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The Mathematical Work of S.-Y. Alice Chang and Paul C. Yang

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Dedicated to Professors Sun-Yung Alice Chang and Paul C. Yang on their 70th birthdays

1 Introduction

Writing an article about the mathematical work of Alice Chang and Paul Yang is simultaneously a great honor and an almost hopeless task. How does one begin to summarize the work of two accomplished mathematicians, whose combined careers span almost a century? First, there is the sheer volume of output: since receiving their PhDs in the early 1970s, Paul and Alice have each published over 100 articles (about half on which they were co-authors). Then there is the diversity of fields: complex geometry, nonlinear PDEs, spectral geometry, calculus of variations, conformal geometry, CR geometry, functional inequalities, etc. And finally, there are the many students and postdocs Paul and Alice have mentored over the years (the author of this article included). Consequently, even a cursory overview could easily fill several issues of the *Journal of Mathematical Study*.

However, I also know that Paul and Alice would vastly prefer reading research articles about topics that interest them, rather than a lengthy paean in their honor. I will therefore turn necessity into a virtue, and attempt to distill two rich mathematical careers into a few pages. There will necessarily be gaps; some research topics I will barely touch, while for others I will try to give enough context in order to better appreciate their contributions. It also behooves me to point out that this is a provisional survey, since Paul and Alice remain very active!

I will proceed chronologically rather than topically, and since Paul is more senior, I will begin with him. Paul (Chien-Ping) Yang was born in 1947, in Changhua, Taiwan. His family immigrated to the United States in 1962, and settled in San Francisco. Paul did his undergraduate and graduate work at UC Berkeley, receiving his PhD in 1973 under

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the direction of Hung-Hsi Wu. From 1974-1976 he was an Evans Instructor at Rice, the first stop on a peripatetic early career that brought him to Maryland, Stanford and IU-Bloomington before arriving at the University of Southern California in 1982. He was at USC almost 20 years, until 2001 when he joined the faculty at Princeton.

Paul's early work was in complex geometry. His first paper [53], which appeared in *Duke Math. J.*, studied complete Kähler metrics with non-positive holomorphic bisectional curvature, a problem that would occupy his interest for some time. In a subsequent *Inventiones* paper co-authored with Y. T. Siu [51], "Compact Kähler-Einstein surfaces of nonpositive bisectional curvature", they conjectured that every compact Kähler-Einstein surface with negative sectional curvature is biholomorphic to a compact quotient of the complex 2-ball. They were able to prove the conjecture assuming a kind of pinching condition on the holomorphic sectional curvatures (which, interestingly, permits non-positive holomorphic bisectional curvature, as opposed to strictly negative).

One of Paul's other papers from this period prefigured some of his later research interests. In "Local boundary regularity of holomorphic mappings" (*Comm. Pure Appl. Math*), joint with S. Webster and L. Nirenberg [48], they gave a simplified proof of Fefferman's result that a biholomorphic map of strictly pseudoconvex domains extends smoothly to the boundaries. Relating the (real) geometry of the boundary of a complex domain to function theory in the domain is one of the origins of CR geometry, a field to which Paul would make beautiful contributions later in his career (as we will see below). On the basis of his work during this period Paul was awarded a Sloan Fellowship in 1981.

Sun-Yung Alice Chang was born in Xi'an, China, in 1948, and grew up in Taiwan. After completing her undergraduate work at National Taiwan University, she came to the US in 1970 at attend UC Berkeley for graduate school. It was during their time at Berkeley that Paul and Alice met, and were married in 1973. Alice received her PhD in 1974 under the direction of Donald Sarason. After Berkeley, Alice had positions at SUNY-Buffalo (1974-75), UCLA (1975-77), and Maryland (1978-1980), before returning to UCLA. She was on the faculty of UCLA from 1980 to 1998.

Early in her career, Alice worked in the field of classical harmonic analysis, operator theory, and algebras of functions of one complex variable. One of her earliest accomplishments was her work on the Douglas Problem, which was introduced to her by Sarason, who did pioneering work on the conjecture. Alice's *Acta* paper, "A characterization of Douglas subalgebras" [9]), when combined with the results of Donald Marshall, completely resolved the conjecture. Over the next decade she continued making important contributions to the field, including two papers that appeared in the *Annals of Mathematics*: "Carleson measure on the bi-disc" [10] and "On a continuous version of duality of H^1 with BMO on the bi-disc" [11]), joint with R. Fefferman. In the latter paper, Chang and Fefferman studied the question of extending H^1-BMO duality to functions on the poly-disc. In particular, they have a much simpler and more natrual description of the dual of H^1 than any appearing in the literature at that time. Alice's work over this period was recognized with an Alfred P. Sloan Fellowship in 1979-1980.

It was Alice's paper with L. Carleson ("On the existence of an extremal function for