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Optimization of Current Carrying Muticables using Topological and Shape Sensitivity

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Abstract. In this paper, we use the topological and shape gradient framework, to optimize a current carrying multicables. The geometry of the multicables is modeled as a coated inclusions with different conductivities and the problem we are interested is the location of the inclusions to get a suitable thermal environnent. We solve numerically the optimization problem using topological and shape gradient strategy. Finally, we present some numerical experiments.

AMS subject classifications: 49Q10, 49Q12, 65K10, 68W25.

Key words: Electrical multicables, heat conduction, shape optimization, topology optimization.

1 Introduction

In modern electrical machines like hybrid and electrical cars, manufacturers reduce cable diameters to save material, space and weight. But smaller diameters of the electrical cables result a higher temperatures in the connecting structures. This may cause overheating and irreparable damages of the machines. The heat transfer in the current carrying multi-cables depend on the position of the cables. Therefore finding the positions of cables which lead to the minimal temperature is of interest.

In the sequel, we will use the following notations:

The k-th single cable $C_k=(x_k,y_k,r_k^i,r_k^e)$ is described by its center (x_k,y_k), the radius of the current carrying part r_k^i and the outer radius r_k^e. It is surrounded by insulation part with thickness $r_k^e-r_k^i$ and boundaries Γ_k^i and Γ_k^e .

The multi-cable $MC=((x_0,y_0,r_0^i,r_0^e),C_1\dots C_N)$ consists of N single cables, has the center coordinates (x_0,y_0), the inner radius r_0^i and the outer radius r_0^e. It is surrounded by

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insulation layer with thickness $r_0^e - r_0^i$ and conductivity σ^e. Each single cable consists of a core part Ω_k^{core} with heat conductivity σ_k^{core}, carrying the current I_k, and an insulation part Ω_k^{iso} with heat conductivity σ_k^{iso}. The gaps between the single cables and the exterior insulation can be of solid material or air. They are modeled by pure conduction with heat conductivity σ^{gap}.

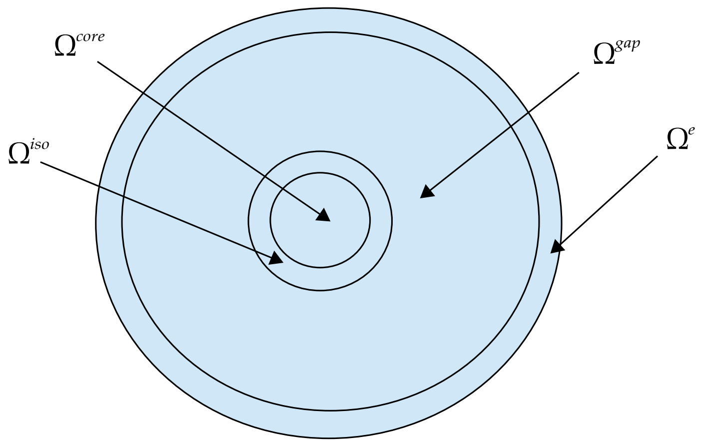

Figure 1: The cross section of a single cable.

The temperature distribution is described by the following Helmoltz equation :

$$\begin{cases} -\mathrm{div}(\sigma\nabla u) - cu = f & \text{in } \Omega\setminus\Gamma^i, \\ \sigma_e \partial_n u + \alpha(u)(u-u_{amb}) = 0 & \text{on } \Gamma^e, \\ [\![u]\!] = 0 & \text{on } \Gamma^i, \\ [\![\sigma\partial_n u]\!] = 0 & \text{on } \Gamma^i, \end{cases} \tag{1.1}$$

where $\Omega = \cup_{k=1}^N (\Omega_k^{core} \cup \Omega_k^{iso}) \cup \Omega^{gap} \cup \Omega^e$ is the two-dimensional cross section of the multi-cable with regular exterior boundary $\partial\Omega = \Gamma^e$ and interface boundaries $\Gamma^i = \bigcup_{k=1}^N (\Gamma_k^e \cup \Gamma_k^i) \cup \Gamma^{gi}$, Γ^{gi} represents the interface between the exterior insulation and the gaps, $\Gamma_k^i = \partial\Omega_k^{core}$ and Γ_k^e represents the external boundary of Ω_k^{iso}.

The heat conductivity σ, the linear temperature coefficient c and the source term f are given respectively by

$$\sigma := \sum_{k=1}^N \left(\sigma_k^{core} \mathbb{1}_{\Omega_k^{core}} + \sigma_k^{iso} \mathbb{1}_{\Omega_k^{iso}} \right) + \sigma^e \mathbb{1}_{\Omega^e} + \sigma^{gap} \mathbb{1}_{\Omega^{gap}}, \tag{1.2}$$

$$c := \sum_{k=1}^N \frac{1}{n_k} \left(\frac{4 I_k}{d_k^{in} \delta_k \pi} \right)^2 \rho_{0,k} \alpha_{\rho,k} \mathbb{1}_{\Omega_k^{core}}, \tag{1.3}$$