops in rural areas, multi-objective SBRP (School Bus Routing Problem) models were developed for the mixed-load and non-mixed-load scenarios. In the non-mixed-load scenario, a model of the SBRP was developed to optimize the students' travel cost and school operating cost, while in the mixed-load scenario, another model of the SBRP was developed to consider the input cost and operation cost of the school bus. Several heuristic algorithms were compared, based on which the simulated annealing algorithm was selected to solve the models, and the horizontal comparison of the solution results based on genetic algorithm were determined. Tests were conducted on an international bench mark case and the constructed models were solved by introducing different search operators into the simulated annealing algorithm, then the proposed approach was applied to the optimal design of school bus routes in Wulian county, Rizhao, Shandong province. The results showed that in the non-mixed-load scenario, compared with the original school bus operation mode, the school bus input, mileage and travel cost were reduced by 28.6%, 37.8% and 35.6%, respectively, and students' travel cost was reduced by 4.3% considering the students' perception of school bus service. While in the mixed-load scenario, the proposed approach reduced the school bus input, mileage and travel cost by 37.5%, 42.0% and 35.8%, respectively; due to the complexity of the mixed-load scenario, it is difficult to take the travel cost into account, thus the students' travel cost was increased by 0.5%. The proposed SBRP models were verified to be effective and the simulated annealing approach can optimize service quality and reduce operation cost of rural school bus to a greater extent than the genetic algorithm.

AMS subject classifications: 91B32, 90C29

Key words: Highway transportation management, School Bus Routing Problem (SBRP), Mixed-load, Simulated annealing, Multi-objective, Travel cost.

Translated from *Journal of Nanjing University of Information Science & Technology*, 2023, 15(3): 293-303.

*Corresponding author. *Email addresses:* **likailei2021@163.com** (K. Li), **gushantraffic@163.com** (H. Bai).

1 Introduction

At present, the development of rural traffic in our country is generally backward and the school bus service is not perfect in rural areas. Different from the short-distance line length and high-density station coverage mode of urban school bus stations, rural school bus stations mostly show the demand mode of longitudinal extension and scattered distribution, and rural school bus route planning needs to be improved. School Bus Routing Problem (SBRP) is a combinational optimization problem that can reasonably plan school bus routes to send students from bus station to school (or return from school to bus station) and achieve specific goals under the constraints of school bus capacity and time window. Since Newton et al. [1], the author of the multi-school SBRP problem, generated school bus routes and schedules based on heuristic algorithms and used quadratic programming to plan school bus network, many scholars have been exploring relevant mathematical models, optimization algorithms and their applications. In order to solve the problem of school bus routing, Jaradat et al. [2] adopted Intelligent Water Drops (IWD) algorithm to optimize and solve the problem by targeting school bus capacity, maximum ride time and school time window. Calvete et al. [3] proposed a local assignment local search algorithm to solve the school bus routing problem with parking space selection. Gao Wei et al. [4] focused on the problem of the minimum number of operations of school buses, defined and described the problem of school buses, and divided the problem into limit case and general case. The SBLS (School Bus Limit Situation) algorithm and SBGS (School Bus General Situation) algorithm are designed for different situations. Regarding mixed-load SBRP, Hargroves et al. [5] pointed out the research direction, but did not build relevant models and algorithms to solve it. Hou et al. [6] constructed a hybrid iterative local search (ILS) meta-heuristic algorithm that can be used for SBRPS with multiple planning scenarios, including homogeneous or heterogeneous fleets, single-load or mixed-load operating modes. Park et al. [7] proposed a new mixed-load improvement algorithm that decomposed the multi-school SBRP problem into a single-school SBRP problem, used a scanning algorithm to optimize the single-school route, and then merged the optimized single-school route results. Semba et al. [8] used three meta-heuristic algorithms, Simulated Annealing (SA), Tabu Search (TS) and Ant Colony Optimization (ACO), to solve the model of multi-calibrated SBRP problems. The performance of the three algorithms is compared empirically.

The above literature has studied the multi-school SBRP, but the school bus service in rural areas pays too much attention to the cost of the school bus operator, and the service quality is not deeply discussed. Aiming at student travel cost, that is, students' perception of school bus service, this paper establishes an immixed school bus network layout model based on student travel cost and service coordination. Under the condition of ensuring school bus service quality, school bus routes can be optimized to reduce school bus operating mileage, thereby reducing school operating costs. Under the mixed-load scenario, the mixed-load optimization model is established from two aspects of school bus purchase and operation cost. The test results of simulated annealing Algorithm (SA) and Genetic Algorithm (GA) with different search operators on international benchmark cases show that the SA algorithm has good applicability, and the models constructed under the two scenarios can improve the service quality of school buses in rural areas and reduce the operating costs of school schools.

2 Problem description and data introduction

2.1 Problem Description

There are several schools in a school district, each with one or more school buses, and students are only allowed to get on and off at their own stops. The number and coordinates of schools, bus stations, and yards are known, the number of students at each station and the target school for students at that station are known, and the number and capacity of school buses are known. Each station can only be served by one school bus with at least one student waiting. In the non-mixing scenario, for students who cannot take the same bus to different schools at the same time, the service time of getting on and off the bus to the station should be taken into account, and all students should arrive within the time window stipulated by the school to ensure the minimum travel cost of students and reduce the operating cost of the school. In the mixed-load scenario, each school bus provides services for different schools, and it is also necessary to consider the on-off service time and set the maximum ride time for students.

2.2 Data Introduction

Park et al. [7] proposed a general data test set for school bus network layout in 2012, and summarized the commonality of school bus station and school layout into two types: RSRB and CSCB, where "R" and "C" respectively mean random and clustered, and "S" and "B" respectively mean school and station. That is, the distribution of school and site coordinates of RSRB is random, while CSCB will go to the student sites of different schools for site layout, forming several cluster centers, and different schools are also concentrated in the same area.

Wulian County, Rizhao City, Shandong Province is located in the middle and south of Shandong Peninsula, with a total area of 1 497 km^2 and a permanent population of 499,800. This paper chooses Wulian County urban district to carry on the school bus network layout case study. There are many roads in the study area, but some roads are too narrow and the pavement quality is not good. There are 6 example schools in the study area with a total of 4,522 students, including 795 students who take school buses. As can be seen from Figure 1, the example school district belongs to the case of distributed school sites, that is, RSRB.

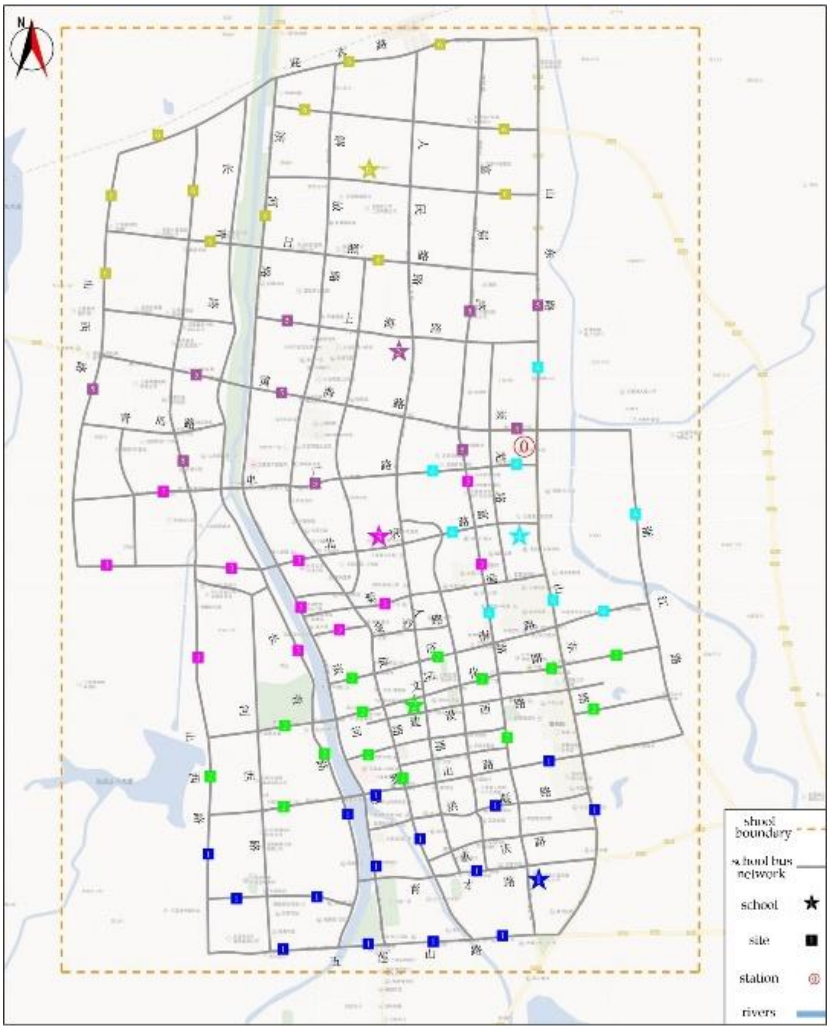

Figure 1: Example site layout

3 Build a mathematical model

3.1 Parameters and decision variables

In the school bus network layout problem, the parameters and variables used in the model are defined symbolically from four aspects: station, student, station and school bus. Because the yard station is set in a school district, if the school buses are all parked at a known yard station, the yard station is a station, denoted by 0; If the school bus is parked at different yards, a collection D of yards can be set up. Usually, P^+ and P^- are used to represent the set of students and school sites respectively, and P refers to the union of students and school sites, expressed as $P = P^+ \cup P^-$. The set of sites, student sites, and school sites is V. Taking a single site 0 as an example, the set can be represented as $V = P \cup \{0\}$. Each station corresponds to the school $s(i)$ that needs to be reached. When serving different schools, the school has its own time window, describing the earliest arrival time e_i and the latest arrival time l_i. The set of school bus k used in school bus operation is K, that is, $k \in K$.

Considering the abstractness of the construction of the school bus network layout model, the bus traveling path from station i to station j is represented as c_{ij}, and the number of people waiting for buses at each station $i \geq 1$. The number of students at each site varies from instance to instance, so the number of students at site i is n_i, which, depending on the nature of site i, represents the number of students who ride or get off the school bus. It takes a certain amount of service time for students to get on and off the bus. t_i is the stagnant service time of the bus at station i, which changes according to the number of students. In addition, in the construction of different school bus network layout models, due to the different optimization objectives, the selected decision variables are also different, and the general model usually selects decision variables such as x_{ijk}, T_{ik}, L_{ik} etc.

To sum up, parameters and decision variables commonly used in the school bus network layout problem model are sorted out, as shown in Table 1.

Table 1: Description of common symbols

argument	meaning statement
P^+	student site collection
P^-	school site collection
P	a union of student sites and school sites
V	collection of all sites
$s(i)$	the school site corresponding to the student site
K	vehicle assembly
Q	capacity of vehicle
c_{ij}	distance traveled from site to site,
e_i, l_i	the earliest and latest arrival time of the school corresponding to the station
n_i	the number of students on (off) buses at the site
t_i	the service time required by the site
v	the average speed at which a school bus operates
α_1	price per unit school bus
α_2	the driving cost per mile
α_3	stagnant cost per unit of time
t_{max}	maximum ride time for students
decision variable	
x_{ijk}	1 if the KTH car passes through stations i to j, 0 otherwise
T_{ik}	the time when the k car arrives at station i
L_{ik}	the number of people in the car after the k car visits site i
y_{ik}	if the k school bus service station i is 1, otherwise it is 0

3.2 Model Construction

3.2.1 Mathematical model of linear programming under unmixed load

In the construction of the model, the two perspectives of school and students (passengers) are selected, and the optimization goal is to minimize school operating costs and student travel costs. The school only considers the variable costs of operating the vehicle and the fixed costs associated with the purchase of the school bus. The variable cost in vehicle operation is the sum of the running cost of the school bus and the stagnant service cost caused by students getting on and off when the school bus arrives at the station. Since the maintenance of school buses and the construction of yard stations are not considered, the dominant factor in the change of fixed costs is the number of school buses, so the school operating costs F_1 can be expressed as

$$F_1 = \alpha_1 \sum_{i\in P^+} \sum_{i\in P^-} x_{0jk} + \alpha_2 \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} x_{ijk} \cdot c_{ij} + \alpha_3 \left(\sum_{i\in P^+} |n_i| \cdot t_i - \sum_{i\in P^-} |n_i| \cdot t_i\right). \tag{1}$$

the service time t_i after school bus k arrives at station i is related to the n_i of the station and the service nature of station i, wherein t_i is defined according to the developed regression model [9].

In order to ensure that n_i conforms to the definition domain, n_i at the waiting station represents the number of people getting on, n_i' is a positive number, and $t_i = 19 + 2.6 \cdot n_i'$ at this time; n_i at the school station represents the number of people getting off, n_i' is negative, and $t_i = 29 - 1.9 \cdot n_i'$ is the stopping time of students getting off. To wit:

$$\begin{cases} i \in P^+, n_i' = |n_i|, t_i = 19 + 2.6 \cdot n_i', \\ i \in P^-, n_i' = -|n_i|, t_i = 29 - 1.9 \cdot n_i'. \end{cases} \tag{2}$$

when analyzing from the perspective of students as passengers, considering students' satisfaction with school bus service level, which is difficult to determine due to various factors, this paper draws inspiration from the similarity between school bus and public transport, introduces the concept of generalized travel cost into the school bus network layout [10-11], and selects time and fare as evaluation indicators. The students' satisfaction with the service is reduced to a perceived response to time and fare.

Travel time includes the time spent waiting for the bus at the station and the time spent in the bus when the student takes the bus to school. In order to simplify the model, the in-car time is approximated as the ratio of the total mileage and speed of the school bus, and the service time of the school bus is equal to the stagnation time after the school bus arrives at the station. For students, the time value generated by waiting time and in-car time is different in terms of time consumption, so the time weight is added to the processing of riding time and waiting time, and the waiting time weight is three times that of in-car time. To sum up, in order to minimize the travel cost for students, taking into account the cost of passengers' in-car time, waiting time and fare cost, student travel cost F_2 can be expressed as

$$\begin{cases} F_2 = (z\mu_1/v) \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} x_{ijk} \cdot c_{ij} + 3z\mu_1 \left(\sum_{i\in P^+} |n_i| \cdot t_i - \sum_{i\in P^-} |n_i| \cdot t_i\right) + M\mu_2 \sum_{i\in P^+} |n_i|, \\ z = (m \times \eta)/3600, \\ 0 < \mu_1, \mu_2 < 1, \mu_1 + \mu_2 = 1, \end{cases} \tag{3}$$

where: z is the unit time value of students; m is local per capita hourly income; η is the value ratio of different travel purposes, which is 0.15; M is the ticket price of the unit student, which is the fixed value; μ_1 and μ_2 are the weight coefficients of travel time cost and fare cost, respectively [12].

The function constraint is carried out from four perspectives: site, time window, capacity and site.

1) Station constraint: each station is served by only one school bus, ensuring that when school bus k enters the station and leaves the station after completing the service, and considering the pick-up order of students when school bus k serves students, ensuring that school bus k first serves the station in operation, and then drives to the corresponding school station, and finally, the constraint on the decision variable x_{ijk}.

$$\begin{gathered}\textstyle\sum_{k\in K}\sum_{i\in V} x_{ijk} = 1, \forall j \in P^+,\\ \sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, i \in P,\\ T_{ik} + t_i + T_{is(i)} \le T_{s(i)k}, i \in P^+, s(i) \in P^-, k \in K,\\ x_{ijk} \in \{0,1\}, i, j \in P, k \in K. \end{gathered} \tag{4}$$

2) Time window constraint: within the service scope of school bus k, ensure that all school buses will deliver students within the earliest arrival time e_i and the latest arrival time l_i stipulated by the school.

$$e_i \le T_{ik} \le l_i, i \in P^-, k \in K. \tag{5}$$

3) Capacity constraint: ensure that the total number of students on the bus is greater than or equal to the number of students on the bus and is less than or equal to the specified capacity of the bus Q when the bus line k arrives at the service station to complete the service.

$$|n_i| \le L_{ik} \le Q, i \in P^+, k \in K. \tag{6}$$

4) Station constraint: ensure that the school bus k will return to the original station after completing the student pickup service from the station 0 to form a closed loop.

$$\textstyle\sum_{j\in P} x_{d(k)j} - \sum_{j\in P} x_{jd(k)} = 1, k \in K. \tag{7}$$

In summary, the mathematical model of school bus path planning under the unmixed load scenario is

$$\begin{aligned} \min F = {} & \alpha_1 \sum_{i\in P^+}\sum_{i\in P^-} x_{0jk} + (\alpha_2 + z\mu_1/v)\sum_{i\in V}\sum_{j\in V}\sum_{k\in K} x_{ijk}\cdot c_{ij}\\ & + (\alpha_3 + 3z\mu_1)\left(\sum_{i\in P^+}|n_i|\cdot t_i - \sum_{i\in P^-}|n_i|\cdot t_i\right) + M\mu_2\sum_{i\in P^+}|n_i|, \end{aligned}$$

$$\text{s.t.}\begin{cases}\sum_{k\in K}\sum_{i\in V} x_{ijk} = 1, \forall j \in P^+,\\ \sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, i \in P,\\ T_{ik} + t_i + T_{is(i)} \le T_{s(i)k}, i \in P^+,\\ s(i) \in P^-, k \in K,\\ x_{ijk} \in \{0,1\}, i, j \in P, k \in K,\\ e_i \le T_{ik} \le l_i, i \in P^-, k \in K,\\ |n_i| \le L_{ik} \le Q, i \le P^+, k \in K,\\ \sum_{j\in P} x_{d(k)j} - \sum_{j\in P} x_{jd(k)} = 1, k \in K.\end{cases} \tag{8}$$

3.2.2 Mathematical model of linear programming under mixed load scenario

The optimization model of school bus network layout under the mixed load scenario is limited by the complexity of constraints, and the optimization goal of the minimum travel cost introduced in the model construction cannot be achieved, so the acquisition cost and operating cost of school bus are adopted as the optimization goals. Among them, the school bus purchase cost is related to the school bus input, the number of school buses multiplied by the school bus unit price α_1 is the purchase cost, while the school operating cost is affected by the total operating mileage, the product of the total mileage and the unit mileage driving cost α_2 is the school operating cost:

$$F_3 = \alpha_1 \sum_{j\in P^+} \sum_{k\in K} x_{0jk} + \alpha_2 \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} x_{ijk} \cdot c_{ij}. \tag{9}$$

The function constraint is carried out from four angles: site, time, capacity and site.

1) Station constraint: it is guaranteed that there is only one school bus service at each station, and when the school bus k enters the station and ends the service, it leaves. Since school bus k can pick up students from different schools at the same time in the process of service, it is necessary to restrict some dynamic changes in school bus operation to ensure that when school bus k serves station i, it needs to pass through the school corresponding to station i. Finally, it is the constraint on decision variables x_{ijk}, y_{ik}.

$$\begin{aligned}
&\sum_{k\in K} y_{ik} = 1, i \in P^+,\\
&\sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, i \in P, k \in K,\\
&\sum_{j\in V} x_{jik} - \sum_{j\in V} x_{js(i)k} = 0, i \in P^+, k \in K,\\
&T_{ik} + t_i + T_{is(i)} \le T_{s(i)k}, i \in P^+, s(i) \in P^-, k \in K,\\
&x_{ijk} \in \{0,1\}, i,j \in P, k \in K,\\
&y_{ik} \in \{0,1\}, i \in P, k \in K.
\end{aligned} \tag{10}$$

2) Time constraints: taking into account the differences in school bells between different schools, ensure that all school buses deliver students within the earliest arrival time e_i and the latest arrival time l_i specified by the school. Determine a maximum ride time $t_{\max}$ to prevent students from spending too much time in the car and the school bus service level is too low.

$$\begin{gathered}
e_i \le T_{ik} \le l_i, i \in P^-, k \in K,\\
T_{s(i)k} - T_{ik} \le t_{\max}, i \in P^+, k \in K.
\end{gathered} \tag{11}$$

3) Capacity constraint: ensure that the number of people on board at any time node is within the capacity Q and constrain the changes in the capacity of the vehicle between different stations.

$$\begin{gathered}
|n_i| \le L_{ik} \le Q, i \in P^+, k \in K,\\
x_{ijk}\left(L_{ik} + |n_i| - L_{jk}\right) = 0, i,j \in P, k \in K,\\
0 \le L_{s(i)k} \le Q + |n_i|, i \in P^+, k \in K.
\end{gathered} \tag{12}$$

4) Station constraints: ensure that there are no students in the bus when k departs from the station, and return to the original station after completing the student pickup service.

$$L_{0k} = 0, k \in K,$$
$$\sum_{j\in P} x_{d(k)j} = \sum_{j\in P} x_{jd(k)} = 1, k \in K. \tag{13}$$

In summary, the mathematical model of school bus path planning under mixed load conditions can be expressed as

$$\min F_3 = \alpha_1 \sum_{j\in P^+} \sum_{k\in K} x_{0jk} + \alpha_2 \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} x_{ijk} \cdot c_{ij},$$
$$\text{s.t.} \begin{cases} \sum_{k\in K} y_{ik} = 1, i \in P^+, \\ \sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, i \in P, k \in K, \\ \sum_{j\in V} x_{jik} - \sum_{j\in V} x_{js(i)k} = 0, i \in P^+, k \in K, \\ T_{ik} + t_i + T_{is(i)} \le T_{s(i)k}, i \in P^+, \\ s(i) \in P^-, k \in K, \\ x_{ijk} \in \{0,1\}, i, j \in P, k \in K, \\ y_{ik} \in \{0,1\}, i \in P, k \in K, \\ e_i \le T_{ik} \le l_i, i \in P^-, k \in K, \\ T_{s(i)k} - T_{ik} \le t_{max}, i \in P^+, k \in K, \\ |n_i| \le L_{ik} \le Q, i \in P^+, k \in K, \\ x_{ijk}\left(L_{ik} + |n_i| - L_{jk}\right) = 0, i, j \in P, k \in K, \\ 0 \le L_{s(i)k} \le Q + |n_i|, i \in P^+, k \in K, \\ L_{0k} = 0, k \in K, \\ \sum_{j\in P} x_{d(k)j} = \sum_{j\in P} x_{jd(k)} = 1, k \in K. \end{cases} \tag{14}$$

4 Solution algorithm

4.1 Selection of algorithm framework

Genetic algorithm is the most commonly used in school bus problem, because the objective function of genetic algorithm is set to search information, so solving multiple objective functions has advantages. When searching, genetic algorithm follows probability, has strong global, randomness and flexibility, and can greatly reduce the interference of parameters on the results. However, it is prone to premature convergence, incomplete expression of constraints, strong dependence on the initial population and other shortcomings, which affect the accuracy of the results of multi-calibration problems [13-14]. Although Ant Colony Optimization (ACO) is more flexible in use and can be combined with other heuristic algorithms to improve its solving ability, it is unable to adapt to large-scale problems due to its large computational load and long solving time. Moreover, when searching, it is easy to terminate the operation due to the consistency of the solution obtained by all individuals, which is not conducive to obtaining the optimal solution [15]. Compared with the above two algorithms, the simulated annealing algorithm has higher operation efficiency and shorter operation time, and is not affected by the initial solution. Moreover, the algorithm can make the complex constraints in the model intuitively and clearly displayed in the algorithm structure [16-18]. Of course, the simulated annealing algorithm is easy to fall into the local optimal in the search process, and it is difficult to guarantee the optimal solution at one time, but this problem can be solved by multiple substituting solutions. Considering the large amount of research data, many constraints of the model, complex calculation and other factors, the models constructed under unmixed load and mixed load scenarios are based on simulated annealing algorithm as the framework, and different neighborhood search operators are introduced to solve the models. In order to better verify the superiority of simulated annealing algorithm, the results of solving the model were compared with those of genetic algorithm.

4.2 Neighborhood search operator

Three conventional search operators, shift, swap and 2-opt, are used in the non-mixed load scenario, while the neighborhood search operators between and within paths move in pairs when the algorithm is solved. Therefore, three neighborhood operators, PD-Shift, PD-Exchange and PD-Rearrange, are introduced based on the characteristics of the problem [19], which are mainly described as follows:

1) PD-Shift: move a pair of points "P" and "D" from route 1 to route 2, subject to all constraints in the optimization target model when moving, and prohibit infeasible moves, as shown in Figure 2.

2) PD-Exchange: exchanges point pairs "P" and "D" in two lines. As shown in Figure 3, "P1" and "D1" are point pairs in line 1, "P2" and "D2" are point pairs in line 2. First, these two sets of point pairs are deleted from line 1 and line 2, and then "P1" and "D1" are inserted into feasible positions in line 2, and "P2" and "D2" are inserted into feasible positions in line 1.

3) PD-Rearrange: in the same path, the "P" and "D" point pairs are placed to the best position through rearrangement, thus minimizing the value of the objective function. As shown in Figure 4, "P" and "D" are a set of point pairs of a certain line, which are deleted in the line through PD-Rearrange operation, and then inserted into a new feasible position in the same line.

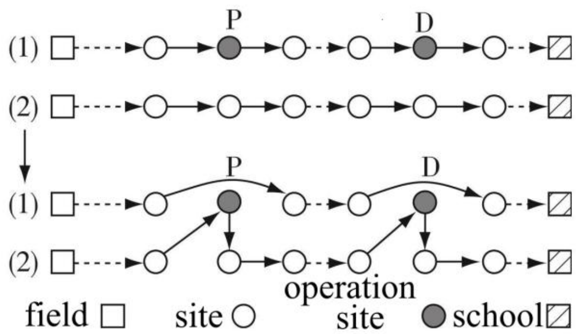

Figure 2: PD-Shift

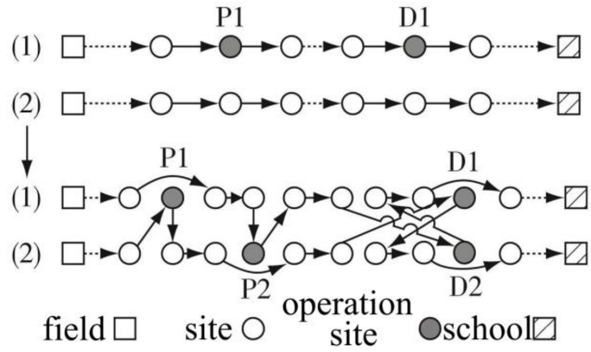

Figure 3: PD-Exchange

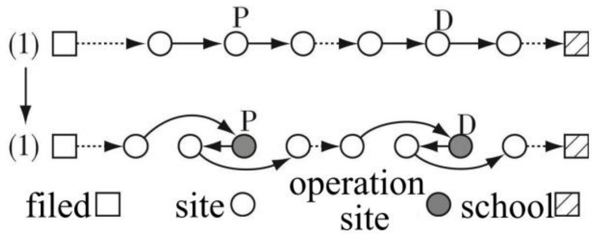

Figure 4: PD-Rearrange

The final result of neighborhood search often pays too much attention to the total mileage [20]. Compared with the school bus route optimization, the number of school bus input is the primary factor affecting the school operation cost. Therefore, after each local search operator completes the search and generates a new neighborhood solution, the Metropolis judgment criterion [21] is realized, and the evaluation function is

$$C(R) = \alpha|R| + \beta(-\sum_{r\in R}|r|^2) + \gamma\sum_{r\in R} c(r), \tag{15}$$

where, $C(R)$ is the total operating cost of school buses on the road network, $|R|$ is the number of paths in the solution, $|r|^2$ is the number of stops for school buses on the path r in the solution, and $c(r)$ is the mileage of the path r. Where $\alpha \gg \beta \gg \lambda$, in that the three quantities are evaluated in lexicographic order. With the increase of $|r|^2$, some paths in the optimization plan are developing in the direction of serving more sites, which is more conducive to the merger of paths.

5 Experimental verification and data testing

5.1 Test cases and parameter Settings

The parameters are set as the capacity Q of the school bus is sixty-six people, the average speed v of the school bus is about 32 km/h, taking into account the service level, the maximum time allowed for students on the school bus is 2 700 s, and finally a school bus station 0 is set in the center. The data set provides school coordinates, site coordinates, and site requirements, as shown in Table 2.

Table 2: Some experimental data

living example	number of sites /	number of schools/schools	total number of students/student	maximum ride time /s
RSRB01	250	6	3 409	2 700
RSRB02	250	12	3 670	2 700
RSRB03	500	12	6 794	2 700
RSRB04	500	25	6 805	2 700
CSCB01	250	6	3 907	2 700
CSCB02	250	12	3 204	2 700
CSCB03	500	25	6 813	2 700
CSCB04	500	25	7 541	2 700

Considering the randomness of the simulated annealing algorithm, the optimal value of the test results is taken from ten times. The algorithm was programmed using MATLAB2016a version. The experimental computer was configured as Intel Core i5-7360U, CPU2.3 GHz, and the system was Windows 10.

The parameters are set as follows: the purchase price of the unit school bus α_1 is 200,000/vehicle, the school bus unit distance driving cost α_2 is 0.001 95 yuan/m, the unit time stagnation cost α_3 is 0.010 5 yuan/s, and the time value z is 0.000 27 yuan/s. The cost of taking the school bus in the whole region is 150 yuan/month, the one-way ticket price is 1.25 yuan, the average speed of the school bus is 40 km/h, the maximum travel time of each school bus is set at 40 min, the student travel cost weight coefficient μ_1 is 0.37, the fare cost weight coefficient μ_2 is 0.63.

5.2 Analysis of experimental results under unmixed loading

Since there is no data calculated by the test set in the closed loop service mode with no mixing and no splitting of site requirements under the same scenario in the existing literature, the comparison data of the solution results of the constructed model are the results obtained by the general model with only operating mileage as the optimization objective under the same scenario, as shown in Table 3. Where $N_1, D_{m1}, D_{c1}, T_{c1}$ respectively represent the number of school buses, driving miles, driving and student travel costs in the solution results of the general model; $N_2, D_{m2}, D_{c2}, T_{c2}$ respectively represent the number of school buses, driving miles, driving and student travel costs in the solution results of the model.

As can be seen from Table 3, the constructed model comprehensively considers the running cost of school buses and the traveling cost of students during school bus operation. The number of school bus input is reduced by 14.1% on average, the mileage is reduced by 14.4% on average, and the running cost is reduced by 14.2% on average. Most of the school's operating cost comes from the purchase of school buses. Therefore, the reduction of school bus investment also fundamentally reduces the school's investment costs. Considering the optimal travel cost of students as far as possible, each school bus service in the arrival mode has a strong pertinency, so the average travel cost of students is reduced by 0.26%.

5.3 Analysis of model experiment results under mixed load scenario

The comparison data of the solution results of the model constructed under the multi-school mixed load scenario selects the test set and the solution results of a single model under the same scenario, as shown in Table 4. The meanings of $N_i, D_{mi}, D_{ci}, T_{ci}$ $(i = 1,2)$ are the same as above.

As can be seen from Table 4, the model optimized with the number of school buses and mileage as the target has more advantages than the single target model in general, and the addition of PD three search operators enables more feasible exchanges between stations and significantly improves route optimization. Therefore, the average input of school buses decreases by 7.9% and the average mileage of school buses decreases by 7.6%. Trip costs decreased by 7.5% on average, and student travel costs increased by 0.11% on average.

5.4 Example verification: Wulian County school bus route optimization design

Data such as road network situation and site information in the example region were obtained through field investigation, as shown in Table 5. In the "existing routes" column, 0 represents the field station, 1,2,⋯,6 represents different schools, 1.1,1.2,⋯,2.1,⋯,6.12 represents the bus service station. The existing school bus service mode is a single school, six schools are equipped with a total of fourteen school buses, each school bus is arranged with a line to pick up students, a total of fourteen lines, the mileage of 265.224 km, the driving cost is 587.85 yuan, the travel cost is 630.301 yuan. The example area has a school bus parking lot, and all school buses leave from station 0 and return to the original station after service. Each station has a fixed number of students going to the designated school, all students on the station are on the bus, no student retention phenomenon. In addition, the parameters set are the same as the test case except that the standard capacity of the school bus is forty-six people.

Table 3: Comparison of solving results between general model and the proposed model in non-mixed-load scenario

case	N_1/ vehicle	N_2/vehicle	D_{m1}/m	D_{m2}/m	D_{c1}/ yuan	D_{c2}/ yuan	T_{c1}/ yuan	T_{c2}/yuan
RSRB01	50	44	5 070 122	4 616 358	14 864. 76	13 548. 04	2 733. 55	2 729. 60
RSRB02	57	47	5 340 931	5 008 415	15 664. 64	14 696. 93	2 941. 87	2 938. 91
RSRB03	81	68	12 592 608	9 753 674	36 818. 63	28 593. 10	5 469. 20	5 444. 50
RSRB04	87	79	12 738 576	10 472 738	37 244. 77	30 680. 92	5 479. 25	5 459. 55
CSCB01	52	46	4 994 927	4 528 193	14 671. 60	13 317. 34	3 125. 77	3 121. 68
CSCB02	53	46	5 515 893	5 205 749	16 147. 13	15 245. 71	2 575. 78	2 573. 02
CSCB03	76	65	11 386 935	9 446 182	33 326. 47	27 702. 26	5 473. 70	5 456. 78
CSCB04	89	73	12 546 379	11 074 925	36 724. 12	32 456. 06	6 058. 15	6 045. 24

Table 4: Comparison of solving results between general model and the proposed model in mixed-load scenario

case	N_1/vehicle	N_2/vehicle	D_{m1}/m	D_{m2}/m	D_{c1}/yuan	D_{c2}/yuan	T_{c1}/yuan	T_{c2}/yuan
RSRB01	32	29	4 414 207	3 806 322	12 956. 75	11 195. 35	2 727. 68	2 722. 375
RSRB02	29	27	4 897 815	4 816 728	14 367. 66	14 131. 90	2 937. 70	2 936. 973
RSRB03	61	55	8 922 373	8 319 293	26 182. 87	24 433. 88	5 437. 20	5 431. 896
RSRB04	65	58	9 382 672	8 657 549	27 518. 02	25 415. 19	5 449. 92	5 443. 543
CSCB01	33	30	4 358 148	4 149 746	12 818. 49	12 213. 63	3 120. 03	3 118. 184
CSCB02	29	28	5 049 424	4 419 578	14 784. 56	12 960. 60	2 571. 44	2 565. 967
CSCB03	55	53	8 502 611	7 662 908	24 965. 50	22 533. 47	5 448. 47	5 441. 172
CSCB04	64	59	10 849 938	10 270 998	31 800. 16	30 121. 56	6 043. 17	6 038. 084

Table 5: Related regional data

site attribute	total number of students/student	number of school buses/vehicles	corresponding site	existing path	site number/name
station 0					
school 1	156	3	1. 1, 1. 2, ⋯, 1. 15	0→1. 1→1. 3→1. 2→1→1. 15→1. 6→1. 4→1→0	61
				0→1. 7→1. 10→1. 8→1. 5→1→0	45
				0→1. 13→1. 11→1. 9→1→1. 14→1. 12→1→0	50
school 2	144	2	2. 1, 2. 2, ⋯, 2. 14	0→2. 14→2. 1→2. 2→2. 4→2→2. 3→2. 6→2. 7→2→0	72
				0→2. 9→2. 11→2. 13→2. 12→2→2. 5→2. 10→2. 8→2→0	72
school 3	123	2	3. 1, 3. 2, ⋯, 3. 11	0→3. 1→3. 2→3. 3→3→3. 7→3. 6→3. 4→3→0	69
				0→3. 8→3. 9→3. 5→3→3. 11→3. 10→3→0	54
school 4	108	2	4. 1, 4. 2, ⋯, 4. 8	0→4. 7→4. 4→4. 5→4→0	44
				0→4. 8→4. 6→4→4. 1→4. 2→4. 3→4→0	64
school 5	129	2	5. 1, 5. 2, ⋯, 5. 10	0→5. 1→5. 2→5. 6→5→5. 8→5. 5→5→0	64
				0→5. 3→5. 4→5. 7→5→5. 9→5. 10→5→0	65
school 6	135	3	6. 1, 6. 2, ⋯, 6. 12	0→6. 7→6. 8→6. 12→6→0	33
				0→6. 6→6. 9→6. 5→6→0	32
				0→6. 1→6. 11→6. 10→6→6. 2→6. 3→6. 4→6→0	70
total	795	14	70		795

5.4.1 Horizontal comparison results

The results of genetic algorithm and simulated annealing algorithm are shown in Table 6 under non-mixing conditions, and the results of genetic algorithm and simulated annealing algorithm under mixing conditions are shown in Table 7.

The results of simulated annealing algorithm were compared with those of simulated annealing algorithm. Under the unmixed load scenario, the number of school buses, mileage, driving cost and travel cost decreased by 14.28%, 19.67%, 20.96% and 3.60% respectively. In the mixed load scenario, the number of school buses, mileage, driving cost and travel cost of simulated annealing algorithm decreased by 7.14%, 15.17%, 17.14% and 2.72%, respectively, compared with genetic algorithm. Compared with the original school bus service mode, the number of school buses, mileage and driving cost have been improved, but the advantages of genetic algorithm are slightly insufficient compared with simulated annealing algorithm.

5.4.2 Results of Vertical Comparison

The results of school bus network layout under non-mixed load and mixed load scenarios are shown in Figure 5 and Figure 6 respectively. The output network layout is a straight-line connection between stations, which needs to be combined with the current passable network in the instance area, and adjustments are made on the basis of the output network layout results to exclude impassable station connection sections.

Table 6: Planning results solved by GA and SA in non-mixed-load scenario

GA						SA					
ID	path length /m	travel time /s	service time /s	running cost/yuan	travel cost/yuan	ID	path length /m	travel time /s	service time /s	running cost/yuan	travel cost/yuan
1	17 452. 12	1 571	969	51. 65	62. 853	1	14 051. 76	1 265	780	35. 59	91. 670
2	19 235. 41	1 731	450	42. 23	59. 132	2	18 022. 23	1 622	422	39. 57	55. 403
3	18 841. 27	1 696	426	41. 20	46. 246	3	17 755. 94	1 598	401	38. 83	43. 582
4	18 642. 97	1 678	565	42. 29	62. 584	4	17 921. 61	1 613	543	40. 65	60. 162
5	17 884. 90	1 610	734	42. 57	48. 818	5	17 594. 48	1 584	722	41. 88	81. 473
6	15 671. 45	1 411	472	35. 52	51. 378	6	14 971. 90	1 348	451	33. 93	49. 085
7	17 642. 35	1 588	550	40. 18	59. 727	7	16 835. 98	1 515	525	38. 34	56. 997
8	17 854. 61	1 608	447	39. 51	42. 776	8	15 882. 64	1 430	398	35. 15	38. 052
9	19 876. 71	1 789	617	52. 95	63. 085	9	15 469. 35	1 392	480	35. 21	49. 097
10	16 678. 92	1 501	685	39. 71	41. 066	10	16 561. 25	1 491	680	39. 43	77. 515
11	18 518. 11	1 666	561	32. 55	45. 568						
12	18 933. 02	1 704	777	41. 43	42. 496						
total	217 231. 83	19 552	7 253	501. 79	625. 729	total	165 067. 14	14 858	5 402	378. 59	603. 063

Table 7: Planning results solved by GA and SA in mixed-load scenario

GA						SA					
ID	path length /m	travel time /s	service time /s	running cost/yuan	travel cost/yuan	ID	path length /m	travel time /s	service time /s	running cost/yuan	travel cost/yuan
1	19 880. 37	1 789	1 158	50. 93	61. 593	1	16 167. 90	1 455	942	41. 42	72. 862
2	19 256. 63	1 733	1 049	48. 56	56. 783	2	16 563. 68	1 491	902	41. 77	72. 066
3	22 306. 73	2 008	1 187	55. 96	67. 261	3	16 481. 91	1 484	877	41. 35	68. 908
4	18 608. 66	1 675	917	45. 92	76. 947	4	16 661. 14	1 500	821	41. 11	68. 894
5	16 973. 05	1 528	739	40. 87	65. 692	5	16 571. 92	1 492	722	39. 90	64. 139
6	19 114. 06	1 721	865	46. 36	79. 303	6	17 172. 60	1 546	777	41. 65	71. 248
7	18 259. 33	1 644	592	41. 82	63. 426	7	18 937. 25	1 705	614	43. 37	72. 004
8	19 813. 22	1 784	870	47. 76	66. 706	8	17 359. 96	1 563	762	41. 85	75. 970
9	20 680. 73	1 676	1 085	51. 73	59. 588	9	17 951. 56	1 455	942	44. 90	67. 349
10	19 211. 06	1 730	1 022	48. 20	53. 318						
total	194 103. 84	17 289	9 484	478. 10	650. 616	total	153 867. 92	13 689	7 361	377. 32	633. 441

Compared with the original school bus service mode, the school bus network layout under the non-mixed load scenario shows a 28.6% reduction in the number of school buses, and a large part of the school bus operation cost comes from the purchase of school buses, so the decrease in the school bus investment also fundamentally reduces the school bus investment cost. In terms of operating mileage, the connection between different school service routes is optimized under the non-mixed load scenario, which reduces the mileage of school buses by 37.8% and the driving cost by 35.6%. Considering the optimal travel cost of students as much as possible, so that students can get a better ride experience, but in the arrival mode, each school bus service has a strong pertinency, so the student travel cost is reduced by 4.3%.

Compared with the original school bus service mode, the school bus network layout under the mixed load scenario has greater advantages in both school bus investment and school bus operation. The number of school bus investment and mileage are reduced by 37.5% and 42.0% respectively. In this scenario, the school bus is more flexible, and the alternate transformation between route schemes also produces more possibilities, and the driving cost is reduced by 35.8%. However, due to the complexity of the mixing scenario, it is difficult to take into account the travel cost at the same time, and also produces the most travel time, which reduces the service level of school buses to a certain extent, so the travel cost of students is slightly higher than the original school bus service mode, increasing by 0.5%.

Under two scenarios of unmixed and mixed load, the results of school bus network layout based on simulated annealing algorithm and genetic algorithm were compared with the original school bus service mode in vertical and horizontal direction, as shown in Figure 7.

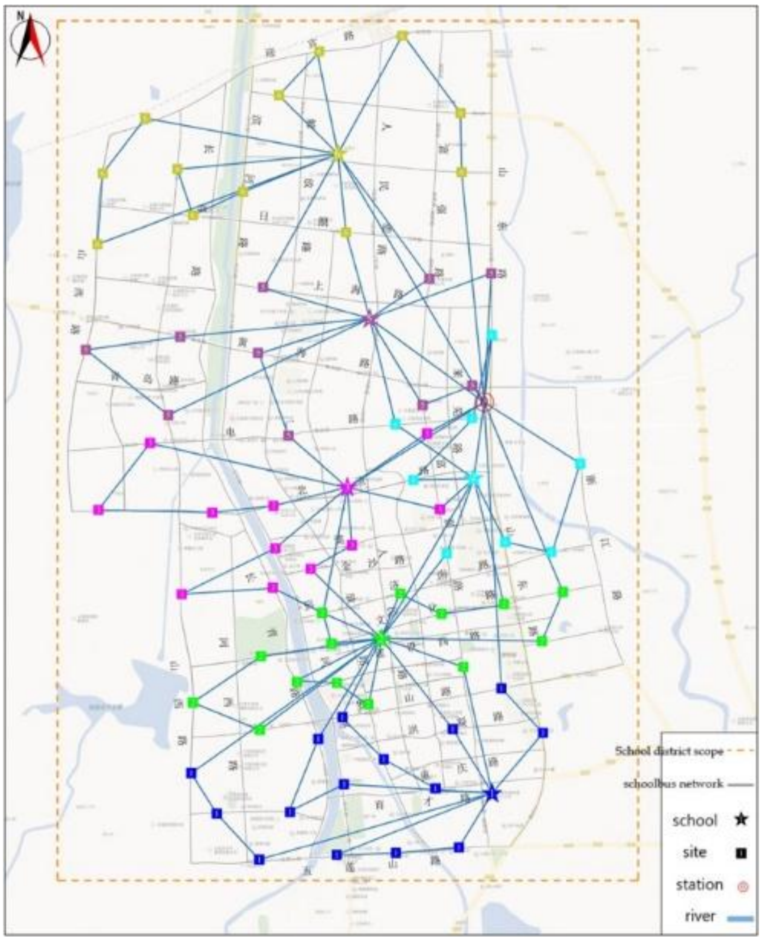

Figure 5: Road network layout of school bus in non-mixed-load scenario

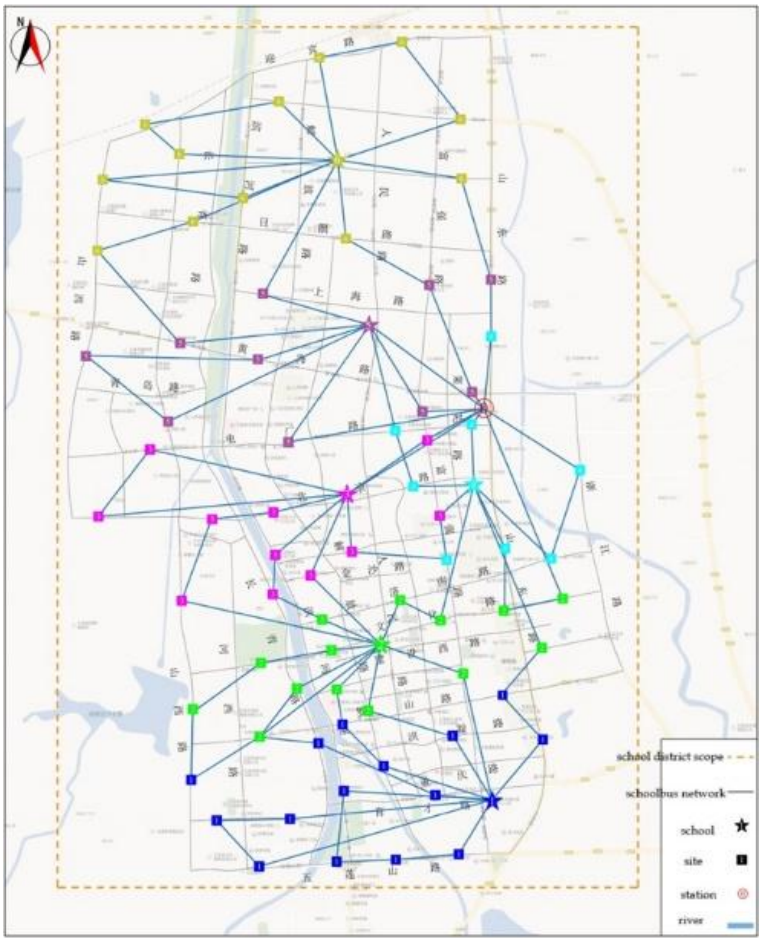

Figure 6: Road network layout of school bus in mixed-load scenario

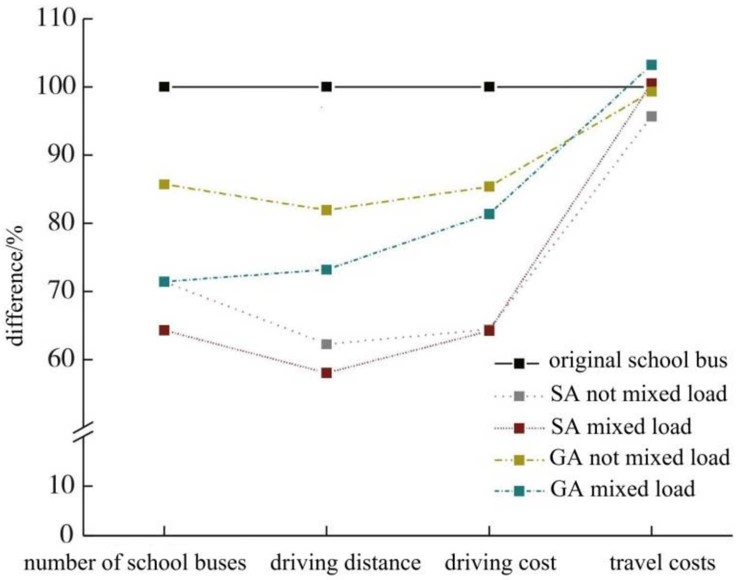

Figure 7: Vertical and horizontal comparisons

6 Conclusions

Taking multi-school SBRP in rural areas as the research object, considering different scenarios and optimization objectives, a mathematical model of immiscible SBRP based on school operating cost and student travel cost is established. Considering the school operating cost, a mixed-load SBRP mathematical model with the goal of minimizing the school operating cost and input cost is established. Using simulated annealing algorithm and genetic algorithm to analyze international benchmark test cases and domestic examples, the following conclusions are drawn.

1) Horizontal comparative analysis shows that the number of school buses, driving distance, driving cost and travel cost of simulated annealing algorithm have decreased compared with genetic algorithm in both unmixed and mixed load scenarios. Simulated annealing algorithm with different search operators has more advantages than genetic algorithm in solving the constructed model. It is more pertinence to construct the model considering multiple optimization objectives in this paper.

2) Longitudinal comparative analysis shows that the immixed SBRP model established in this paper can effectively improve the service level of rural school buses while compatible with reducing operating costs, ensure the optimization of the investment and total mileage of school buses, reduce the average mileage by 37.8%, and reduce the operating costs of school schools by 28.6% and 35.6% respectively. Meanwhile, students' travel perception is considered. Student travel costs are reduced by 4.3% and students get a better ride experience. The established mixed-load SBRP model can reduce the operating cost by 37.5% and 35.8% respectively, and the driving distance is reduced by 42.0%, and the school bus is more flexible, and the alternate transformation between the route schemes produces more possibilities, which is more conducive to the school operation.

3) It is worth pointing out that it will be the next research direction for the scenarios of station demand splitting and multi-vehicle types of school buses, the further improvement of the algorithm for solving the model, and the influence of different school bus network layout on students' waiting time under mixed load scenarios.

Acknowledgments

This work is supported by the Natural Science Foundation of Shandong Province (Grant No. ZR2021QF110).

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] R. M. Newton, and W. H. Thomas. Bus routing in a multi-school system, *Comput. Oper. Res.*, 1974, 1(2): 213-222.

[2] A. S. Jaradat, and M. Q. Shatnawi. Solving school bus routing problem by intelligent water drops algorithm, *J. Comput. Sci.*, 2020, 16(1): 25-34.

[3] H. I. Calvete, C. Galé, J. A. Iranzo, et al., A partial allocation local search matheuristic for solving the school bus routing problem with bus stop selection, *Mathematics*, 2020, 8(8): 1214.

[4] W. Gao, Z. Y. Chen, and D. Z. Li, Optimization of the model of unmixed school bus route based on the number of school bus, *J. Shenyang Univ. Chem. Technol.*, 2021, 35(1): 82-89.

[5] B. T. Hargroves, and M. J. Demetsky, A computer assisted school bus routing strategy: a case study, *Socio-Econ. Plan. Sci.*, 1981, 15(6): 341-345.

[6] Y. Hou, N. Zhao, L. Dang, et al., A hybrid metaheuristic algorithm for the school bus routing problem with multi-school planning scenarios, *Eng. Lett.*, 2021, 29(4): 1-10.

[7] J. Park, H. Tae, and B. I. Kim, A post-improvement procedure for the mixed load school bus routing problem, *Eur. J. Oper. Res.*, 2012, 217(1): 204-213.

[8] S. Semba, and E. Mujuni, An empirical performance comparison of meta-heuristic algorithms for school bus routing problem, *Tanzan. J. Sci.*, 2019, 45(1): 81-92.

[9] J. Braca, J. Bramel, B. Posner, et al., A computerized approach to the New York City school bus routing problem, *IIE Trans.*, 1997, 29(8): 693-702.

[10] M. Q. Liu, and H. Z. Qu, Study on the route choice using the combination of rail and bus transit, *J. Transp. Eng. Inf.*, 2018, 16(4): 63-68.

[11] H. H. Feng, and J. H. Deng, Model of public transport choice based on prospect value and passenger optimal theory, *Sci. Technol. Eng.*, 2019, 19(5): 307-311.

[12] M. W. Levin, and S. D. Boyles, Practice summary: improving bus routing for KIPP charter schools, *Interfaces*, 2016, 46(2): 196-199.

[13] Y. Hong, L. P. Yin, Genetic algorithm-based stochastic distribution control for non-Gaussian systems, *J. Nanjing Univ. Inf. Sci. Technol. (Nat. Sci. Ed.)*, 2020, 12(4): 504-509.

[14] B. Minocha, and S. Tripathi, Solving school bus routing problem using hybrid genetic algorithm: a case study, *Adv. Intell. Syst. Comput.*, New Delhi: Springer India, 2014: 93-103.

[15] Y. Li, J. N. Ji, J. L. Shen, et al., Mobile robot path planning based on improved ant colony algorithm, *J. Nanjing Univ. Inf. Sci. Technol. (Nat. Sci. Ed.)*, 2021, 13(3): 298-303.

[16] S. Q. Deng, Z. J. Guo, F. Li, et al., Adaptive simulated annealing particle swarm optimization algorithm based on metropolis, *Softw. Guid.*, 2022, 21(6): 85-91.

[17] C. Q. Li, and J. C. Pei, Application of new simulated annealing genetic algorithm in path optimization, *Modular Mach. Tool Autom. Manuf. Tech.*, 2022(3): 52-55.

[18] V. F. Yu, P. Jewpanya, A. A. N. P. Redi, et al., Adaptive neighborhood simulated annealing for the heterogeneous fleet vehicle routing problem with multiple cross-docks, *Comput. Oper. Res.*, 2021, 129: 105205.

[19] H. B. Li, and A. Lim, A metaheuristic for the pickup and delivery problem with time windows, *Proc. 13th IEEE Int. Conf. Tools Artif. Intell. (ICTAI 2001)*, November 7-9, 2001, Dallas, TX, USA, IEEE, 2001: 160-167.

[20] R. Bent, and P. V. Hentenryck, A two-stage hybrid algorithm for pickup and delivery vehicle routing problems with time windows, *Comput. Oper. Res.*, 2006, 33(4): 875-893.

[21] W. W. Liu, X. Li, X. N. Qin, et al., A metropolis criterion based fuzzy Q-learning flow controller for high-speed networks, *Appl. Mech. Mater.*, 2012, 241/242/243/244: 2327-2330.