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Prescribed-time Adaptive Fuzzy Tracking Control Of UAV Swarms Under Deception Attack

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Abstract. In addressing the tracking control problem for unmanned aerial vehicle (UAV) swarms, we consider several challenges: the unmeasurable state of the swarm system, potential deception attacks on actuators, external random disturbances, and the nonlinear dynamics of each UAV. To tackle these issues, we first introduce a time-varying function and utilize a coordinate transformation method to convert the time tracking problem into an error variable constraint problem. Next, we propose an adaptive time tracking control method employing one-to-one mapping and inversion techniques, aimed at achieving system convergence to a specified accuracy within a designated time frame. To mitigate the impact of possible deception attacks on actuators, we design an attack compensator that removes disturbances caused by time-varying attack gains. Additionally, we implement an observer to estimate the unmeasurable state of the system and utilize a fuzzy logic system to manage unknown functions. Finally, we validate the effectiveness of our control method through simulations.

AMS subject classifications: 93D15, 93C10

Key words: UAV swarm control, Deception attack, Prescribed-time control, Fuzzy logic system.

1 Introduction

With the rapid development of unmanned air vehicle technology, the application of UAV swarms has become increasingly widespread in fields such as military, agriculture, and logistics. However, the complexity and diversity of UAV swarm systems have led to significant control issues, which are increasingly important and challenging to address. The

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utilization of trajectory tracking technology enables multiple UAVs to commence their journey from a variety of arbitrary initial positions and subsequently maintain the desired trajectory motion. This technology has already achieved significant results [1, 2] in practical systems. However, due to various disturbances in the external environment, there are inevitably uncertain parts in the modeling of (unmanned aerial vehicle)UAVs. Swarm control methods are constrained by nonlinear systems, requiring the internal nonlinear dynamics of the system to be known. Currently, a plethora of solutions exists, among which neural networks and fuzzy logic systems (FLS) [3, 4] are efficacious instruments for the management of the unknown and uncertain components of the controlled system. For example, in [5], an adaptive neural control method was designed for UAV formation. Nevertheless, in the majority of practical systems, the state of the controlled UAVs cannot be directly measured. Furthermore, the high costs associated with obtaining this data during the control design process render it impractical. Therefore, for nonlinear systems with unmeasurable states, appropriate fuzzy network state observers are employed. These observers are used to propose an observer-based leader-follower consensus controller [6]. The aforementioned control scheme only considers deterministic nonlinear systems. Due to the extreme complexity of the actual environment, additional consideration of stochastic disturbances brought by the environment is necessary.

It is well-known that stochastic disturbances can destabilize the entire system. Consequently, there has been a notable increase in the focus on the design of controllers for stochastic nonlinear systems. Wu et al. [7, 8] respectively studied the adaptive control of stochastic nonlinear systems with measurable and unmeasurable states. Subsequently, Ren et al. [9] further considered fuzzy leader-follower control based on local information.

Compared to traditional control systems, UAVs need to operate in network environments, where most of their data is susceptible to various types of cyber-attacks, such as stochastic attacks [10], denial-of-service (DoS) attacks [11], and deception attacks [12]. Once the system is attacked, it becomes difficult to transmit signals completely. Therefore, the cybersecurity of UAV systems is an important research topic. This paper focuses on deception attacks, which primarily aim to intercept the signals of the system's actuators and inject false signals [13], thereby destabilizing the system. In their study, Han et al. [14] devised a consensus control scheme on the premise that the probability of deception attacks was a constant, known quantity. In addition, Han et al. [15] considered the case where deception attacks are multiplicative gains. Subsequently, it was proposed that a leaderless consensus controller be employed under direct communication conditions, with the objective of ensuring that all closed-loop signals are bounded. To mitigate attacks on the actuator channels, Jin et al. [16] designed an adaptive control method for linear systems.

In addition to considering abnormal factors in the system (such as stochastic disturbances and deception attacks), the convergence speed of the system is also an important criterion in the design of controllers. Bhat and Bernstein [17] proposed a finite-time control method, which has stronger anti-disturbance capability and better robustness compared to asymptotic convergence controllers. Min et al. [18] studied the trajectory tracking problem based on finite-time control. Within a limited sensing range, Liao et al. [19] studied the finite-time consensus of second-order multi-agent systems, while Yang et al [20] investigated the finite-time formation control problem of multiple UAVs. The limitation of finite-time control is contingent upon the settling time, which is a function of the initial conditions of the system. In practical applications, particularly in larger systems, it is not feasible for users to obtain the specific initial conditions of the system. Therefore, Zhang et al. [21] proposed a fixed-time controller that ensures the closed-loop system reaches stability within a fixed time. Sun et al. [22] studied the timed control problem of Euler-Lagrange type multi agent systems under limited communication range. It is worth noting that although fixed-time control does not require obtaining the initial state of the system, it still relies on certain control parameters. The boundary estimation of the settling time is overly conservative, resulting in an estimated settling time that is significantly longer than the actual settling time. Due to the time-sensitive nature of certain tasks, higher control precision is required for the system to ensure that it reaches consensus within the user-prescribed time. Consequently, researchers in [23] devised a prescribed-time controller for nonlinear systems and constructed an adaptive fuzzy prescribed-time consensus protocol to preserve connectivity for nonlinear systems. In recent years, the security control of multi-UAV networks has emerged as a prominent research area within the field of control, garnering significant attention from the academic community. However, due to the unknown gain of deception attacks on the system, ensuring stable operation under such circumstances remains challenging.

Inspired by the aforementioned achievements, this paper designs a prescribed-time adaptive fuzzy controller for UAV swarm with nonlinear dynamics, considering external environmental noise and deception attacks. The principal contributions are as follows.

(1) Unlike [23], due to the inevitability of external noise, in addition to considering deception attacks on the system, the impact of environmental noise on the internal UAVs and the nonlinear dynamics within the UAVs are also considered.

(2) In order to address the unknown time-varying gain of deception attacks on the actuators, a new attack compensator has been designed with the objective of eliminating the impact of deception attacks, thereby ensuring the stable operation of the system in a network environment.

(3) By designing a controller based on the attack compensator, it is ensured that all signals in the stochastic nonlinear system are bounded under closed-loop conditions and can achieve precise tracking within the prescribed time.

The paper proceeds systematically to address the challenge of prescribed-time tracking control for UAV swarms under deception attacks. Section 2 introduces the graph-theoretic framework and formalizes the problem, including the stochastic UAV swarm dynamics and attack model. Building on this foundation, Section 3 develops the adaptive fuzzy control strategy, integrating a time-varying attack compensator, a state observer, and a backstepping-based prescribed-time convergence proof. Section 4 validates the theoretical results through numerical simulations, demonstrating the method's effectiveness in achieving secure tracking under adversarial conditions. The paper concludes in Section 5 with a discussion of contributions and future research directions, particularly in scaling the approach to heterogeneous swarm systems.

2 Preliminaries

2.1 Graph Theory

Consider a directed graph $\mathcal{G}=\{V,E,A\}$ containing $n$nodes, where $V=\{v_1,v_2,...,v_n\}$ represents the set of nodes and $\varepsilon\subseteq\{V\times V\}$ represents the set of edges. The matrix $A=[a_{ij}]$ is the weighted adjacency matrix of the graph $\mathcal{G}$. For a directed edge $\varepsilon_{ij}=(\mathrm{v}_j,\mathrm{v}_i)$, there exists a flow of information from UAV j to UAV i if and only if $\varepsilon_{ij}\in\varepsilon$. If $\varepsilon_{ij}\in\varepsilon$, then UAV $j$is considered a neighbor of UAV i. The set of all neighbors of UAV $i$is denoted by $N_i=\{\mathrm{v}_j\in V:\varepsilon_{ij}\in\varepsilon\}$. The adjacency matrix $A=[a_{ij}]$ is used to describe the communication weights between UAVs, where $a_{ij}>0$ indicates communication, otherwise $a_{ij}=0,a_{ii}=0$. The Laplacian matrix L of the graph G is defined as $L=D-A$, where $D=diag\{d_1,d_2,...,d_n\}$ and $d_n=\sum\limits_{j=1}^{n}a_{ij}$.

2.2 Problem Statement

In this paper, the following stochastic first-order UAV swarm is considered:

$$\begin{cases} dx_{i,k}=x_{i,k+1}+f_{i,k}(\bar{x}_{i,k})dt+g_{i,k}(\bar{x}_{i,k})dw, \\ dx_{i,n}=\upsilon_i+f_{i,n}(\bar{x}_{i,n})dt+g_{i,n}(\bar{x}_{i,n})dw, \\ \quad y_i=x_{i,1}, \quad 1\leq k\leq n-1, \end{cases} \tag{2.1}$$

where $\bar{x}_{i,k}=[x_{i,1},x_{i,2},...,x_{i,k}]^T\in R^k$ represents the state vector for $i=1,...,n$. The variables υ_i and y_i represent the system's input and output, respectively. Both $f_{i,k}(\cdot)$ and $g_{i,k}(\cdot)$ are unknown smooth nonlinear functions. The term w denotes a standard Brownian motion defined in a complete probability space.

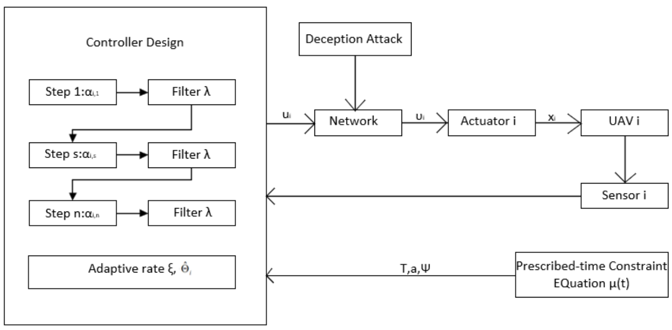

Figure 1: System Control Diagram

In practical networks, the system environment is intricate and complex. Therefore, it is assumed that each UAV actuator in the system is subject to different deception attacks, as illustrated in Figure 1.

Therefore, similar to the approach in reference [24], the deception attack on the actuator can be modeled as

$$\upsilon_i = B_i(t)u_i(t) + \mathcal{K}_i(t)\gamma_i(x_i), \tag{2.2}$$

where $u_i(t)$ is the designed controller. Both $B_i(t)$ and $\mathcal{K}_i(t)$ are unknown time-varying gains, and $\gamma_i(x_i)$ represents an unknown nonlinear function.

Assumption 1: There exist two unknown positive constants $\bar{B}$ and $\underline{B}$ such that $\bar{B} \leq |B_i(t)| \leq \underline{B}$.

Assumption 2: Let y_0 be the leader, and the overall system is controlled by adjusting the leader's trajectory. If the i-th UAV can directly access the information of y_0, then $\iota_i = 1$; otherwise, $\iota_i = 0$. Additionally, the directed graph $\mathcal{G}$ is balanced and weakly connected, with at least one UAV having direct access to all the information of y_0.

Let $F(X)$ be a continuous function defined on a compact set $\prod$. Then, there exists a fuzzy logic system (FLS) $\hat{F}(X) = \hat{W}^{*T}H(X)$ such that

$$FX = W^{*T}H(X) + \hbar.$$

Furthermore, we can obtain $\sup_{X\in\prod} |F(X) - W^{*T}H(X)| \leq \varepsilon$ with an approximation error $\varepsilon > 0$, and $W^* = \left[W_1^*, W_2^*, \ldots, W_m^*\right]$ represents the ideal weight vector. Additionally, $H(X) = \frac{[H_1,\ldots,H_n]^T}{\sum_{i=1}^{N} H_i}$ denotes the fuzzy basis function vector.

Lemma 1 [25]: The second-order sliding mode integral filter is designed as

$$\begin{cases} \dot{\lambda}_{p1} = -\frac{\lambda_{p1} - P(t)}{I_{p1}} - \frac{Q_{p1}(\lambda_{p1} - P(t))}{\lambda_{p1} - P(t) + B_{p1}}, \\ \dot{\lambda}_{p2} = -\frac{\lambda_{p1} - \dot{\lambda}_{p1}}{I_{p2}} - \frac{Q_{p2}(\lambda_{p2} - \dot{\lambda}_{p1})}{\lambda_{p2} - \dot{\lambda}_{p1} + B_{p2}}. \end{cases}$$

Lemma 2 [26]: For any vectors $x, y \in R^n$,there exists an inequality $xy \leq \frac{a^b}{b}|x|^b + \frac{1}{ca^c}|y|^c$, where $a > 0, b > 1, c > 1$, and $(b-1)(c-1) = 1$.

3 Main Results

3.1 Controller Design

To ensure that the system achieves the desired tracking accuracy within a limited time frame at a predetermined convergence rate, the following nonlinear transformation functions are introduced to obtain the actual prescribed-time tracking performance:

$$\mu(t) = \begin{cases} \frac{T^2 e^{\beta t}}{(1-\frac{\psi}{a})(T-t)^2 + \frac{\psi}{a}T^2 e^{\beta t}}, 0 \leq t < T \\ \frac{a}{\psi}, t \geq T \end{cases} \tag{3.1}$$

where the preset settling time T satisfies $T>0$, and the controller design parameter $\beta>0$ represents the convergence rate. Additionally, $a>\psi>0$ and ψ denotes the tracking accuracy. The function has the following properties:

(1) $\mu(t)$ is a bounded and differentiable function, and its derivative is also bounded.

(2) $\mu(t)$ is strictly increasing on the interval [0,T], with $\mu(0)=1$.When $t=T$, the function can approach its maximum value and maintain this constant.

At the same time, using adaptive fuzzy control, the following coordinate transformations are defined:

$$z_{i,1}=y_i-\iota_i y_0-(1-\iota_i)\hat{y}_{i,0},$$

$$\chi_1=\mu(t)z_{i,1}, \tag{3.2}$$

$$z_{i,s}=x_{i,s}-\alpha_{i,s-1}, s=2,...,n. \tag{3.3}$$

In this context, $\hat{y}_{i,0}$ represents the estimate of y_0, χ_1 denotes the virtual error, and α_i is the designed virtual controller.

Note 1: From the above nonlinear transformation of the error, it is required that when $t>T$, $|z_{i,1}|<\psi$, i.e., when $t>0, |\chi_1|<a$.

Step 1. First, by differentiating χ_1, we obtain

$$\begin{aligned}
d\chi_1 &= \mu dz_{i,1}+z_{i,1}d\mu\\
&= \mu d(y_i-\iota_i y_0-(1-\iota_i)\hat{y}_{i,0})+z_{i,1}\dot{\mu}dt\\
&= \Big[z_{i,1}\dot{\mu}+\mu(z_{i,2}+\alpha_{i,1}-f_{i,1}-\iota_1\dot{y}_0-(1-\iota_1)\hat{y}_0)\Big]dt+\mu g_{i,1}dw.
\end{aligned} \tag{3.4}$$

Consider the following candidate Lyapunov function:

$$V_1=\frac{1}{4}\ln\frac{a^4}{a^4-\chi_1^4}+\frac{1}{2m_1}\tilde{\xi}_1^2.$$

Here, m_1 is the design parameter, $\hat{\xi}_1$ represents the estimate of ξ_1, and $\tilde{\xi}_1=\xi_1-\hat{\xi}_1$ denotes the approximation error. By applying $It\hat{o}'s$ lemma and equations (3.3) and (3.4), we obtain

$$\mathcal{L}V_1=\frac{\chi_1^3}{a^4-\chi_1^4}[z_1\dot{\mu}+\mu(z_{i,2}+\alpha_{i,1}+f_{i,1}-\iota_1\dot{y}_0-(1-\iota_1)\dot{\hat{y}}_{i,0}]+\frac{\chi_1^3}{a^4-\chi_1^4}\mu^2 g_{i,1}^T g_{i,1}-\frac{1}{m_1}\tilde{\xi}_1\dot{\hat{\xi}}_1$$

By applying Lemma 2, the terms in the equation satisfy the following conditions:

$$\frac{\chi_1^3}{a^4-\chi_1^4}\mu z_{i,2}\le\frac{3\chi_1^4\mu}{4(a^4-\chi_1^4)^{\frac{4}{3}}}+\frac{1}{4}\mu z_{i,2}, \tag{3.5}$$

$$\frac{\chi_1^3(3a^4+\chi_1^4)}{2(a^4-\chi_1^4)^2}\mu g_{i,1}^T g_{i,1}\le\frac{\chi_1^4(3a^4+\chi_1^4)^2}{8(a^4-\chi_1^4)^{\frac{4}{3}}}\mu^4 {g_{i,1}}^4+\frac{1}{2}.$$

Assuming $\bar{f}_1$ is an unknown continuous function, it is approximated by the fuzzy logic system (FLS) as $\bar{f}_1=\mu f_{i,1}+\frac{\chi_1(3a^4+\chi_1^4)^2}{8(a^4-\chi_1^4)^3}\mu^4 g_{i,1}{}^4$, and $\xi_1=\|W_1^*\|^2$. The design parameters b_1 and ε_1^* satisfy $\varepsilon_1^*\geq\|\varepsilon_1\|$. Therefore, we can obtain

$$
\begin{aligned}
a^4-\chi_1^4\bar{f}_1&=\frac{\chi_1^3}{a^4-\chi_1^4}\left(W_1^{*T}F_1(x_{i,1})+\varepsilon_1(x_{i,1})\right)\\
&\leq\frac{1}{2b_1^2}\left(\frac{\chi_1^3}{a^4-\chi_1^4}\right)^2\|W_1^*\|^2F_1^TF_1+\frac{1}{2}b_1^2+\frac{1}{2}\left(\frac{\chi_1^3}{a^4-\chi_1^4}\right)^2+\frac{1}{2}\varepsilon_1^{*2}\\
&\leq\frac{1}{2b_1^2}\left(\frac{\chi_1^3}{a^4-\chi_1^4}\right)^2\xi_1F_1^TF+\frac{1}{2}\left(\frac{\chi_1^3}{a^4-\chi_1^4}\right)^2+\frac{1}{2}(b_1^2+\varepsilon_1^{*2}).
\end{aligned}
$$

Then, design the first-order filter $\alpha_{i,1}$ as:

$$
\begin{aligned}
\alpha_{i,1}=&-k_1z_{i,1}+\iota_1\dot{y}_0+(1-\iota_1)\dot{\hat{y}}_{i,0}-\frac{1}{\mu}z_{i1}\dot{\mu}\\
&-\frac{1}{2\mu}\frac{\chi_1^3}{a^4-\chi_1^4}-\frac{3\chi_1}{4(a^4-\chi_1^4)^{1/3}}-\frac{1}{\mu}\frac{1}{2b_1^2}\frac{\chi_1^3}{a^4-\chi_1^4}\hat{\xi}_1F_1^TF_1,
\end{aligned}
\tag{3.6}
$$

where k_1 is a system design parameter. Substituting it into equation (3.5), we obtain

$$
\mathcal{L}V_1\leq\frac{\tilde{\xi}_1}{m_1}\left(\frac{m_1}{2b_1^2}\frac{\chi_1^6}{(a^4-\chi_1^4)^2}F_1^TF_1-\dot{\hat{\xi}}_1\right)+\frac{1}{2}(1+b_1^2+\varepsilon_1^{*2}+\frac{1}{2}\mu z_{i,2}^4)-k_1\frac{\chi_1^4}{a^4-\chi_1^4}.
$$

Thus, the adaptive controller can be designed as

$$
\dot{\hat{\xi}}_1=\frac{m_1}{2b_1^2}\frac{\chi_1^6}{(a^4-\chi_1^4)^2}F_1^TF_1-\eta_1\hat{\xi}_1,
\tag{3.7}
$$

where η_1 is a positive definite design parameter. Thus, we further obtain

$$
\mathcal{L}V_1\leq-k_1\frac{\chi_1^4}{a^4-\chi_1^4}+\frac{\eta_1}{m_1}\tilde{\xi}_1\hat{\xi}_1+\frac{1}{4}\mu z_{i,2}^4+d_1,
\tag{3.8}
$$

where $d_1=\frac{1}{2}(1+b_1^2+\varepsilon_1^{*2})$.

Step 2. From equation (3.3), we can obtain

$$
z_{i,2}=x_{i,2}-\alpha_{i,1},
$$

Complexity explosion refers to the phenomenon where, in dealing with certain complex systems or problems, the difficulty of computation and analysis increases sharply as the system scale or the number of variables increases. To avoid the problem of complexity explosion, according to Lemma 1, a second-order sliding mode integral filter is introduced:

$$
\begin{cases}
\dot{\lambda}_{21}=-\dfrac{\lambda_{21}-\alpha_{i,1}}{I_{21}}-\dfrac{Q_{21}(\lambda_{21}-\alpha_{i,1})}{\|\lambda_{21}-\alpha_{i,1}\|+B_{21}},\\
\dot{\lambda}_{22}=-\dfrac{\lambda_{21}-\dot{\lambda}_{21}}{I_{22}}-\dfrac{Q_{22}(\lambda_{21}-\dot{\lambda}_{21})}{\|\lambda_{21}-\dot{\lambda}_{21}\|+B_{22}}.
\end{cases}
$$

$$dz_{i,2} = (z_{i,3} + \alpha_{i,2} + f_2 - L\alpha_{i,1})dt + \left(g_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_1} g_{i,1}\right)^T dw.$$

At the same time, the virtual signal is represented as $L\alpha_{i,1} = \lambda_{22} - \lambda_{r_1}$, where λ_{r_1} is the error estimate of the second-order sliding mode filter, and there exists $\lambda_{r_1}^*$ such that $\lambda_{r_1} \leq \lambda_{r_1}^*$. Next, we choose the Lyapunov function:

$$V_2 = V_1 + \frac{1}{4} z_{i,2}^4 + \frac{1}{2m_2} \tilde{\xi}_2^2.$$

Similar to equation (3.8), m_2 is a positive definite design parameter. Let $G_2 = g_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_1} g_{i,1}$. By applying Lemma 4, we obtain

$$\begin{aligned} \mathcal{L}V_2 = & -k_1 \frac{\chi_1^4}{a^4 - \chi_1^4} + \frac{\eta_1}{m_1} \tilde{\xi}_1 \hat{\xi}_1 + d_1 + \frac{3}{2} z_{i,2}^2 G_2^T G_2 \\ & + \frac{1}{4} \mu z_{i,2}^4 + z_{i,2}^3 (z_{i,3} + f_{i,2} + \alpha_{i,2} - \lambda_{22} + \lambda_{r_1}) - \frac{1}{m_2} \tilde{\xi}_2 \dot{\hat{\xi}}_2. \end{aligned} \tag{3.9}$$

By applying Young's inequality, we obtain

$$z_{i,2}^3 z_{i,3} \leq \frac{3}{4} z_{i,2}^4 + \frac{1}{4} z_{i,3}^4, \tag{3.10}$$

$$z_{i,2}^3 \lambda_{r_1} \leq \frac{3}{4} z_{i,2}^4 + \frac{1}{4} \lambda_{r_1}^{*4}, \tag{3.11}$$

$$\frac{3}{2} z_{i,2}^2 G_2^T G_2 \leq \frac{9}{8} z_{i,2}^4 \|G_2\|^4 + \frac{1}{2}. \tag{3.12}$$

Similar to equation (3.6), we select $\bar{f}_2$ and approximate it using FLS as $\bar{f}_2 = f_{i,2} + \frac{9}{8} z_{i,2} \|G_2\|^4$. From this, we can further obtain

$$\begin{aligned} z_{i,2}^3 \bar{f}_2 &= z_{i,2}^3 (W_2^{*T} F_2 + \varepsilon_2) \\ &\leq \frac{1}{2b_2^2} z_{i,2}^6 \xi_2 F_2^T F_2 + \frac{1}{2} b_2^2 + \frac{1}{2} z_{i,2}^6 + \frac{1}{2} \varepsilon_2^{*2}. \end{aligned} \tag{3.13}$$

Here, b_2 is the design parameter, ε_2 represents the bounded approximation error, and it satisfies $\varepsilon_2^* \geq \|\varepsilon_2\|$. Furthermore, we construct the adaptive controller $\dot{\hat{\xi}}_2$ and the virtual controller $\alpha_{i,2}$.

$$\alpha_{i,2} = -k_2 z_{i,2} - \frac{1}{4} \mu z_{i,2} - \frac{3}{2} z_{i,2} + \lambda_{22} - \frac{1}{2} z_{i,2}^3 - \frac{1}{2b_2^2} z_{i,2}^3 \hat{\xi}_2 F_2^T F_2, \tag{3.14}$$

$$\dot{\hat{\xi}}_2 = \frac{m_2}{2b_2^2} z_{i,2}^6 F_2^T F_2 - \eta_2 \hat{\xi}_2. \tag{3.15}$$

The design parameters k_2 and η_2 are required to be positive.

From equations (3.9) to (3.15), we obtain

$$\mathcal{L}V_2 \le -k_1\frac{\chi_1^4}{a^4-\chi_1^4} - k_2 z_{i,2}^4 + \frac{1}{4}z_{i,3}^4 + \frac{1}{4}\varepsilon_2^{*2} + d_2 + \sum_{i=1}^{2}\frac{\eta_i}{m_i}\tilde{\xi}_i\hat{\xi}_i,$$

where $d_2 = \frac{1}{2}i + \sum_{i=1}^{2}(\frac{b_i^2}{2} + \frac{\varepsilon_i^{*2}}{2})$.

Step 3 ($2 < s \le n-1$). By applying the coordinate transformation (3.2), we obtain

$$z_{i,s} = x_{i,s} - \alpha_{i,s-1}, \tag{3.16}$$

$$dz_{i,s} = (z_{i,s+1} + \alpha_{i,s} + f_{i,s} - L\alpha_{i,s-1})dt + (g_{i,s} - \sum_{j=1}^{s-1}\frac{\partial\alpha_{i,j}}{\partial x_j}g_{i,j})^T dw. \tag{3.17}$$

Furthermore, the virtual controller $\alpha_{i,s-1}$ is designed as

$$L\alpha_{i,s-1} = \lambda_{s-1,2} - \lambda_{r_{s-1}}, \tag{3.18}$$

the integral filter $\lambda_{r_{s-1}}$ is required to satisfy $\lambda_{r_{s-1}} \le \lambda_{r_{s-1}}^*$, and $\lambda_{r_{s-1}}^*$ must be a positive number. We select the candidate Lyapunov function as

$$V_s = V_{s-1} + \frac{1}{4}z_{i,s}^4 + \frac{1}{2m_s}\tilde{\xi}_s^2. \tag{3.19}$$

In this equation, $\tilde{\xi}_s = \xi_s - \hat{\xi}_s$ represents the estimate of ξ_s, and m_s is a positive definite design parameter. Combining equations (3.16) to (3.19), we obtain

$$\begin{aligned}\mathcal{L}V_s \le & -k_1\frac{\chi_1^4}{a^4-\chi_1^4} - \sum_{j=2}^{s-1}k_j z_{i,j}^4 + \frac{1}{4}z_{i,s}^4 + d_{s-1} + \sum_{i=1}^{s-1}\frac{\eta_i}{m_i}\tilde{\xi}_i\hat{\xi}_i + \sum_{i=1}^{s-2}\frac{1}{4}\varepsilon_{r_i}^{*4} \\ & + z_{i,s}^3(z_{i,s+1} + f_{i,s} + \alpha_{i,s} + \lambda_{r_{s-1}} - \lambda_{s,2}) + \frac{3}{2}z_{i,s}^2 G_s^T G_s - \frac{1}{m}\tilde{\xi}_s\dot{\hat{\xi}}_s,\end{aligned} \tag{3.20}$$

where $G_s = g_{i,s} - \sum_{j=1}^{s-1}\frac{\partial\alpha_{i,j}}{\partial x_{i,j}}g_{i,j}$. Similar to equations (3.11) to (3.14):

$$\begin{aligned} z_{i,s}^3 z_{i,s+1} &\le \frac{3}{4}z_{i,s}^4 + \frac{1}{4}z_{i,s+1}^4, \\ z_{i,s}^3\lambda_{r_{s-1}} &\le \frac{3}{4}z_{i,s}^4 + \frac{1}{4}\lambda_{r_{s-1}}^{*4}, \\ \frac{3}{2}z_{i,s}^2 G_s^T G_s &\le \frac{9}{8}z_{i,s}^4\|G_s\|^4 + \frac{1}{2}. \end{aligned} \tag{3.21}$$

Similarly, by approximating the unknown function $\bar{f}_s = \frac{9}{8}z_{i,s}\|G_s\|^4 + f_{i,s}$ using FLS, and similar to equation (3.13), we obtain

$$z_{i,s}^3\bar{f}_s = z_{i,s}^3(W_s^{*T}F_s + \varepsilon_s) \le \frac{1}{2b_s^2}z_{i,s}^6\xi_s F_s^T F_s + \frac{1}{2}b_s^2 + \frac{1}{2}z_{i,s}^6 + \frac{1}{2}\varepsilon_s^{*2}. \tag{3.22}$$

Here, b_s is a positive definite design parameter. Furthermore, the virtual controller is designed as

$$\alpha_{i,s}=-k_s z_{i,s}-\frac{7}{4}z_{i,s}-\frac{1}{2}z_{i,s}^3-\frac{1}{2b_s^2}z_{i,s}^3\hat{\xi}_s F_s^T F_s+\lambda_{s2}, \tag{3.23}$$

and construct the adaptive control rate $\dot{\hat{\xi}}_s$ as

$$\dot{\hat{\xi}}_s=\frac{m_s}{2b_s^2}z_{i,s}^6 F_s^T F_s-\eta_s\hat{\xi}_s. \tag{3.24}$$

Here, k_s and η_s are design parameters, and both are positive numbers. By combining equations (3.20) to (3.24), we obtain

$$\mathcal{L}V_s\le -k_1\frac{\chi_1^4}{a^4-\chi_1^4}-\sum_{j=2}^{s}k_j z_{i,j}^4+\frac{1}{4}z_{i,s+1}^4+d_s+\sum_{i=1}^{s}\frac{\eta_i}{m_i}\tilde{\xi}_i\hat{\xi}_i+\sum_{i=1}^{s-1}\frac{1}{4}\varepsilon_{r_i}^{*4},$$

where $d_s=d_{i-1}+\frac{1}{2}+\frac{b_i^2}{2}+\frac{\varepsilon_i^{*2}}{2}$.

Step 4 ($s=n$). Considering the coordinate transformation, we have:

$$z_{i,n}=x_{i,n}-\alpha_{i,n-1},$$

$$dz_{i,n}=(\upsilon_i+f_{i,s}-L\alpha_{i,s-1})dt+\left(g_{i,n}-\sum_{j=1}^{n-1}\frac{\partial\alpha_{i,j}}{\partial x_j}g_{i,j}\right)^T dw.$$

Similarly, the virtual controller is constructed in the following form:

$$L\alpha_{i,n-1}=\lambda_{n2}-\lambda_{r_{n-1}}, \tag{3.25}$$

the estimation error of the second-order sliding mode filter is $\lambda_{r_{n-1}}$, and it satisfies $\lambda_{r_{n-1}}\le\lambda_{r_{n-1}}^*$. From equation (2.2), we know that the system is subject to a deception attack $\upsilon_i=B_i(t)u_i(t)+\mathcal{K}_i(t)\gamma_i(x_i)$. There exists a constant $\bar{\mathcal{K}}_i$ such that $\bar{\mathcal{K}}_i\ge\mathcal{K}_i(t)$.

$$\begin{aligned} z_{i,n}^3\mathcal{K}_i(t)\gamma_i(x_i)&\le\bar{\mathcal{K}}_i z_{i,n}^3(W_\phi^{*T}F_\phi+\varepsilon_\phi)\\ &\le\frac{1}{2b_\phi^2}z_{i,n}^6\xi_\phi F_\phi^T F_\phi+\frac{1}{2}b_\phi^2+\frac{1}{2}z_{i,n}^6+\frac{1}{2}\varepsilon_\phi^{*2}. \end{aligned} \tag{3.26}$$

Further, we make the following provisions:

$$Q_i=\inf_{t\ge0}|b_i(t)|>0,\Theta_i=\frac{1}{Q_i}. \tag{3.27}$$

Here, Θ_i is an unknown parameter. We design a new attack compensator Ξ_i, as well as a controller u_i based on the attack compensator:

$$\Xi_i=\frac{1}{2b_\phi^2}z_{i,n}^3\xi_\phi F_\phi^T F_\phi+\frac{1}{2b_n^2}z_{i,n}^3\xi_n F_n^T F_n+z_{i,n}^3-\lambda_{n2}+z_{i,n},$$

$$u_i = -\frac{\Xi_i^2 \hat{\Theta} i^2}{\sqrt{\Xi_i^2 \hat{\Theta} i^2 + \sigma^2}}, \tag{3.28}$$

where σ is a positive definite design parameter. We select the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{4} z_{i,n}^4 + \frac{1}{2m_n} \tilde{\xi}_n^2 + \frac{Q_i}{2} \tilde{\Theta}_i^2.$$

In this equation, $\tilde{\xi}_n = \xi_n - \hat{\xi}_n$, and $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$. Additionally, by applying $It\hat{o}'s$ lemma, we obtain

$$\mathcal{L}V_n \leq -k_1 \frac{\chi_1^4}{a^4 - \chi_1^4} - \sum_{j=2}^{n-1} k_j z_{i,j}^4 + \frac{1}{4} z_{i,n}^4 + d_{n-1} + \sum_{i=1}^{n-1} \frac{\eta_i}{m_i} \tilde{\xi}_i \hat{\xi}_i + \sum_{i=1}^{s-2} \frac{1}{4} \varepsilon_{r_i}^{*4}$$
$$+ z_{i,n}^3 (\upsilon_i + f_{i,n} - \lambda_{n2} + \lambda_{r_{n-1}}) + \frac{3}{2} z_{i,n}^2 G_n^T G_n - \frac{1}{m_n} \tilde{\xi}_n \dot{\hat{\xi}}_n - Q_i \tilde{\Theta}_i \dot{\hat{\Theta}}_i,$$

where $G_n = g_{i,n} - \sum\limits_{j=1}^{n-1} \frac{\partial \alpha_{i,j}}{\partial x_{i,j}} g_{i,j}$. Similar to equations (3.16) to (3.25), by applying Young's inequality, we obtain

$$z_{i,s}^3 \lambda_{r_{n-1}} \leq \frac{3}{4} z_{i,n}^4 + \frac{1}{4} \lambda_{r_{n-1}}^{*4},$$
$$\frac{3}{2} z_{i,n}^2 G_n^T G_n \leq \frac{9}{8} z_{i,n}^4 \|G_n\|^4 + \frac{1}{2},$$

$$z_{i,n}^3 \bar{f}_n = z_{i,n}^3 (W_n^{*T} F_n + \varepsilon_n) \leq \frac{1}{2b_n^2} z_{i,n}^6 \xi_n F_n^T F_n + \frac{1}{2} b_n^2 + \frac{1}{2} z_{i,n}^6 + \frac{1}{2} \varepsilon_n^{*2}.$$

By approximating the unknown function

$$\bar{f}_n = \frac{9}{8} z_{i,n} \|G_n\|^4 + f_{i,n},$$

using FLS, we define the adaptive rates $\dot{\hat{\xi}}_n$ and $\dot{\hat{\Theta}}_i$ as

$$\dot{\hat{\Theta}}_i = z_{i,n}^3 \Xi_i - \sigma \hat{\Theta}_i, \dot{\hat{\xi}}_n = \frac{m_n}{2b_n^2} z_{i,n}^6 F_n^T F_n - \eta_n \hat{\xi}_n. \tag{3.29}$$

By applying Young's inequality, we obtain

$$Q_i \sigma \tilde{\Theta}_i \hat{\Theta}_i \leq -\frac{1}{2} Q_i \sigma \tilde{\Theta}_i^2 + \frac{1}{2} Q_i \sigma \Theta_i^2.$$

By combining equations (3.26) to (3.29), we obtain

$$\mathcal{L}V_n \leq -k_1 \frac{\chi_1^4}{a^4 - \chi_1^4} - \sum_{j=2}^{n} k_j z_{i,j}^4 + \sum_{i=1}^{n} \frac{\eta_i}{m_i} \tilde{\xi}_i \hat{\xi}_i + d + \sum_{i=1}^{n-1} \frac{1}{4} \beta_{ri}^{*4} - \frac{1}{2} Q_i \sigma \hat{\Theta}_i^2, \tag{3.30}$$

$$d = d_{n-1} + \frac{1}{2} + \frac{b_i^2}{2} + \frac{\varepsilon_i^{*2}}{2} + \frac{b_\phi^2}{2} + \frac{\varepsilon_\phi^{*2}}{2} \frac{1}{2} Q_i \sigma \Theta_i^2.$$

3.2 Stability Analysis

Theorem 3.1. *For the stochastic system* (2.1)*, by designing the adaptive laws* (3.7)*,* (3.15)*,* (3.24)*, and* (3.29)*, the virtual controllers* (3.6)*,* (3.14)*, and* (3.23)*, and the controller* (3.28)*, it can be shown that all signals in the closed-loop system are bounded, and the tracking error can reach the predetermined accuracy set within a prescribed-time.*

Proof. By applying the quadratic formula, we obtain $\tilde{\xi}_i\hat{\xi}_i \leq \frac{1}{2}\xi_i^2 - \frac{1}{2}\tilde{\xi}_i^2$, inequality (3.30) can be further written as

$$\mathcal{L}V_n \leq -k_1\frac{\chi_1^4}{a^4-\chi_1^4} - \sum_{j=2}^{n} k_j z_{i,j}^4 - \sum_{i=1}^{n}\frac{\eta_i}{2m_i}\tilde{\xi}_i^2 + \sum_{i=1}^{n}\frac{\eta_i}{2m_i}\xi_i^2 + d + \sum_{i=1}^{n-1}\frac{1}{4}\lambda_{ri}^{*4} - \frac{1}{2}Q_i\tilde{\Theta}_i^2.$$

At the same time, by selecting $\Gamma = \min\{4k_1, 4k_2, ..., 4k_n, \eta_1, ..., \eta_n, \sigma\}$, $\Omega = \sum_{i=1}^{n}\frac{\eta_i}{2m_i}\xi_i^2 + d + \sum_{i=1}^{n-1}\frac{1}{4}\lambda_{ri}^{*4}$, we obtain

$$\mathcal{L}V_n \leq -\Gamma V + \Omega.$$

It satisfies $E(V(t)) \leq (V(0) - \frac{\Omega_0}{\Gamma_0})e^{-\Gamma_0 t} + \frac{\Omega_0}{\Gamma_0}$, indicating that for $i = 1, 2, ..., n, y_{i,0}\Theta_i, \xi_s, \varepsilon_s$are bounded. Furthermore, it can be shown that $\alpha_{i,s}$ is also a bounded function, and thus $x_{i,s}$ is bounded as well. In summary, all signals in the closed-loop system are bounded.

By applying the coordinate transformations (3.1) and (3.2), considering that $z_2, ..., z_n, \tilde{\xi}_1, ..., \tilde{\xi}_n$ are bounded, we can conclude that

$$|z_1| = \mu_1^{-1}(t)|\chi_1| = \begin{cases} \psi + (a-\psi)(T-t)^2, & 0 \leq t < T, \\ \psi, & t \geq T. \end{cases}$$

Therefore, it can be concluded that within the predetermined settling time, the error variable z_1 converges to the predefined region $\{z_1 \in |z_1| < \psi\}$, demonstrating its tracking performance. □

Remark 3.1. While the proposed method demonstrates robust control performance and reduced communication overhead, the employment of time-varying impulsive signals may require additional computational resources. This trade-off between computational load and performance efficiency is an important consideration in real-time implementations, especially for resource-constrained systems.

4 Simulation Example

In this section, the effectiveness of the designed controller for the studied multi-agent system is verified through numerical simulations. Therefore, a nonlinear system with four agents is selected:

$$\begin{cases} dx_{i,1} = x_{i,2} + \sin(x_{i,2})\cos(x_{i,1}))dt + \frac{\cos(x_{i,1})}{10}dw, \\ dx_{i,2} = x_{i,3} + \sin(x_{i,2})\cos(x_{i,3}))dt + \frac{\sin(x_{i,2})}{10}dw, \\ dx_{i,3} = x_{i,4} + \sin(x_{i,4})\cos(x_{i,3}))dt + \frac{\cos(x_{i,3})}{10}dw, \\ dx_{i,4} = \upsilon_i + \cos(x_{i,4})\sin(x_{i,4}))dt + \frac{\sin(x_{i,4})}{10}dw, \\ y_i = x_{i,1}, \end{cases}$$

Here, x represents the state of the agent. The initial state of the system is set to $\beta = 0.9, \psi = 0.1, a = 2, m_1 = 1, b_1 = 1.1, k_1 = 1.51, \eta_1 = 0.1, Q_{21} = Q_{22} = 2, B_{21} = 4.5, B_{22} = 3$, $T = 3$, and the reference signal is set along the line $y_0 = 0$.The actuator deception attack parameters are chosen as $\upsilon_i = b_i(t)u_i(t) + \mathcal{K}_i(t)\gamma_i(x_i), b_1(t) = 1 - 0.1\sin(t), \mathcal{K}_i(t) = 0.1\cos(t), \gamma_1(x_1) = 0.1x_1e^{x_1}$. Based on the design method of Theorem 1, the simulation results of the system are shown in Figures 2-6.

5 Conclusion

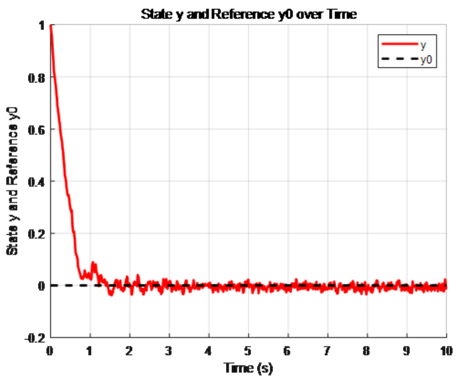

Figure 2: Trajectory of System State y and Reference State y_0 Over Time

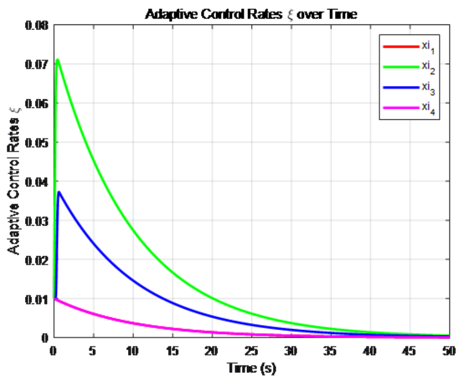

Figure 3: Trajectory of Adaptive Control Parameters ξ

This paper explores the prescribed-time fuzzy adaptive control of multi-agent systems affected by stochastic noise. Unlike previous works, it considers the nonlinear dynamics within agents and the complex network environment where each system actuator is subjected to different deception attacks. Based on backstepping techniques, combined with sliding mode filters and adaptive controllers, a time-varying constraint function is introduced to eliminate the impact of deception attacks on system actuators using attack compensators. For stochastic nonlinear systems, a controller based on attack compensators is designed, ensuring prescribed-time tracking performance and that all signals in the closed-loop system are bounded. Finally, the theoretical results are validated through simulation examples. Currently, there are still many important issues to be studied in the prescribed-time fuzzy adaptive control of stochastic nonlinear systems. Future research could extend the method to handle multiple types of cyber-attacks, adapt to dynamic environments, and support heterogeneous UAV swarms. Exploring the scalability of the method in large-scale systems also remains an important direction for future work.

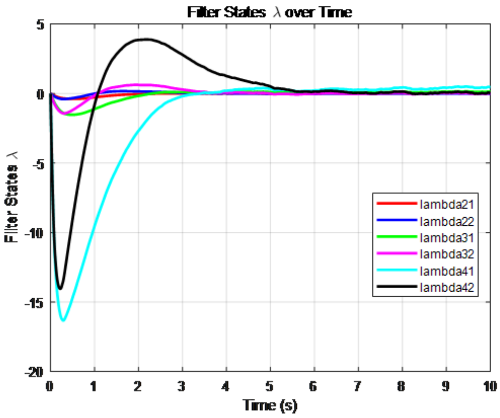

Figure 4: Trajectory of the Filter λ

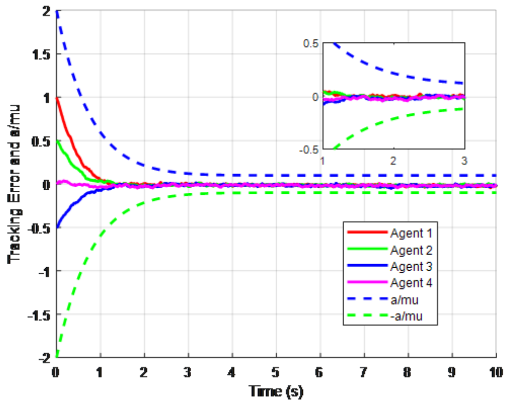

Figure 5: Trajectory Tracking and Constraint Boundaries

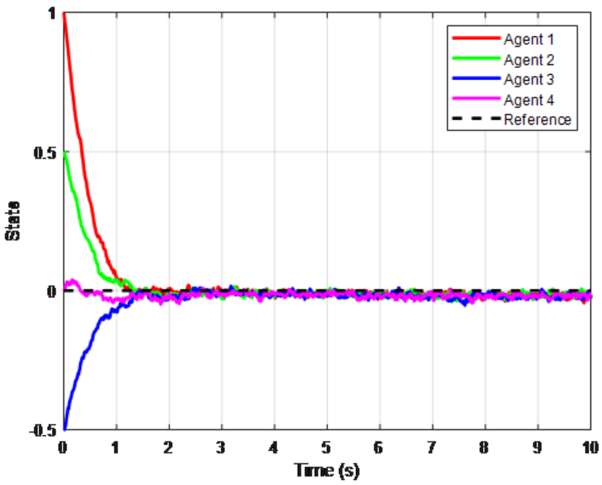

Figure 6: State Trajectories of Each Agent under the Controller

Figure 2 shows the system trajectory y and the reference signal y_0. It can be visually observed that the system output y can track the reference signal well and achieve trajectory tracking within the prescribed-time.Figure 3 presents the curves of the adaptive controllers ξ_1,ξ_2,ξ_3,ξ_4. Figure 4 displays the integral filter λ_{21}. Figure 5 shows the error variable curve of the system, clearly indicating that the overall system does not exceed the range of $\frac{a}{\mu}$. Figure 6 illustrates the state trajectories of each agent and the reference signal in an unknown attack environment using the attack compensation controller. By analyzing the above simulation results, it can be demonstrated that in a noisy environment, with actuators subjected to different deception attacks, the use of the prescribed-time attack compensation controller can ensure the stable operation of the system.

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